CHAPTER 6

Lesson 6.1

Think & Discuss (p. 319)

1. As $m\angle B$ increases, A and D move apart. Therefore AD increases.

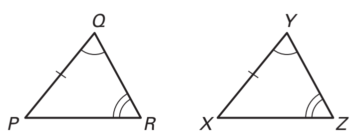
2. The platform of the scissors lift will be raised higher.

Skill Review (p. 320)

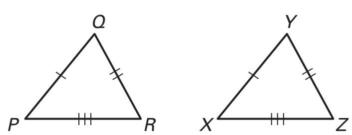
1. If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary (Theorem 3.5 Consecutive Interior Angles).

2. If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent (Theorem 3.4 Alternate Interior Angles).

3. AAS Congruence Theorem



4. SSS Congruence Postulate



5.
$$\begin{aligned} AB &= \sqrt{(-3-2)^2 + (4-(-8))^2} \\ &= \sqrt{(-5)^2 + (12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$\begin{aligned} m &= \frac{-8-4}{2-(-3)} \\ &= -\frac{12}{5} \\ &= \left(\frac{(2+(-3))}{2}, \frac{(-8+4)}{2}\right) \\ &= \left(-\frac{1}{2}, -2\right) \end{aligned}$$

Developing Concepts Activity 6.1 (p. 321)

Drawing Conclusions

1. squall, snow, altostratus cloud, hail, showers, lightning

2. squall, hail, showers

3. Process, input/output, manual operation, decision, and extract are polygons because they are made up of line segments and each line segment intersects exactly two other line segments, one at each endpoint.

4. The fewest number of sides a polygon can have is three, because each segment must intersect exactly two other line segments. There is no limit other than this on the number of sides.

6.1 Guided Practice (p. 325)

1. vertices **2.** octagon, 15-gon

3. Yes; no; if the polygon were convex, the string would wrap exactly around the polygon with no gaps. If the polygon were concave, there would be gaps and the length of the string would be less than the perimeter of the polygon.

4. yes **5.** No, because one side is not a line segment.

6. No, because two sides intersect only one other side.

7. equilateral **8.** none of these **9.** regular

10.
$$\begin{aligned} m\angle A &= 360° - m\angle B - m\angle C - m\angle D \\ &= 360° - 125° - 60° - 70° \\ &= 105° \end{aligned}$$

11.
$$\begin{aligned} m\angle A &= 360° - m\angle B - m\angle C - m\angle D \\ &= 360° - 105° - 113° - 75° \\ &= 67° \end{aligned}$$

6.1 Practice and Applications (pp. 325–328)

12. yes **13.** no **14.** no **15.** no **16.** yes **17.** no

18. pentagon; convex **19.** heptagon; concave

20. heptagon; concave **21.** octagon

22. *Sample answers: MNPQRSTL, NPQRSTLM,*

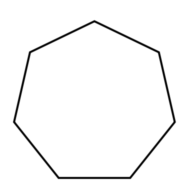
23. $\overline{MP}, \overline{MQ}, \overline{MR}, \overline{MS}, \overline{MT}$ **24.** regular **25.** equilateral

26. equiangular **27.** quadrilateral; regular

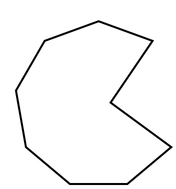
28. pentagon; none of these **29.** triangle; regular

30. octagon; regular

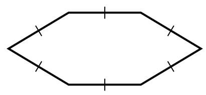
31. *Sample answer:*



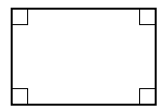
32. *Sample answer:*



33. *Sample answer:*



34. *Sample answer:*



35. Yes; to be concave, a polygon must have an angle with measure greater than 180°.

36. $m\angle ABC$ is 90° because a regular quadrilateral has four equal angles whose sum is 360°.

$x° + x° + x° + x° = 360°$

$4x° = 360°$

$x = 90$

37. $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

$m\angle A + 95° + 100° + 90° = 360°$

$m\angle A + 285° = 360°$

$m\angle A = 75°$

38. $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

$m\angle A + 55° + 110° + 124° = 360°$

$m\angle A + 289° = 360°$

$m\angle A = 71°$

39. $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

$m\angle A + 85° + 63° + 87° = 360°$

$m\angle A + 235° = 360°$

$m\angle A = 125°$

40. The sum of the measures of the interior angles remains constant (always 360°).

41. $x° + 100° + 87° + 106° = 360°$

$x° + 293° = 360°$

$x = 67$

42. $3x° + 150° + 60° + 90° = 360°$

$3x° + 300° = 360°$

$3x° = 60°$

$x = 20$

43. $2x° + 2x° + 84° + 100° = 360°$

$4x° + 184° = 360°$

$4x° = 176°$

$x = 44$

44. $(4x + 10)° + 108° + 3x° + 67° = 360°$

$4x° + 10° + 108° + 3x° + 67° = 360°$

$7x° + 185° = 360°$

$7x° = 175°$

$x = 25$

45. $82° + (25x - 2)° + (20x - 1)° + (25x + 1)° = 360°$

$82° + 25x° - 2° + 20x° - 1° + 25x° + 1° = 360°$

$70x° + 180° = 360°$

$70x° = 280°$

$x = 4$

46. $(x^2)° + 90° + 90° + 99° = 360°$

$(x^2)° + 279° = 360°$

$(x^2)° = 81°$

$x = 9 \text{ or } -9$

47. Tri–means three; *sample answers:* triangle is a polygon with three angles, tricycle is a child's vehicle with three wheels, tripod is an object with three legs, and triathlon is an athletic competition with three events.

