CHAPTER 7

Think & Discuss (p. 393)

1. The image in box A is flipped to get the image in box B. The image in box C is turned to get the image in box D.
2. *Sample answer:* If you look at the picture as a whole, the right half is the image of the left half flipped over the center vertical red line.

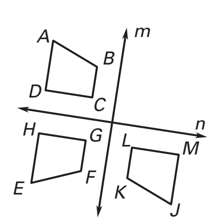
Skill Review (p.394)

1. $AB = \sqrt{(1-(-6))^2 + (3-4)^2}$
$= \sqrt{7^2 + (-1)^2}$
$= \sqrt{49 + 1}$
$= \sqrt{50}$
$= \sqrt{25} \cdot \sqrt{2}$
$= 5\sqrt{2}$
$BC = \sqrt{(8-1)^2 + (4-3)^2}$
$= \sqrt{7^2 + 1^2}$
$= \sqrt{49 + 1}$
$= \sqrt{50}$
$= \sqrt{25}\sqrt{2}$
$= 5\sqrt{2}$
$\overline{AB} \cong \overline{BC}$
2. $AB = \sqrt{(3-0)^2 + (1-3)^2}$
$= \sqrt{3^2 + (-2)^2}$
$= \sqrt{9 + 4}$
$= \sqrt{13}$
$BC = \sqrt{(7-3)^2 + (4-1)^2}$
$= \sqrt{4^2 + 3^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$
$\overline{AB}$ and $\overline{BC}$ are not congruent.
3. $AB = \sqrt{(4-1)^2 + (6-1)^2}$
$= \sqrt{3^2 + 5^2}$
$= \sqrt{9 + 25}$
$= 34$
$BC = \sqrt{(7-4)^2 + (1-6)^2}$
$= \sqrt{3^2 + (-5)^2}$
$= \sqrt{9 + 25}$
$= \sqrt{34}$
$\overline{AB} \cong \overline{BC}$
4. $XZ = 10$ **5.** $m\angle X = 35°$ **6.** $m\angle Q = 55°$
7. $m\angle Z + m\angle Y + m\angle X = 180°$
$m\angle Z + 55° + 35° = 180°$
$m\angle Z + 90° = 180°$
$m\angle Z = 90°$
8. $\overline{YZ} \cong \overline{QR}$
9. $(QR)^2 + (RP)^2 = (PQ)^2$
$(QR)^2 + 10^2 = 12.2^2$
$(QR)^2 + 100 = 148.84$
$(QR)^2 = 48.84$
$QR \approx 7.0$

Lesson 7.1

Developing Concepts Activity 7.1 (p. 395)

Exploring the Concept

1. **a.** $\overline{FH}$ corresponds to $\overline{KL}$.
$\overline{FG}$ corresponds to $\overline{KJ}$.
$\overline{GH}$ corresponds to $\overline{JL}$.
b. $\overline{AB}$ corresponds to $\overline{JK}$.
$\overline{BC}$ corresponds to $\overline{KL}$.
$\overline{CD}$ corresponds to $\overline{LM}$.
$\overline{DE}$ corresponds to $\overline{MN}$.
$\overline{AE}$ corresponds to $\overline{JN}$.
c. $\overline{WX}$ corresponds to $\overline{MN}$.
$\overline{XY}$ corresponds to $\overline{NP}$.
$\overline{YZ}$ corresponds to $\overline{PQ}$.
$\overline{WZ}$ corresponds to $\overline{MQ}$.
d. $\overline{NP}$ corresponds to $\overline{TU}$.
$\overline{PQ}$ corresponds to $\overline{UV}$.
$\overline{QR}$ corresponds to $\overline{VW}$.
$\overline{RS}$ corresponds to $\overline{WX}$.
$\overline{NS}$ corresponds to $\overline{TX}$.
2. **a.** Turn $\triangle FGH$ to get $\triangle LKJ$.
b. Flip figure $ABCDE$ to get figure $JKLMN$.
c. Slide figure $WXYZ$ to get figure $MNPQ$.
d. Turn figure $NPQRS$ to get figure $TUVWX$.
3. The three types of motion that preserve the congruence of a figure when it is moved in the plane are flip, slide, and turn.
4. Figure $ABCD$ is flipped over line n to get figure $EFGH$.
5. 

Yes, $EFGH$ is congruent to $JKLM$.

6. *ABCD* is turned to get *JKLM*.

Yes, *ABCD* is congruent to *JKLM*.

7. No; there is no line over which *ABCD* can be flipped to give *JKLM*.

7.1 Guided Practice (p. 399)

1. An operation that maps a preimage onto an image is called a transformation.

2. The preimage and image of a transformation are sometimes congruent.

3. A transformation that is an isometry always preserves length.

4. An isometry never maps an acute triangle onto an obtuse triangle.

5. translation **6.** reflection **7.** rotation **8.** $\overline{ST}$ **9.** $\overline{VW}$

10. *Sample answer:* $\angle QRS$ and $\angle VWX$ **11.** $\triangle WXY$

7.1 Practice and Applications (pp. 399–402)

12. Figure $ABCDE \to$ Figure $JKLMN$.

13. This transformation is a rotation about the origin. The figure *ABCDE* is turned about the origin.

14. *Sample answer:* $\overline{AE}$ and $\overline{JN}$ **15.** *Sample answer:* $\angle L$ and $\angle C$

16. (2, 4).

17. *Sample answer:*

$\overline{AB}$ corresponds to $\overline{JK}$.

$$AB = \sqrt{(2-1)^2 + (3-1)^2} = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

$$JK = \sqrt{(-3-(-1))^2 + (2-1)^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$$

So, $AB = JK$.

18. true **19.** false **20.** true

21. reflection in the line $x = 1$; flip over the line $x = 1$, $B'(3, 4)$, $C'(3, -1)$, and $D'(6, -1)$.

22. translation, slide 6 units to the right; $L'(2, -2)$, $N'(5, -2)$, and $M'(3, 4)$

23. Yes; the preimage and image appear to be congruent.

24. Yes; the preimage and image appear to be congruent.

25. No; the preimage and image are not congruent.

26. $\triangle ABC \to \triangle PQR$ **27.** $\triangle DEF \to \triangle LKJ$

28. $\triangle KJL \to \triangle EFD$ **29.** $\triangle PRQ \to \triangle ACB$

30. $\triangle LJK \to \triangle DFE$ **31.** $\triangle RQP \to \triangle CBA$

32.
$$FG = \sqrt{(-4-(-1))^2 + (2-1)^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$RS = \sqrt{(2-1)^2 + (2-(-1))^2} = \sqrt{1^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}$$

So, $FG = RS$.

$$GH = \sqrt{(-1-(-4))^2 + (4-2)^2} = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$ST = \sqrt{(4-2)^2 + (-1-2)^2} = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$$

So, $GH = ST$.

$$FH = \sqrt{(-1-(-1))^2 + (4-1)^2} = \sqrt{0^2 + 3^2} = \sqrt{0 + 9} = \sqrt{9} = 3$$

$$RT = \sqrt{(4-1)^2 + (-1-(-1))^2} = \sqrt{3^2 + 0^2} = \sqrt{9 + 0} = \sqrt{9} = 3$$

So, $FH = RT$.

33.
$$AB = \sqrt{(-5-(-2))^2 + (1-(-2))^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

—CONTINUED—

33. —CONTINUED—

$XY = \sqrt{(3-0)^2 + (1-(-2))^2}$
$= \sqrt{3^2 + 3^2}$
$= \sqrt{9+9}$
$= \sqrt{18}$
$= \sqrt{9}\sqrt{2}$
$= 3\sqrt{2}$

So, $AB = XY$.

$BC = \sqrt{(-2-(-5))^2 + (2-1)^2}$
$= \sqrt{3^2 + 1^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

$YZ = \sqrt{(0-3)^2 + (2-1)^2}$
$= \sqrt{(-3)^2 + 1^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

So, $BC = YZ$.

$AC = \sqrt{(-2-(-2))^2 + (2-(-2))^2}$
$= \sqrt{0^2 + 4^2}$
$= \sqrt{0+16}$
$= \sqrt{16}$
$= 4$

$XZ = \sqrt{(0-0)^2 + (2-(-2))^2}$
$= \sqrt{0^2 + 4^2}$
$= \sqrt{0+16}$
$= \sqrt{16}$
$= 4$

So, $AC = XZ$.

34. $2a° = 96°$
$a = 48$
$b = 92$
$c = 7$
$3d = 6$
$d = 2$

35. $2y = 6$
$y = 3$
$3x + 1 = 14$
$3x = 13$
$x = \frac{13}{3} = 4\frac{1}{3}$
$2w° = 70°$
$w = 35$

36. translation **37.** translation **38.** reflection

39. rotation

40. Yes, a point or a line segment can be its own preimage when it is rotated or when it is reflected. Points or line segments on a line of reflection are their own preimages. A center of rotation is its own preimage.

41. From *A* to *B*, the stencil is reflected. From *A* to *C*, the stencil is reflected. From *A* to *D*, the stencil is either rotated or reflected twice.

42. The letters *b, d, p,* and *q* can be formed from each other by reflection or rotation; the letters *n* and *u* can be formed from each other by rotation or repeated reflection.

43. *Sample answer:* The lower right corner is the horizontal reflection of the upper right corner. Then reflect the lower right corner vertically to get the pattern for the lower left corner. From there, reflect horizontally to get the upper left corner.

44. B **45.** D

46.

Statements	Reasons
1. $\triangle ABC \to \triangle PQR$ and $\triangle PQR \to \triangle XYZ$ are isometries.	**1.** Given
2. $AB = PQ$, $BC = QR$, $AC = PR$, $PQ = XY$, $QR = YZ$, and $PR = XZ$	**2.** Definition of isometry
3. $AB = XY$, $BC = YZ$, $AC = XZ$	**3.** Transitive property of equality
4. $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$	**4.** Definition of congruent segments
5. $\triangle ABC \to \triangle XYZ$ is an isometry.	**5.** Definition of isometry

7.1 Mixed Review (p. 402)

47. $AB = \sqrt{(3-(-2)^2 + (10-(-2))^2}$
$= \sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

48. $CD = \sqrt{(5-(-11))^2 + (-7-6)^2}$
$= \sqrt{16^2 + (-13)^2}$
$= \sqrt{256 + 169}$
$= \sqrt{425}$
$= \sqrt{25} \cdot \sqrt{17}$
$= 5\sqrt{17}$

49. $EF = \sqrt{(0-(-8))^2 + (8-3)^2}$
$= \sqrt{8^2 + 5^2}$
$= \sqrt{64 + 25}$
$= \sqrt{89}$

50. $GH = \sqrt{(0-6)^2 + (-7-3)^2}$
$= \sqrt{(-6)^2 + (-10)^2}$
$= \sqrt{36 + 100}$
$= \sqrt{136}$
$= \sqrt{4} \cdot \sqrt{34}$
$= 2\sqrt{34}$

51. polygon **52.** polygon

53. Not a polygon because one side is not a line segment.

54. Not a polygon because one side is not a line segment.

55. Not a polygon because two of the sides intersect only one other side.

56. polygon

57. *Sample answers:*

(1) slope of $\overline{PQ} = \frac{6-4}{7-0} = \frac{2}{7}$

slope of $\overline{RS} = \frac{-4-(-2)}{1-8} = \frac{-2}{-7} = \frac{2}{7}$

slope of $\overline{QR} = \frac{6-(-2)}{7-8} = \frac{8}{-1} = -8$

slope of $\overline{PS} = \frac{4-(-4)}{0-1} = \frac{8}{-1} = -8$

$\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$

So, *PQRS* is a parallelogram because both pairs of opposite sides are parallel.

(2) $PQ = \sqrt{(0-7)^2 + (4-6)^2}$
$= \sqrt{(-7)^2 + (-2)^2}$
$= \sqrt{49 + 4}$
$= \sqrt{53}$

$RS = \sqrt{(8-1)^2 + (-2-(-4))^2}$
$= \sqrt{7^2 + 2^2}$
$= \sqrt{49 + 4}$
$= \sqrt{53}$

$QR = \sqrt{(7-8)^2 + (6-(-2))^2}$
$= \sqrt{(-1)^2 + 8^2}$
$= \sqrt{1 + 64}$
$= \sqrt{65}$

$PS = \sqrt{(0-1)^2 + (4-(-4))^2}$
$= \sqrt{(-1)^2 + 8^2}$
$= \sqrt{1 + 64}$
$= \sqrt{65}$

$PQ = RS$ and $QR = PS$

PQRS is a parallelogram because opposite sides are congruent.

58. *Sample answers:*

(1) slope of $\overline{WX} = \frac{5-5}{9-1} = \frac{0}{8} = 0$

slope of $\overline{YZ} = \frac{-1-(-1)}{6-(-2)} = \frac{0}{8} = 0$

slope of $\overline{XY} = \frac{5-(-1)}{9-6} = \frac{6}{3} = 2$

slope of $\overline{WZ} = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = 2$

$\overline{WX} \parallel \overline{YZ}$ and $\overline{XY} \parallel \overline{WZ}$

WXYZ is a parallelogram because opposite sides are parallel.

(2) $WX = \sqrt{(9-1)^2 + (5-5)^2}$
$= \sqrt{8^2 + 0^2}$
$= \sqrt{64 + 0}$
$= \sqrt{64}$
$= 8$

$YZ = \sqrt{(6-(-2))^2 + (-1-(-1))^2}$
$= \sqrt{8^2 + 0^2}$
$= \sqrt{64 + 0}$
$= \sqrt{64}$
$= 8$

$XY = \sqrt{(9-6)^2 + (5-(-1))^2}$
$= \sqrt{3^2 + 6^2}$
$= \sqrt{9 + 36}$
$= \sqrt{45}$
$= \sqrt{9} \cdot \sqrt{5}$
$= 3\sqrt{5}$

$WZ = \sqrt{(1-(-2))^2 + (5-(-1))^2}$
$= \sqrt{3^2 + 6^2}$
$= \sqrt{9 + 36}$
$= \sqrt{45}$
$= \sqrt{9} \cdot \sqrt{5}$
$= 3\sqrt{5}$

$WX = YZ$ and $XY = WZ$

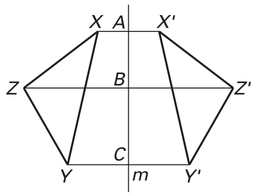
WXYZ is a parallelogram because opposite sides are congruent.

Lesson 7.2

Developing Concepts Activity 7.2 (p. 403)

Exploring the Concept

Sample answer:

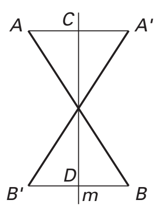


Investigate

1. Measurements will vary, but $\overline{XA} \cong \overline{AX'}$, $\overline{ZB} \cong \overline{BZ'}$, and $\overline{YC} \cong \overline{CY'}$.
2. The measure of each angle is 90°.
3. The line m is the perpendicular bisector of each segment.

Exploring the Concept

Sample answer:



Make a Conjecture

4. Line m is the perpendicular bisector of each segment; *Sample answer:* $m\angle ACD = m\angle BDC = 90°$ and $\overline{AC} \cong \overline{A'C}$ and $\overline{BD} \cong \overline{B'D}$.
5. The line of reflection is the perpendicular bisector of the segment connecting a point and its image.

7.2 Guided Practice (p. 407)

1. A line of symmetry is a line in which a figure can be reflected onto itself.
2. When a point is reflected in the x-axis, the x-coordinates of the point and its image are the same and the y-coordinates are opposites.
3. not a reflection **4.** not a reflection **5.** reflection

6. $\overline{AB} \to \overline{EF}$ **7.** $\angle DAB \to \angle DEF$

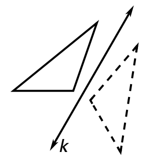
8. $C \to G$ **9.** $D \to D$ **10.** $\angle CBA \to \angle GFE$

11. $\overline{DC} \to \overline{DG}$ **12.** 3 lines of symmetry

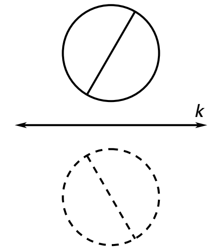
13. 4 lines of symmetry **14.** 5 lines of symmetry

7.2 Practice and Applications (pp. 407–410)

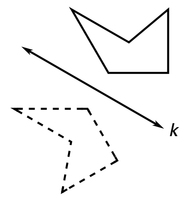
15.



16.



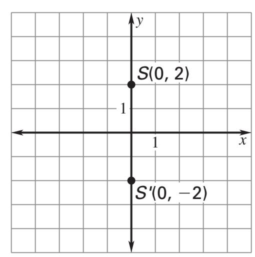
17.



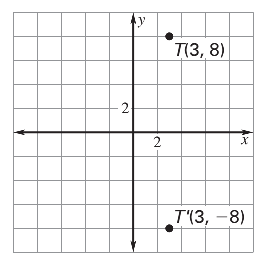
18. True; N is 2 units above the line $y = 2$, so its image is 2 units below the line.
19. True; M is 3 units to the right of the line $x = 3$, so its image is 3 units to the left of the line.
20. False; W is one unit below the line $y = -2$, so its image is one unit above the line. Its image should be $W'(-6, -1)$.
21. True; U is 4 units to the right of the line $x = 1$, so its image is 4 units to the left of the line.

22. $\overline{GH}$ **23.** $\overline{CD}$ **24.** $\overline{FE}$ **25.** $\overline{EF}$

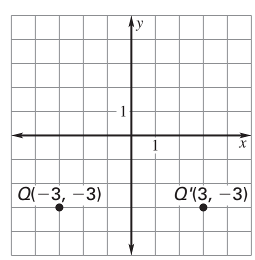
26. $S'(0, -2)$



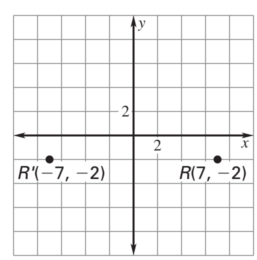
27. $T'(3, -8)$



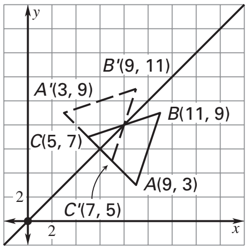
28. $Q'(3, -3)$



29. $R'(-7, -2)$

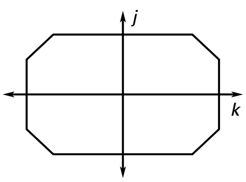


30. *Sample answer:*

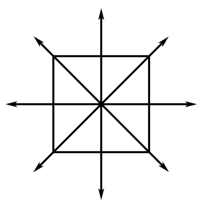


The coordinates of the vertices of the image of (x, y) are reversals of the coordinates of the vertices of the preimage, (y, x).

31.



32.



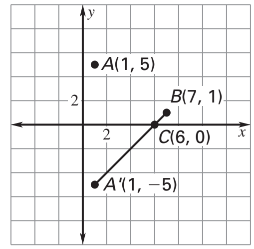
33. Draw $\overline{PP'}$ and $\overline{QQ'}$ intersecting line m at points S and T. $\overline{PQ}$ (and $\overline{P'Q'}$) intersects line m at R. By the definition of reflection, $\overline{P'S} \cong \overline{PS}$ and $\overline{RS} \perp \overline{PP'}$ and $\overline{Q'T} \cong \overline{QT}$ and $\overline{RT} \perp \overline{QQ'}$. $\overline{RT} \cong \overline{RT}$ and $\overline{SR} \cong \overline{SR}$ by the Reflexive Property of Congruence. Angles $P'SR$, RSP, RTQ' and RTQ are right angles (definition of perpendicular) and are congruent (all right angles are congruent). It follows that $\triangle P'SR \cong \triangle PSR$ and $\triangle Q'TR \cong \triangle QTR$ by the SAS Congruence Postulate. Since corresponding parts of $\cong$ ⚠ are $\cong$, $\overline{P'R} \cong \overline{PR}$ and $\overline{Q'R} \cong \overline{QR}$. So $P'R = PR$ and $Q'R = QR$. Since $P'Q' = P'R + Q'R$ and $PQ = PR + QR$ by the Segment Addition Postulate, we get by substitution $PQ = P'Q'$, or $\overline{PQ} \cong \overline{P'Q'}$.