48. hexagon; convex; regular

49. octagon; concave; equilateral

50. pentagon; convex; none of these

51. 17–gon; concave; none of these

52. **a.** 18–gon; no, it is concave.

b. Step 1: 18–gon, concave; Step 2: decagon; convex; Step 3: heptagon, convex; Step 4: quadrilateral, convex

c. The notches allow the paper to be folded into right or straight angles without overlapping the paper.

53. $3x° + 3x° + 3y° + 3y° = 360°$

$6x° + 6y° = 360°$

$6(x + y)° = 360°$

$x° + y° = 60°$

$x° = 60° - y°$

$(4x + 5)° + (4x + 5)° + (3y - 20)° + (3y - 20)° = 360°$

$4x° + 5° + 4x° + 5° + 3y° - 20° + 3y° - 20° = 360°$

$8x° + 6y° - 30° = 360°$

$8x° + 6y° = 390°$

Substituting $60° - y°$ for $x°$ gives

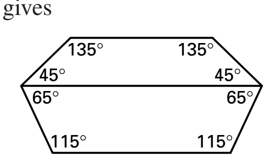
$8(60° - y°) + 6y° = 390°$

$480° + 8y° + 6y° = 390°$

$-2y° = -90°$

$y = 45$

$x = 60 - 45 = 15$



6.1 Mixed Review (p. 328)

54. 120 **55.** 63

56. $x° + 2x° = 180°$

$3x° = 180°$

$x = 60$

57. $(20x + 2)° + (9x + 4)° = 180°$

$$20x° + 2° + 9x° + 4° = 180°$$
$$29x° + 6° = 180°$$
$$29x° = 174°$$
$$x = 6$$

58. $x° + x° = 180°$

$$2x° = 180°$$
$$x = 90$$

59. $25x° + 11x° = 180°$

$$36x° = 180°$$
$$x = 5$$

60. Plot the midpoints of the triangle on a graph. Connect the midpoints to form the midsegments. $\overline{LN}$ will be parallel to the side containing point M. $\overline{LM}$ will be parallel to the side containing point N. $\overline{MN}$ will be parallel to the side containing point L. Find the slope of $\overline{LN}$ and draw a line through point M which is parallel to $\overline{LN}$.

$$\text{slope of } \overline{LN} = \frac{8 - 7}{-8 - (-3)} = -\frac{1}{5}$$

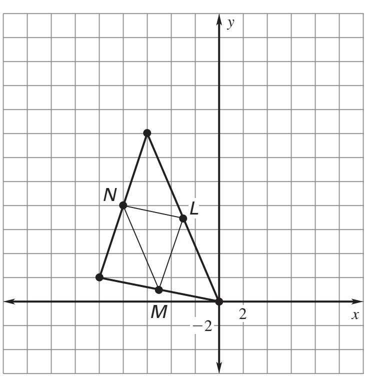
Find the slope of $\overline{LM}$ and a draw a line through point N which is parallel to $\overline{LM}$.

$$\text{slope of } \overline{LM} = \frac{1 - 7}{-5 - (-3)} = 3$$

Find the slope of $\overline{MN}$ and draw a line through point L which is parallel to $\overline{MN}$.

$$\text{slope of } \overline{MN} = \frac{8 - 1}{-8 - (-5)} = -\frac{7}{3}$$

The points where the lines intersect are the vertices of the triangle: (0, 0), (−10, 2) and (−6, 14).



61. Plot the midpoints of the triangle on a graph. Connect the midpoints to form the midsegments. $\overline{LN}$ will be parallel to the side containing point M. $\overline{LM}$ will be parallel to the side containing point N. $\overline{MN}$ will be parallel to the side containing point L. Find the slope of $\overline{LN}$ and draw a line through point M which is parallel to $\overline{LN}$.

$$\text{slope of } \overline{LN} = \frac{-8 - (-1)}{-2 - (-4)} = -\frac{7}{2}$$

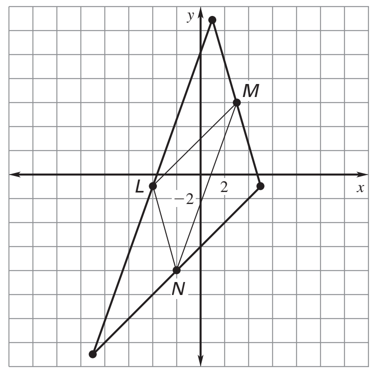
Find the slope of $\overline{LM}$ and draw a line through point N which is parallel to $\overline{LM}$.

$$\text{slope of } \overline{LM} = \frac{6 - (-1)}{3 - (-4)} = 1$$

Find the slope of $\overline{MN}$ and draw a line through point L which is parallel to $\overline{MN}$.

$$\text{slope of } \overline{MN} = \frac{-8 - 6}{-2 - 3} = \frac{14}{5}