34. Since P is on m, then $P = P'$ by definition of a reflection. By definition of reflection, $m \perp \overline{QQ'}$ and $\overline{QR} \cong \overline{Q'R}$ where R is the point where m and $\overline{QQ'}$ intersect. Since $m \perp \overline{QQ'}$, $\angle PRQ$ and $\angle PRQ'$ are right angles. $\angle PRQ \cong \angle PRQ'$ because all right angles are congruent. $\overline{PR} \cong \overline{PR}$ by the Reflexive Property of Congruence. So $\triangle PRQ \cong \triangle PRQ'$ by SAS Congruence Postulate. Therefore $\overline{PQ} \cong \overline{PQ'}$ or $\overline{PQ} \cong \overline{P'Q'}$ because corresponding parts of congruent triangles are congruent. Finally, $PQ = P'Q'$ by definition of congruent segments.

35. By definition of a reflection, m is the perpendicular bisector of $\overline{PP'}$, Q is on m, and $Q = Q'$. Then $PQ = P'Q$ by the definition of reflection. But $Q = Q'$, so $PQ = P'Q'$.

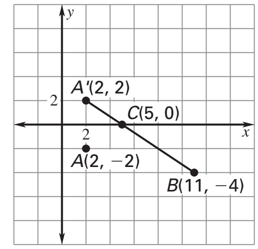
36. Reflect H in line n to obtain its image H'. Then draw a line $\overleftrightarrow{H'J}$. This will intersect n in a point K. Then the distance traveled, $HK + KJ$, will be as small as possible.

37.



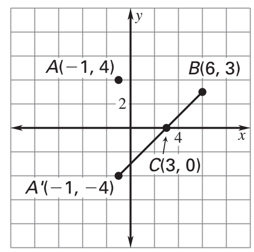
(6, 0)

38.



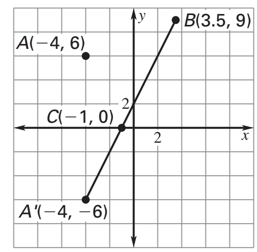
(5, 0)

39.



(3, 0)

40.



(−1, 0)

41. The two molecules are reflections of each other.

42. Triangle 2 is a reflection of triangle 1; triangle 3 is a translation of triangle 1.

43. Triangles 2 and 3 are reflections of triangle 1. Triangle 4 is a rotation of triangle 1.

44. $n(m\angle A) = 180°$

$4(m\angle A) = 180°$

$m\angle A = 45°$

45. $n(m\angle A) = 180°$

$2(m\angle A) = 180°$

$m\angle A = 90°$

46. $n(m\angle A) = 180°$

$3(m\angle A) = 180°$

$m\angle A = 60°$

47. Drawings will vary. The distance between each vertex of the preimage and line m is equal to the distance between the corresponding vertex of the image and line m.

48. $3x = 4$

$x = \frac{4}{3}$

$\frac{1}{2}y - 10 = 8$

$\frac{1}{2}y = 18$

$y = 36$

$2z - 1 = 5$

$2z = 6$

$z = 3$

49. $2u + 1 = 13$

$2u = 12$

$u = 6$

$5v - 10 = 19$

$5v = 29$

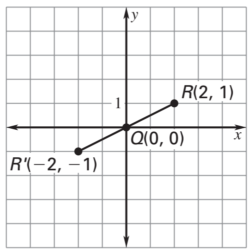
$v = \frac{29}{5}$

$3w = 15$

$w = 5$

50. B **51.** B

52.



53. $Q = \left(\frac{-2 + 2}{2}, \frac{-1 + 1}{2}\right)$

$Q = \left(\frac{0}{2}, \frac{0}{2}\right)$

$Q = (0, 0)$

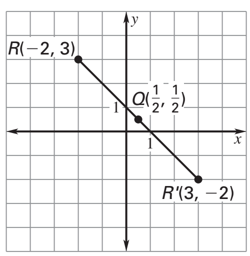
54. slope of $\overline{RR'} = \frac{1 - (-1)}{2 - (-2)} = \frac{2}{4} = \frac{1}{2}$

The slope of $\overline{RR'}$ is $\frac{1}{2}$. So the slope of the line perpendicular to $\overline{RR'}$ is -2 because $-2 \cdot \frac{1}{2} = -1$.

55. $y - 0 = -2(x - 0)$

$y = -2x$

56. (52)



(53) $Q = \left(\frac{3 + (-2)}{2}, \frac{-2 + 3}{2}\right)$

$Q = \left(\frac{1}{2}, \frac{1}{2}\right)$

(54) slope of $\overline{RR'} = \frac{3 - (-2)}{-2 - 3} = \frac{5}{-5} = -1$

The slope of $\overline{RR'}$ is -1 so a line perpendicular to $\overline{RR'}$ has slope 1 because $-1 \cdot 1 = -1$.

(55) $y - \frac{1}{2} = 1\left(x - \frac{1}{2}\right)$

$y - \frac{1}{2} = x - \frac{1}{2}$

$y = x$

7.2 Mixed Review (p. 410)

57. $\angle A \cong \angle P$ **58.** $PQ = AB = 12$ **59.** $\overline{QR} \cong \overline{BC}$

60. $m\angle C = m\angle R = 35°$ **61.** $m\angle Q = m\angle B = 101°$

62. $\angle R \cong \angle C$

63. $b - a < c < a + b$

$17 - 7 < c < 7 + 17$

$10 < c < 24$

64. $b - a < c < a + b$

$21 - 9 < c < 9 + 21$

$12 < c < 30$

65. $b - a < c < a + b$

$33 - 12 < c < 12 + 33$

$21 < c < 45$

66. $a - b < c < a + b$

$26 - 6 < c < 26 + 6$

$20 < c < 32$

67. $a - b < c < a + b$

$41.2 - 15.5 < c < 41.2 + 15.5$

$25.7 < c < 56.7$

68. $b - a < c < a + b$

$11.9 - 7.1 < c < 7.1 + 11.9$

$4.8 < c < 19$

69. $m\angle C = m\angle D = 61°$

$m\angle A = m\angle B = 180° - m\angle D$

$= 180° - 61°$

$= 119°$

70. $m\angle D = 90°$

$m\angle A = 180° - m\angle B$

$= 180° - 115° = 65°$

71. $m\angle C + m\angle B = 180°$ $m\angle A + m\angle D = 180°$

$m\angle C + 119° = 180°$ $m\angle A + 74 = 180$

$m\angle C = 61°$ $m\angle A = 106°$

Lesson 7.3

Technology Activity 7.3 (p. 411)

1. $\triangle ABC \cong \triangle A''B''C''$.
2. a rotation about the point of intersection of the lines
3. Answers will vary.
4. The measure of the acute angle is half the measure of $\angle APA'$.
5. $m\angle BPB'' = m\angle CPC'' =$ twice the measure of the acute angle formed by lines m and k
6. The measure of the angle of rotation is twice the measure of the acute angle formed by the two lines.

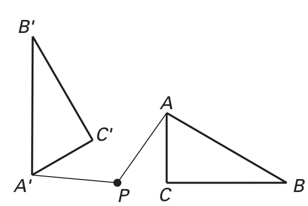
Extension

The conjecture is correct.

Activity 7.3 (p. 413)

Answers may vary.

Sample answer:



7.3 Guided Practice (p. 416)

1. A center of rotation is the fixed point about which a figure being rotated is turned.

2. counterclockwise

3. Yes, $AB = A'B'$ because a rotation is an isometry.

4. No, $AA' \neq BB'$ because the distance between any point and its image after a rotation is not fixed.

5. The measure of the acute angle between k and m would be half the measure of the angle of rotation. So it would be $\frac{1}{2} \cdot (90°) = 45°$.

6. A clockwise rotation of 60° about P maps R onto S.

7. A counterclockwise rotation of 60° about P maps R onto Q.

8. A clockwise rotation of 120° about Q maps R onto W.

9. A counterclockwise rotation of 180° about P maps V onto R.

10. The figure has rotational symmetry about its center with a rotation of 180°, either clockwise or counterclockwise.

11. The figure has rotational symmetry about its center with a rotation of 180°, either clockwise or counterclockwise.

12. The figure does not have rotational symmetry.

7.3 Practice and Applications (pp. 416–419)

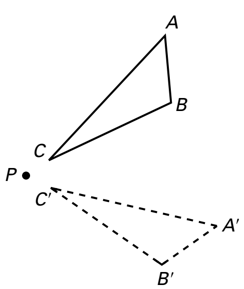
13. $\overline{CD}$ **14.** $\overline{LH}$ **15.** $\overline{GE}$ **16.** $\overline{BM}$ **17.** $\triangle MAB$

18. $\triangle FGL$ **19.** $\triangle CPA$

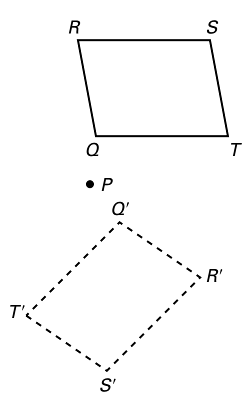
20. By definition of a rotation, $\overline{PR} \cong \overline{PR'}$ and $\overline{PQ} \cong \overline{PQ'}$. By the definition of congruent segments $PR = PR'$ and $PQ = PQ'$. By the Segment Addition Postulate, $PR + RQ = PR' + R'Q'$, so $RQ = R'Q'$ by the subtraction property of equality. Finally $\overline{RQ} \cong \overline{R'Q'}$ by definition of congruent segments.

21. By definition of rotation $\overline{QP} \cong \overline{Q'P}$. Since P and R are the same point and R and R' are the same point, then $\overline{QR} \cong \overline{Q'R'}$.

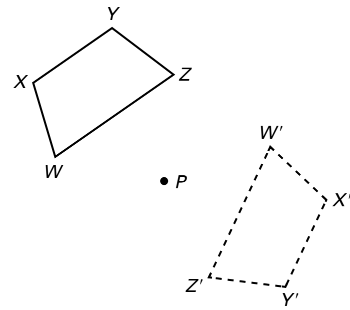
22.



23.



24.



25. $J'(1, 2), K'(4, 1), L'(4, -3), M'(1, -3)$

26. $P'(-1, -3), Q'(-3, -5), R'(-4, -2), S'(-2, 0)$.

27. $D'(4, 1), E'(0, 2), F'(2, 5)$.

28. $A'(1, 1), B'(4, -2), C'(2, -5)$; the x-coordinate of the image is the y-coordinate of the preimage. The y-coordinate of the image is the opposite of the x-coordinate of the preimage.

29. $O'(0, 0)$, $X'(2, 3)$, and $Z'(-3, 4)$; the x-coordinate of the image is the opposite of the x-coordinate of the preimage. The y-coordinate of the image is the opposite of the y-coordinate of the preimage.

30. The measure of the angle of rotation from $\triangle ABC$ to $\triangle A''B''C''$ is twice the measure of the acute angle of the intersecting lines, which is $2 \cdot 35°$ or 70°.

31. The measure of the angle of rotation from $\triangle ABC$ to $\triangle A''B''C''$ is twice the measure of the acute angle of the intersecting lines, which is $2 \cdot 15°$ or 30°.

32. The measure of the angle of rotation about D is $2 \cdot 36°$ or 72°.

33. The measure of the acute angle between lines m and n is $\frac{1}{2} \cdot 162°$ or 81°.

34.
$$4e - 2 = 5$$
$$4e = 7$$
$$e = \frac{7}{4}$$
$$3b = 12$$
$$b = 4$$
$$d + 2 = 10$$
$$d = 8$$
$$\frac{c}{2} = 7$$
$$c = 14$$
$$2a° = 110°$$
$$a = 55$$

35.
$$2q° = 60°$$
$$q = 30$$
$$3t = 3$$
$$t = 1$$
$$2r = 10$$
$$r = 5$$
$$2u = 4$$
$$u = 2$$
$$s = 11$$

36. The wheel hub can be mapped onto itself by a clockwise or counterclockwise rotation of 45°, 90°, 135°, or 180° about its center.

37. The wheel hub can be mapped onto itself by a clockwise or counterclockwise rotation of $51\frac{3}{7}°$, $102\frac{6}{7}°$, or $154\frac{2}{7}°$ (which is $360° \cdot \frac{1}{7}$, $360° \cdot \frac{2}{7}$, and $360° \cdot \frac{3}{7}$, respectively) about its center.

38. The wheel hub can be mapped onto itself by a clockwise or counterclockwise rotation of 72° and 144° (which is $360 \cdot \frac{1}{5}$ and $360 \cdot \frac{2}{5}$, respectively) about its center.

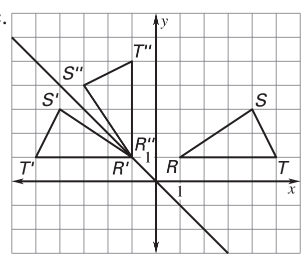
39. Yes, the image can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about its center.

40. Yes; the answer would change to a clockwise or counterclockwise rotation of 90° or 180° about its center. This is because the white figures can be mapped onto the black figures.

41. The center of rotation is the point of intersection of the diagonals of the square.

42. Yes, it is possible for the piece to be hung upside down because the rotational symmetry has an angle of rotation of 180°. This would make the picture the same right side up and upside down.

43. a. Graph for a–c.



b. $R'(-1, 1)$, $S'(-4, 3)$, $T'(-5, 1)$

c. $R''(-1, 1)$, $S''(-3, 4)$, $T''(-1, 5)$

d. A single transformation that maps $\triangle RST$ onto $\triangle R''S''T''$ would be a counterclockwise rotation of 90° about the origin.

e. Any polygon can be rotated 90° counterclockwise about the origin by doing two reflections of the polygon. First, reflect the polygon in one of the axis. Then reflect the result of the first reflection in the line $y = -x$ or $y = x$. Then the measure of the acute angle between the two lines is 45° and the angle of rotation is 90°.

44. a. By definition of a reflection, $k \perp \overline{QQ'}$ and $\overline{QA} \cong \overline{Q'A}$ where A is the point of intersection of k and $\overline{QQ'}$. $\angle QAP$ and $\angle Q'AP$ are right angles, and $\angle QAP \cong \angle Q'AP$ because all right angles are congruent. By the Reflexive Property of Congruence, $\overline{AP} \cong \overline{AP}$. So, $\triangle QAP \cong \triangle Q'AP$ by the SAS Congruence Postulate. By corresponding parts of congruent triangles are congruent, $\overline{PQ} \cong \overline{PQ'}$. By definition of a reflection, $m \perp \overline{Q'Q''}$ and $\overline{Q'B} \cong \overline{Q''B}$ where B is the point of intersection of m and $\overline{Q'Q''}$. $\angle Q'BP$ and $\angle Q''BP$ are right angles, and $\angle Q'BP \cong \angle Q''BP$ since all right angles are congruent. $\overline{BP} \cong \overline{BP}$ by the Reflexive Property of Congruence. So $\triangle Q'BP \cong \triangle Q''BP$ by the SAS Congruence Postulate. Since corresponding parts of congruent triangles are congruent, $\overline{PQ'} \cong \overline{PQ''}$. Since $\overline{QP} \cong \overline{Q'P}$ then $\overline{QP} \cong \overline{Q''P}$ by the Transitive Property of Congruence. Q'' is a rotation of Q about point P.

b. $\angle QPA \cong \angle Q'PA$ and $\angle Q'PB \cong \angle Q''PB$ because corresponding parts of $\cong$ ⨺ are $\cong$. By the definition of congruent angles, $m\angle Q'PB = m\angle Q''PB$ and $m\angle QPA = m\angle Q'PA$. By the Angle Addition Postulate, $m\angle APB = m\angle Q'PA + m\angle Q'PB$, $m\angle QPQ'' = m\angle QPA + m\angle Q'PA + m\angle Q'PB + m\angle Q''PB$. Then, $m\angle QPQ'' = m\angle Q'PA + m\angle Q'PA + m\angle Q'PB + m\angle Q'PB$ by substitution. By the Distributive property, $m\angle QPQ'' = 2(m\angle Q'PA + m\angle Q'PB)$. Finally by substitution, $m\angle QPQ'' = 2(m\angle APB)$.

7.3 Mixed Review (p. 419)

45. $m\angle 5 = m\angle 1 = 82°$

46. $m\angle 7 = m\angle 5 = 82°$

47. $m\angle 3 = m\angle 1 = 82°$

48. $m\angle 6 + m\angle 5 = 180°$

$m\angle 6 + 82° = 180°$

$m\angle 6 = 98°$

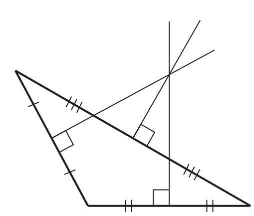
49. $m\angle 4 + m\angle 1 = 180°$

$m\angle 4 + 82° = 180°$

$m\angle 4 = 98°$

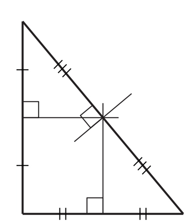
50. $m\angle 8 = m\angle 6 = 98°$

51. *Sample answer:*



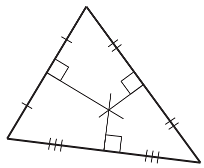
The circumcenter is outside the triangle when the triangle is obtuse.

52. *Sample answer:*



The circumcenter of a right triangle is always on the triangle.

53. *Sample answer:*



The circumcenter of an acute triangle is always inside the triangle.

54. Only one pair of sides are given as parallel, which is not enough information to show that the figure is a parallelogram.

Quiz 1 (p. 420)

1. Figure $ABCD \to$ Figure $RSTQ$.

2. The transformation is a reflection in line m. The figure is flipped over line m.

3. Yes, the reflection is an isometry because it preserves length.

4. $L'(2, -3)$ **5.** $M'(2, -4)$ **6.** $N'(-4, 0)$

7. $P'(-8.2, -3)$

8. The rotations that map the knot onto itself are rotations by multiples of 120° (360° ÷ 3) clockwise or counterclockwise about the center of the knot where the rope starts to unravel.

Math & History (p. 420)

1. The design has 2 lines of symmetry.

2. The design has rotational symmetry. It can be mapped onto itself by a rotation of 180° clockwise or counterclockwise about its center.

Lesson 7.4

7.4 Guided Practice (p. 425)

1. A *vector* is a quantity that has both direction and magnitude.

2. *Sample answer:* The direction is incorrect. $\overrightarrow{PQ}$ starts at P and ends at Q. So the vector from P to Q is $\langle 6, -2 \rangle$.

3. $(x, y) \to (x + 6, y - 2)$. **4.** $(x, y) \to (x + 4, y + 3)$.

5. $(x, y) \to (x - 7, y + 1)$. **6.** $(x, y) \to (x - 5, y - 8)$.

7. If (0, 2) maps onto (0, 0), then (8, 5) maps (8, 3).

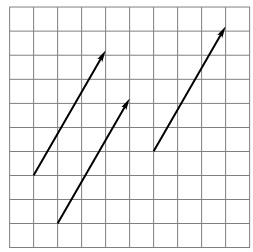
8. If (0, 2) maps onto $(-5, 4)$, then (8, 5) maps onto (3, 7).

9. If (0, 2) maps onto $(-3, -5)$, then (8, 5) maps onto $(5, -2)$.

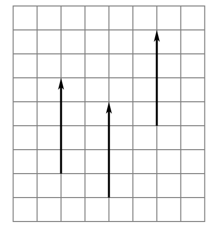
10. If (0, 2) maps onto $(-8, -3)$, then (8, 5) maps onto (0, 0).

11.–14. Sample figures are given.

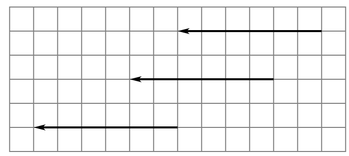
11.



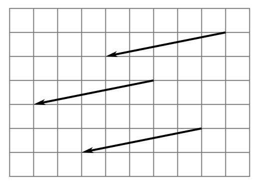
12.



13.



14.



7.4 Practice and Applications (pp.425–428)

15. a. $(x, y) \to (x - 3, y - 4)$ **b.** $\langle -3, -4 \rangle$

16. a. $(x, y) \to (x - 5, y + 2)$ **b.** $\langle -5, 2 \rangle$

17. $\overrightarrow{HJ}$; $\langle 4, 2 \rangle$ **18.** $\overrightarrow{LK}$; $\langle -4, -4 \rangle$ **19.** $\overrightarrow{MN}$; $\langle 5, 0 \rangle$

20. $\triangle A''B''C''$ **21.** k and m

22. *Sample answer:* $\overline{AA''}$ and $\overline{CC''}$

23. $CC'' = 1.4 + 1.4 = 2.8$ inches

24. Yes, the distance from B' to m is the same as the distance from B'' to m because B' is reflected in m onto B'' and by the definition of reflection, the distances are equal.

25. The image of (5, 3) is $(5 + 12, 3 - 7)$ or $(17, -4)$.

26. The image of $(-1, -2)$ is $(-1 + 12, -2 - 7)$ or $(11, -9)$.

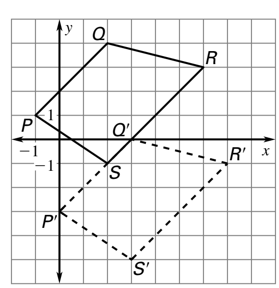
27. The preimage of $(-2, 1)$ is $(-2 - 12, 1 + 7)$ or $(-14, 8)$.

28. The preimage of $(0, -6)$ is $(0 - 12, -6 + 7)$ or $(-12, 1)$.

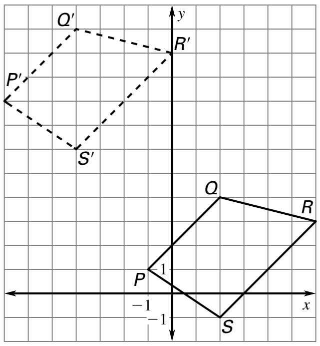
29. The image of (0.5, 2.5) is $(0.5 + 12, 2.5 - 7)$ or $(12.5, -4.5)$.

30. The preimage of $(-5.5, -5.5)$ is $(-5.5 - 12, -5.5 + 7)$ or $(-17.5, 1.5)$.

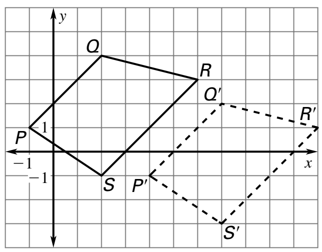
31.



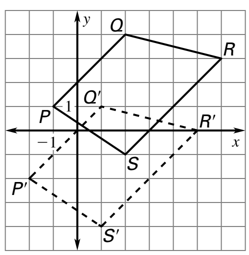
32.



33.

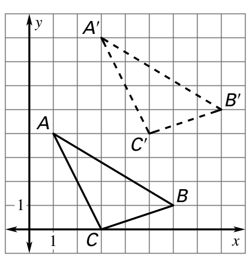


34.

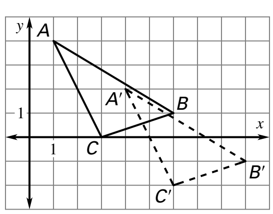


35. true **36.** false **37.** true. **38.** true

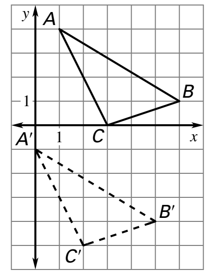
39.



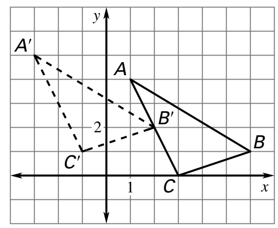
40.



41.



42.



43. We are given $P(a, b)$ and $Q(c, d)$. Suppose P' has coordinates $(a + r, b + s)$. Then

$PP' = \sqrt{(a + r - a)^2 + (b + s - b)^2}$ or $\sqrt{r^2 + s^2}$. The slope of $\overline{PP'} = \dfrac{b + s - b}{a + r - a}$ or $\dfrac{s}{r}$. If $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$ as given, then $QQ' = \sqrt{r^2 + s^2}$ and the slope of $\overline{QQ'}$ is $\dfrac{s}{r}$. So, the coordinates of Q' are $(c + r, d + s)$. By the Distance Formula, $PQ = \sqrt{(a - c)^2 + (b - d)^2}$

and

$P'Q' = \sqrt{[(a + r) - (c + r)]^2 + [(b + s) - (d + s)]^2}$ or $\sqrt{(a - c)^2 + (b - d)^2}$. Thus, by the substitution property of equality, $PQ = P'Q'$.

44. C **45.** D **46.** A **47.** B

48. yes **49.** no **50.** yes

51. Samples might include photographs of floor tiles or fabric patterns.

52. C: $(x, y) \to (x + 12, y)$, D: $(x, y) \to (x, y - 6)$, E: $(x, y) \to (x + 6, y - 6)$. F: $(x, y) \to (x + 12, y - 6)$

53. The two vectors are $\overrightarrow{AB}$ $\langle 6, 4\rangle$ and $\overrightarrow{BC}$ $\langle 4, 6\rangle$.

54. To arrive at D from C, the vector is $\overrightarrow{CD}$ $\langle 8, 2\rangle$.

55. To go straight from town A to town D, the vector would be $\overrightarrow{AD}$ $\langle 18, 12\rangle$.

56. The correct answer is C because a translation preserves length.

57. $AB = \sqrt{(-1-(-2))^2 + (2-6)^2}$
$= \sqrt{1^2 + (-4)^2}$
$= \sqrt{1 + 16}$
$= \sqrt{17}$
$AA' = \sqrt{(3-(-2))^2 + (3-6)^2}$
$= \sqrt{5^2 + (-3)^2}$
$= \sqrt{25 + 9}$
$= \sqrt{34}$
B

58. $BB' = \sqrt{(4-(-1))^2 + (-1-2)^2}$
$= \sqrt{5^2 + (-3)^2}$
$= \sqrt{25 + 9}$
$= \sqrt{34}$
$A'A'' = \sqrt{(8-3)^2 + (3-3)^2}$
$= \sqrt{5^2 + 0^2}$
$= \sqrt{25 + 0}$
$= \sqrt{25}$
$= 5$
A

59. $A'B'' = \sqrt{(9-3)^2 - (-1-3)^2}$
$= \sqrt{6^2 + (-4)^2}$
$= \sqrt{36 + 16}$
$= \sqrt{52}$
$= \sqrt{4} \cdot \sqrt{13}$
$= 2\sqrt{13}$
$A''B' = \sqrt{(4-8)^2 + (-1-3)^2}$
$= \sqrt{(-4)^2 + (-4)^2}$
$= \sqrt{16 + 16}$
$= \sqrt{32}$
$= \sqrt{16} \cdot \sqrt{2}$
$= 4\sqrt{2}$
A

60. $-1 + 4 = 2x + 1$
$3 = 2x + 1$
$2 = 2x$
$1 = x$
$w + 1 = 4$
$w = 3$
$8y - 1 + 4 = 3$
$8y + 3 = 3$
$8y = 0$
$y = 0$
$1 + 1 = 3z$
$2 = 3z$
$\frac{2}{3} = z$

61. $r - 1 + 3 = 3$
$r + 2 = 3$
$r = 1$
$8 - 6 = s + 1$
$2 = s + 1$
$1 = s$
$2t - 2 + 3 = 5$
$2t + 1 = 5$
$2t = 4$
$t = 2$
$u - 6 = -2u$
$-6 = -3u$
$2 = u$

7.4 Mixed Review (p. 428)

62. $m = \frac{-8-(-2)}{-7-0}$
$= \frac{-6}{-7}$
$= \frac{6}{7}$

63. $m = \frac{18-3}{-1-2}$
$= \frac{15}{-3}$
$= -5$

64. $m = \frac{1-1}{-1-(-10)}$
$= \frac{0}{9}$
$= 0$

65. $m = \frac{6-12}{-1-(-2)}$
$= \frac{-6}{1}$
$= -6$

66. $m = \frac{10-0}{0-(-6)}$
$= \frac{10}{6}$
$= \frac{5}{3}$

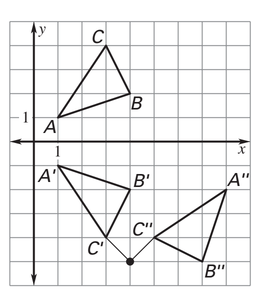
67. $m = \frac{6-(-3)}{9-(-3)}$
$= \frac{9}{12}$
$= \frac{3}{4}$

68. If $JK = 12$, then $SR = 6$. **69.** If $QR = 6$, then $JL = 12$.
70. If $RL = 6$, then $QS = 6$. **71.** true **72.** false **73.** false

Lesson 7.5

Developing Concepts Activity 7.5 (p. 429)

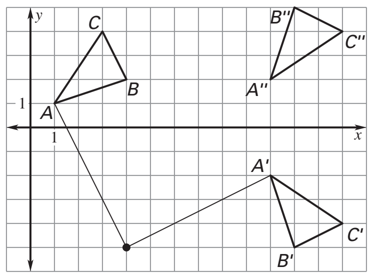
Exploring the Concept



Investigation

1. $A''(8, -2)$, $B''(7, -5)$, $C''(5, -4)$

2.

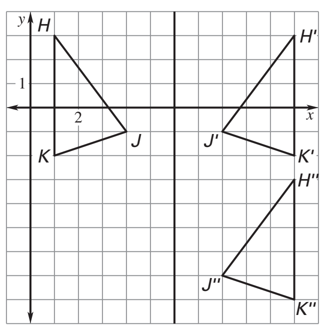


$A''(10, 2)$, $B''(11, 5)$, $C''(13, 4)$

3. Yes, the order in which transformations are completed affects the final image.

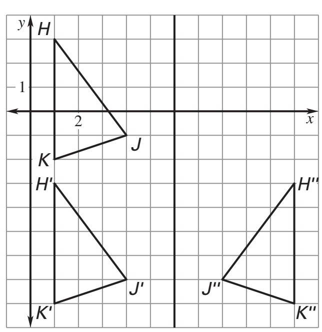
Investigate

4.



$H''(11, -3)$, $J''(8, -7)$, $K''(11, -8)$

5.



$H''(11, -3)$, $J''(8, -7)$, $K''(11, -8)$; the coordinates are the same; switching the order of the transformations did not affect the image in this example.

Extension

$AB = \sqrt{(4 - 1)^2 + (2 - 1)^2}$
$= \sqrt{3^2 + 1^2}$
$= \sqrt{9 + 1}$
$= \sqrt{10}$

$A''B'' = \sqrt{(11 - 10)^2 + (5 - 2)^2}$
$= \sqrt{1^2 + 3^2}$
$= \sqrt{1 + 9}$
$= \sqrt{10}$

So, $AB = A''B''$.

$AC = \sqrt{(3 - 1)^2 + (4 - 1)^2}$
$= \sqrt{2^2 + 3^2}$
$= \sqrt{4 + 9}$
$= \sqrt{13}$

$A''C'' = \sqrt{(13 - 10)^2 + (4 - 2)^2}$
$= \sqrt{3^2 + 2^2}$
$= \sqrt{9 + 4}$
$= \sqrt{13}$

So, $AC = A''C''$.

$BC = \sqrt{(3 - 4)^2 + (4 - 2)^2}$
$= \sqrt{(-1)^2 + 2^2}$
$= \sqrt{1 + 4}$
$= \sqrt{5}$

$B''C'' = \sqrt{(13 - 11)^2 + (4 - 5)^2}$
$= \sqrt{2^2 + (-1)^2}$
$= \sqrt{4 + 1}$
$= \sqrt{5}$

So, $BC = B''C''$.

The transformation that maps $\triangle ABC$ onto $\triangle A''B''C''$ is an isometry, by definition.

7.5 Guided Practice (p. 433)

1. In a glide reflection, the direction of a the translation must be parallel to the line of reflection.

2. The order in which two transformations are performed *sometimes* affects the resulting image.

3. In a glide reflection, the order in which the two transformations are performed *never* matters.

4. A composition of isometries is *always* an isometry.

5. $\overline{A'B'}$ **6.** $\overline{A''B''}$

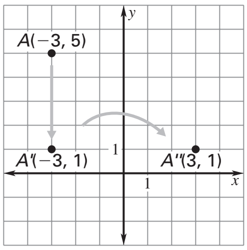
7. The line of reflection is the y-axis.

8. $(x, y) \rightarrow (x, y + 3)$

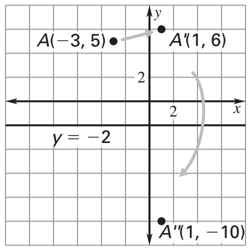
7.5 Practice and Applications (pp. 433–436)

9. A **10.** C **11.** B

12.



13.



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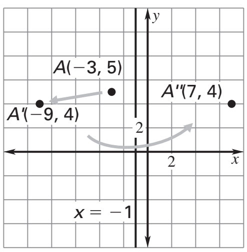

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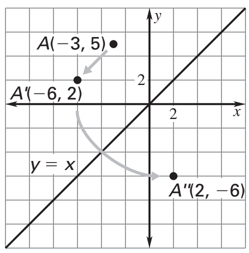

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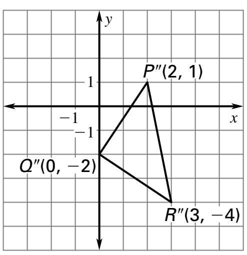

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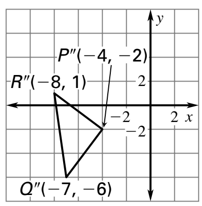

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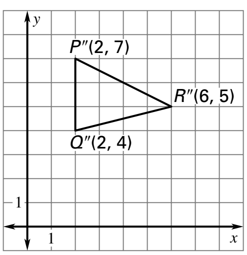

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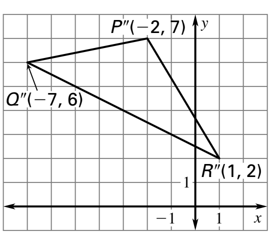

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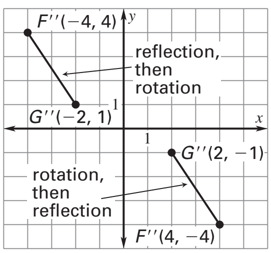
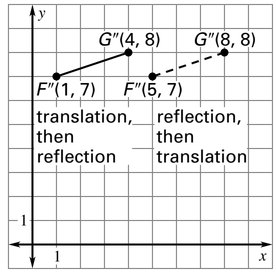


The order does affect the final image.

21.



The order does affect the final image.

22. reflection in the line $y = -\frac{1}{2}$, followed by a clockwise rotation of 90° about the origin

23. reflection in the line $y = 2$, followed by a reflection in the line $x = -2$

24. clockwise rotation of 90° about the origin, followed by the translation $(x, y) \rightarrow (x, y - 3)$

25. counterclockwise rotation of 90° about the point (0, 1), followed by the translation $(x, y) \rightarrow (x + 2, y + 3)$

26. A glide reflection is an isometry because it is a composition of a translation and a reflection, both of which are isometries. The composition of two isometries is an isometry.

27. A, B, and C are preserved in a glide reflection.

28. Answers will vary.

29. $(x, y) \rightarrow (x + 7, y - 2)$

$(x + 7, y - 2) \rightarrow (x + 7 - 1, y - 2 + 3)$

$(x + 7, y - 2) \rightarrow (x + 6, y + 1)$

So, $(x, y) \rightarrow (x + 6, y + 1)$.

30. $(x, y) \rightarrow (x + 9, y + 4)$

$(x + 9, y + 4) \rightarrow (x + 9 + 6, y + 4 - 4)$

$(x + 8, y + 4) \rightarrow (x + 15, y)$

So, $(x, y) \rightarrow (x + 15, y)$

31. After each part was painted, the stencil was moved through a glide reflection. The translation moved it to the right and the reflection in a horizontal line through its center flipped the design.

32. 5 **33.** 1, 4, 5, 6 **34.** 2, 7

35. The pattern can be made by a horizontal translation, 180° rotation, vertical line reflection, or horizontal glide reflection.

36. The pattern can be made by a vertical translation.

37. The pattern can be made by a translation or 180° rotation.

38. *Sample answer:* The X tile needs to be rotated 90° clockwise, then reflected in a horizontal line. The Y tile needs to be reflected in a vertical line, then rotated 90° counterclockwise.

39. **a.** Answers will vary.

b. *Conjecture:* The midpoint of the segment connecting the point and its image is on the x-axis.

c. Let (x, y) be the original point, since the translation must be parallel to the line of reflection, the coordinates of the image are $(x + a, -y)$ for some number a. The coordinates of the midpoint are

$$\left(\frac{x + a + x}{2}, \frac{-y + y}{2}\right) = \left(\frac{2x + a}{2}, 0\right).$$

Then the midpoint is on the x-axis

d. Yes; the midpoint is the point where the segments and the line of reflection intersect.

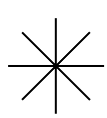
40.

$$\begin{aligned} -2 + 3 &= c + 1 \\ 1 &= c + 1 \\ 0 &= c \\ -(-1) &= -f \\ 1 &= -f \\ -1 &= f \\ 2a &= 4 \\ a &= 2 \\ -4 + 3 &= 5d - 11 \\ -1 &= 5d - 11 \\ 10 &= 5d \\ 2 &= d \\ -4 &= 3g + 5 \\ -9 &= 3g \\ -3 &= g \end{aligned}$$

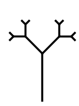
$$\begin{aligned} b - 6 + 3 &= 2 \\ b - 3 &= 2 \\ b &= 5 \\ 6 &= 4e \\ \tfrac{6}{4} &= e \\ \tfrac{3}{2} &= e \\ 2 &= h + 4 \\ -2 &= h \end{aligned}$$

7.5 Mixed Review (p. 436)

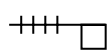
41.



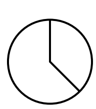
42.



43.



44.



45.

$$\begin{aligned} PQ &= \sqrt{(5 - 1)^2 + (-1 - (-2))^2} \\ &= \sqrt{4^2 + 1^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(6 - 5)^2 + (-5 - (-1))^2} \\ &= \sqrt{1^2 + (-4)^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17} \end{aligned}$$

—CONTINUED—

45. —CONTINUED—

$$\begin{aligned} RS &= \sqrt{(2 - 6)^2 + (-6 - (-5))^2} \\ &= \sqrt{(-4)^2 + (-1)^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \end{aligned}$$

$$\begin{aligned} PS &= \sqrt{(2 - 1)^2 + (-6 - (-2))^2} \\ &= \sqrt{1^2 + (-4)^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17} \end{aligned}$$

$PQ = QR = RS = PS$ so $PQRS$ is a rhombus.

$$\begin{aligned} PR &= \sqrt{(1 - 6)^2 + (-2 - (-5))^2} \\ &= \sqrt{(-5)^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \end{aligned}$$

$$\begin{aligned} QS &= \sqrt{(5 - 2)^2 + (-1 - (-6))^2} \\ &= \sqrt{3^2 + 5^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \end{aligned}$$

Since $PR = QS$, the diagonals of $PQRS$ are congruent, so $PQRS$ is a rectangle. By the Square Corollary, $PQRS$ is a square.

46.

$$\begin{aligned} PQ &= \sqrt{(15 - 10)^2 + (7 - 7)^2} \\ &= \sqrt{5^2 + 0^2} \\ &= \sqrt{25 + 0} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(15 - 15)^2 + (1 - 7)^2} \\ &= \sqrt{0^2 + (-6)^2} \\ &= \sqrt{0 + 36} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} RS &= \sqrt{(10 - 15)^2 + (1 - 1)^2} \\ &= \sqrt{(-5)^2 + 0^2} \\ &= \sqrt{25 + 0} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} PS &= \sqrt{(10 - 10)^2 + (1 - 7)^2} \\ &= \sqrt{0^2 + (-6)^2} \\ &= \sqrt{0 + 36} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

Since opposite sides are congruent, $PQRS$ is a parallelogram.

—CONTINUED—

46. —CONTINUED—

$PR = \sqrt{(10 - 15)^2 + (7 - 1)^2}$

$= \sqrt{(-5)^2 + 6^2}$

$= \sqrt{25 + 36}$

$= \sqrt{61}$

$QS = \sqrt{(15 - 10)^2 + (7 - 1)^2}$

$= \sqrt{5^2 + 6^2}$

$= \sqrt{25 + 36}$

$= \sqrt{61}$

Since $PR = QS$, the diagonals are congruent, so $PQRS$ is a rectangle.

47. $PQ = \sqrt{(10 - 8)^2 + (-7 - (-4))^2}$

$= \sqrt{2^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

$QR = \sqrt{(8 - 10)^2 + (-10 - (-7)^2}$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

$RS = \sqrt{(6 - 8)^2 + (-7 - 10))^2}$

$= \sqrt{(-2)^2 + 3^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

$PS = \sqrt{(6 - 8)^2 + (-7 - (-4))^2}$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

$PR = \sqrt{(8 - 8)^2 + (-4 - (-10))^2}$

$= \sqrt{0^2 + 6^2}$

$= \sqrt{36}$

$= 6$

$QS = \sqrt{(10 - 6)^2 + (-7 - 7))^2}$

$= \sqrt{4^2 + 0^2}$

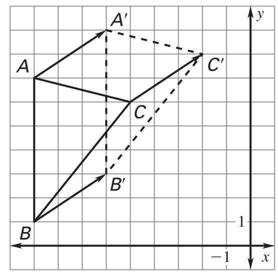
$= \sqrt{16}$

$= 4$

All four sides are congruent, but the diagonals are not congruent, so $PQRS$ is a rhombus but not a rectangle or a square.

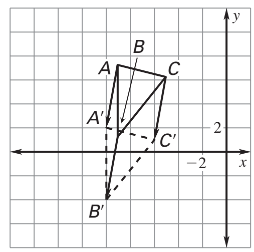
48. (8, 3), (1, 7).

49.



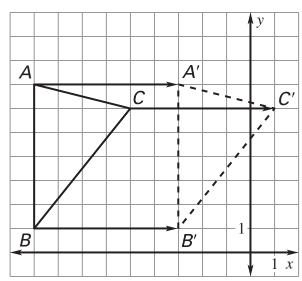
$A'(-6, 9)$, $B'(-6, 3)$, $C'(-2, 8)$

50.



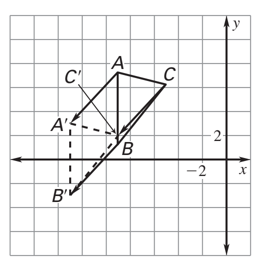
$A'(-10, 2)$, $B'(-10, -4)$, $C'(-6, 1)$

51.



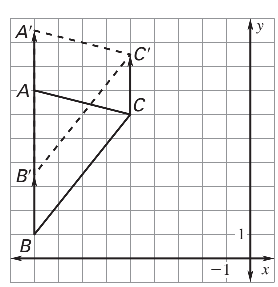
$A'(-3, 7)$, $B'(-3, 1)$, $C'(1, 6)$

52.



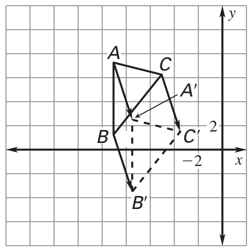
$A'(-13, 3)$, $B'(-13, -3)$, $C'(-9, 2)$

53.



$A'(-9, 9.5)$, $B'(-9, 3.5)$, $C'(-5, 8.5)$

54.



$A'(-7.5, 2.5)$, $B'(-7.5, -3.5)$, $C'(-3.5, 1.5)$

Lesson 7.6

7.6 Guided Practice (p. 440)

1. A *frieze pattern* is a pattern that extends to the left and right in such a way that the pattern can be mapped onto itself by a horizontal translation.

2. The pattern is an example of TV because there is no 180° rotational symmetry. But there is a vertical line symmetry because the triangle is equilateral.

3. translation, vertical line reflection

4. translation, rotation, horizontal line reflection, vertical line reflection, horizontal glide reflection

5. translation, rotation, vertical line reflection, horizontal glide reflection

6. translation, rotation, vertical line reflection, horizontal glide reflection

7. The five possible transformations that can be found in a freeze pattern are translation (T), 180° rotation (R), horizontal glide reflection (G), vertical line reflection (V), and horizontal line reflection (H).

7.6 Practice and Applications (pp. 440–443)

8. C **9.** D **10.** A **11.** B

12. translation, horizontal line reflection, horizontal glide reflection

13. translation, 180° rotation **14.** translation, 180° rotation

15. translation, 180° rotation, horizontal line reflection, vertical line reflection, horizontal glide reflection

16. Yes; there is a reflection in any vertical line that lies midway between two figures.

17. Yes; there is a reflection in a the x-axis.

18. The transformation that maps A onto F is a reflection in the x-axis, followed by a horizontal translation described by $(x, y) \rightarrow (x + 14, y)$.

19. The transformation that maps D onto B is a 180° rotation about (8, 0).

20. The frieze pattern is TRHVG.

21. The pattern on the collar can be classified as TRHVG.

22. The pattern on the collar can be classified as TG.

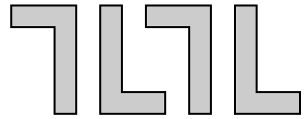
23. The pattern on the collar can be classified as T.

24. Answers will vary.

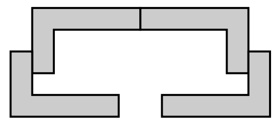
25. Answers will vary.

26.–31. Sample patterns are given.

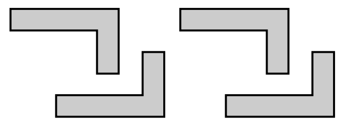
26.



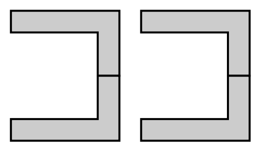
27.



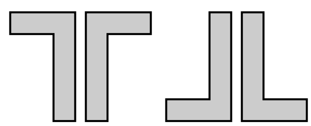
28.



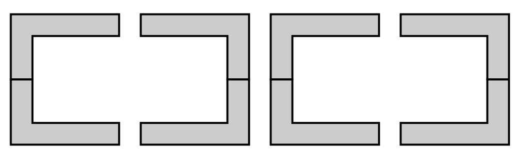
29.



30.



31.



32. T

33. TRHVG

34. TR

35. There are three bands of frieze patterns visible.

36. The patterns near the top and bottom of the jar are T. The pattern in the middle of the jar is TR.

37. Answers will vary.

38. *Sample answer:*

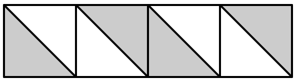
The band around the middle is an example of THG.

39. $d = 9.5$ in.

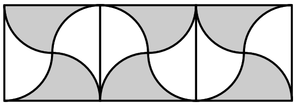
$C = \pi d = 3.14 \cdot 9.5 = 29.83$ in.

The circumference of the base is about 29.83 inches. If you want 10 repetitions of the design, the design should be about $29.83 \div 10$ or 2.98 inches wide.

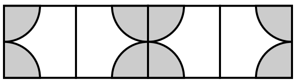
40.



41.



42.



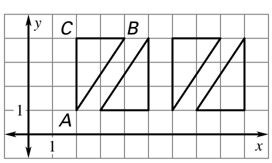
43. *Sample answer:* The design on the tiles limits what classifications of patterns can be made. For instance, in Exercise 40, the design on the tile would not allow the creation of THG in a single row because there is not a horizontal line of symmetry in the tile. The same would be true for the tile in Exercise 41.

44. If a pattern can be mapped onto itself by a horizontal glide reflection and by a vertical line reflection, it can be mapped onto itself by a 180° rotation about the point where the lines of reflection intersect.

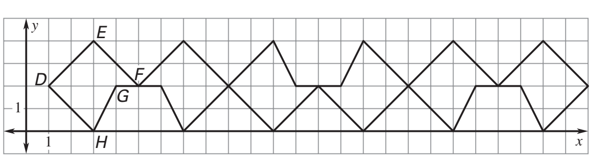
45. If a pattern can be mapped onto itself by a horizontal line reflection and by a vertical line reflection, it can be mapped onto itself by a 180° rotation about the point where the lines intersect. It can also be mapped onto itself by a horizontal glide reflection involving the given horizontal line reflection and any translation.

46. If a pattern can be mapped onto itself by a 180° rotation about a point and a horizontal glide reflection, the center of rotation must be on the line of reflection for the glide reflection. Then the pattern can be mapped onto itself by reflection in a vertical line through the center of rotation.

47. *Sample answer*:

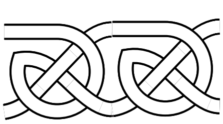


48. *Sample answer:*



49. *Sample answers:*

a.



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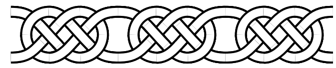
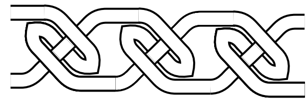
49. —CONTINUED—

b.



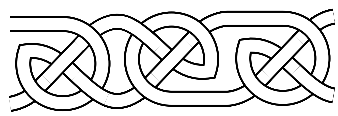
c.



50. a. T **b.** TVG **c.** TRVG

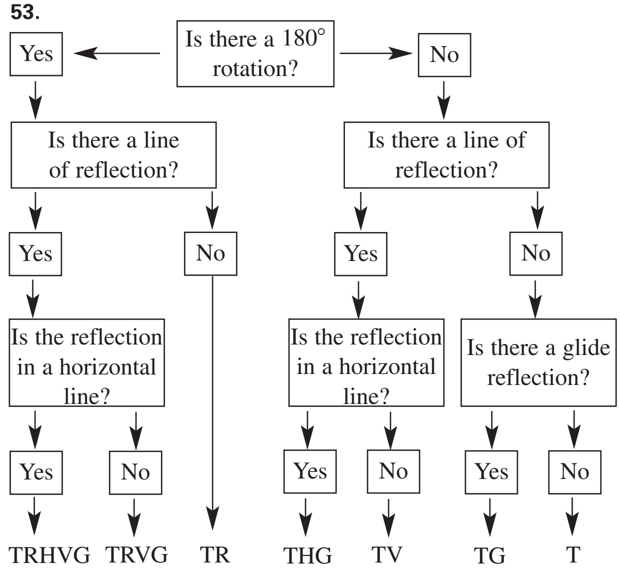
51. Design A does not have a rotational symmetry.

Sample answer:



52. If it has 180° rotational symmetry, then its classification must be at least TR. The 180° symmetry means that the pattern can be mapped onto itself with a 180° rotation. Therefore, it must at least be TR.

53.



7.6 Mixed Review (p. 444)

54. $23 - 12 = 11$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{11}{12}$

55. $21 - 8 = 13$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{13}{8}$

56. $13 - 3 = 10$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{10}{3}$

57. $35 - 19 = 16$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{16}{19}$

58. $18 - 11 = 7$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{7}{11}$

59. $20 - 10 = 10$ girls

$\frac{\text{girls}}{\text{boys}} = \frac{10}{10} = 1$

60. $3x - 5 = \frac{2}{3}(3x - 5 + x)$

$3x - 5 = \frac{2}{3}(4x - 5)$

$3x - 5 = \frac{8}{3}x - \frac{10}{3}$

$3x = \frac{8}{3}x + \frac{5}{3}$

$\frac{1}{3}x = \frac{5}{3}$

$x = 5$

$7z - 2 = \frac{2}{3}(7z - 2 + 3z)$

$7z - 2 = \frac{2}{3}(10z - 2)$

$7z - 2 = \frac{20}{3}z - \frac{4}{3}$

$7z = \frac{20}{3}z + \frac{2}{3}$

$\frac{1}{3}z = \frac{2}{3}$

$z = 2$

61. $2y = \frac{2}{3}(2y + 3y - 4)$

$2y = \frac{2}{3}(5y - 4)$

$2y = \frac{10}{3}y - \frac{8}{3}$

$-\frac{4}{3}y = -\frac{8}{3}$

$y = 2$

$w = \frac{2}{3}(w + w - 4)$

$w = \frac{2}{3}(2w - 4)$

$w = \frac{4}{3}w - \frac{8}{3}$

$-\frac{1}{3}w = -\frac{8}{3}$

$w = 8$

62. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(15 + 15)(12 + 12)$

$= \frac{1}{2} \cdot 30 \cdot 24$

$= 360$ square units

63. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(12 + 12)(4 + 20)$

$= \frac{1}{2} \cdot 24 \cdot 24$

$= 288$ square units

64. $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2} \cdot 17 \cdot (17 + 35)$

$= \frac{1}{2} \cdot 17 \cdot 52$

$= 442$ square units

Quiz 2 (p. 444)

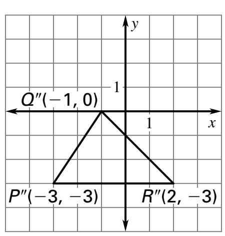
1. $A'(0, 5)$, $B'(5, 6)$, $C'(2, 4)$

2. $A'(-4, 6)$, $B'(1, 7)$, $C'(-2, 5)$

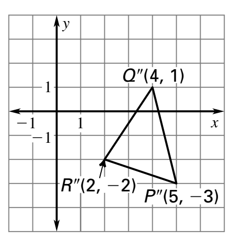
3. $A'(-3, -2)$, $B'(2, -1)$, $C'(-1, -3)$

4. $A'(4, 4)$, $B'(9, 5)$, $C'(6, 3)$

5.



6.



7. Yes, the frieze pattern is TR.

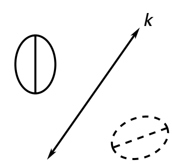
Chapter 7 Review (pp. 446–448)

1. Yes; it is an isometry because the figure and its image appear to be congruent.

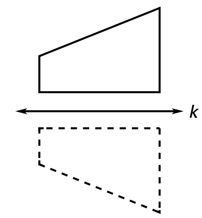
2. No; it is not an isometry because the figure and its image are not congruent.

3. Yes; it is an isometry because the figure and its image appear to be congruent.

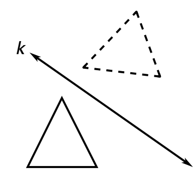
4.



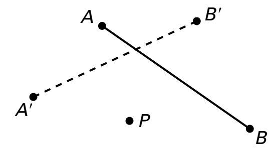
5.



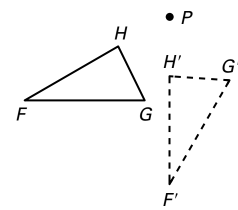
6.



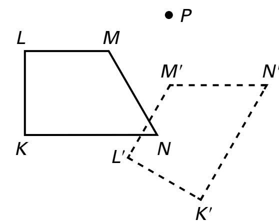
7.



8.



9.



10. C **11.** A **12.** D **13.** B

14. $(x, y) \rightarrow (x + 4, y - 5)$, followed by 90° clockwise rotation about the origin

15. reflection in the x-axis, followed by a 90° counterclockwise rotation about the origin

16. The rainbow boa's snakeskin has a frieze pattern classified as TR.

17. The gray-banded kingsnake's snakeskin has a frieze pattern classified as TRHVG.

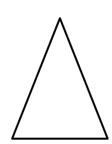
Chapter 7 Test (p. 449)

1. The transformation that maps $\triangle RST$ onto $\triangle XYZ$ is a reflection in the y-axis.

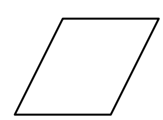
2. Yes, $\overline{RT} \cong \overline{XZ}$ because a reflection preserves length.

3. The image of T is Z. **4.** The preimage of Y is S.

5. *Sample answer:*



6. *Sample answer:*



7. The transformation that maps figure T onto figure T' is a reflection in line m.

8. The transformation that maps figure T onto T'' is a reflection in line m followed by a reflection in line n. Or it is a rotation about the point of intersection of lines m and n.

9. The measure of the angle of rotation is twice the measure of the acute angle formed by lines m and n. So, the measure of the angle of rotation is $2 \cdot 85°$ or 170°.

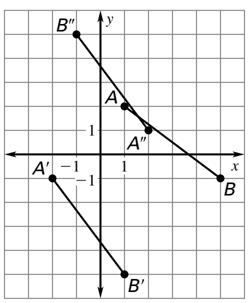
10. The transformation that maps figure R onto figure R' is a reflection in line k.

11. The transformation that maps figure R onto R'' is a reflection in line k, followed by a reflection in line m, or it is a translation.

12. The distance between corresponding parts of figure R and figure R'' is twice the distance between lines k and m. The distance is $2 \cdot 5$ or 10 units.

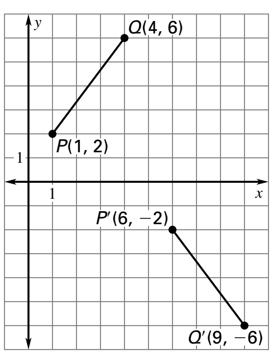
13. A glide reflection is a composition of a translation followed by a reflection in a line parallel to the translation vector.

14. *Sample answer:*



$\overline{A'B'}$ is the final image when $\overline{AB}$ is rotated 90° clockwise about the origin, then reflected in the y-axis. $\overline{A''B''}$ is the final image when $\overline{AB}$ is reflected in the y-axis, then rotated 90° clockwise about the origin.

15. *Sample answer:*



$\overline{PQ}$ is reflected in the x-axis, then translated $(x, y) \rightarrow (x + 5, y)$. The same image results if the transformations are performed in reverse order.

16. The flag of Switzerland has a vertical line of symmetry, a horizontal line of symmetry, two diagonal lines of symmetry and rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about the center.

17. The flag of Jamaica has a vertical line of symmetry, a horizontal line of symmetry, and rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about the center.

18. The flag of the United Kingdom has a vertical line of symmetry, a horizontal line of symmetry, and rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about its center.

19. translation, 180° rotation, horizontal line reflection, vertical line reflection, glide reflection

20. translation, 180° rotation, horizontal line reflection, vertical line reflection, glide reflection

21. translation, vertical line rotation

Chapter 7 Standardized Test (pp. 450–451)

1. B **2.** D **3.** E

4.
$$(8y - 6) = 42°$$
$$8y = 48$$
$$y = 6$$
$$(2x + 19)° = 101°$$
$$2x = 82$$
$$x = 41$$
$$3z = 18$$
$$z = 6$$

A

5. $W(3, 8) \rightarrow (3 - 8, 8 - 10)$ or $(-5, -2)$

$X'(7, 6) \rightarrow (7 - 8, 6 - 10)$ or $(-1, -4)$

$Y'(5, 2) \rightarrow (5 - 8, 2 - 10)$ or $(-3, -8)$

B

6. D

7. A 90° counterclockwise rotation maps $\overline{AB}$ onto $\overline{A'B'}$. A reflection in the x-axis maps $\overline{A'B'}$ onto $\overline{A''B''}$. So the correct answer is D.

8. C

9. With the reflection in $y = 1$, $S(-6, -2)$ is mapped onto $S'(-6, 4)$ and $T(-3, -5)$ is mapped onto $T'(-3, 7)$. A 90° clockwise rotation about the point $(-3, 2)$ maps $S'(-6, 4)$ onto $S''(-1, 5)$ and $T'(-3, 7)$ onto $T''(2, 2)$. So the correct answer is E.

10. The letters that have a vertical line of symmetry are A, H, I, M, O, T, W, X, and Y.

11. The letters that have a horizontal line of symmetry are C, E, H, K, O, and X.

12. The letters with a rotational symmetry are H, N, O, S, X, and Z.

13. **a.** $(x, y) \rightarrow (x - 2, y + 7)$ **b.** $\langle -2, 7 \rangle$

14. The coordinates of the vertices of figure *WXYZ* are $W(2 + 7, 6 - 2)$ or $W(9, 4)$, $X(4 + 7, 6 - 2)$ or $X(11, 4)$, $Y(5 + 7, 3 - 2)$ or $Y(12, 1)$, and $Z(0 + 7, 3 - 2)$ or $Z(7, 1)$.

15. 90° clockwise rotation about the origin

16. *Sample answer:* $y = 2\frac{1}{2}$

Project Chapters 6–7 (pp. 452–453)

Investigation

1. No; the quadrilateral is not a regular polygon.

2. Some transformations that would map the pattern onto itself are translations and rotations.

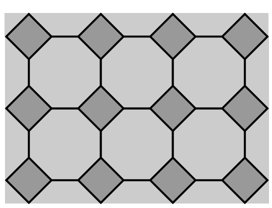
3. The sum of the measures of the angles at any vertex of a quadrilateral tessellation is 360°. Any quadrilateral will tessellate because the sum of the measures of its angles is 360°. If all four angles are placed so they are adjacent, the full 360° rotation is covered.

4. Square and equilateral triangle tessellations are regular tessellations.

5. Tessellations will vary.

A translation and a rotation map the pattern onto itself.

6. *Sample answer:*



This is a semiregular tessellation.

7. These shapes cannot be used together to create a tessellation because there is no combination of multiples of 108° and multiples of 90° that add together to get 360°.

8. These shapes can be used together to create a nonregular tessellation.

Sample answer: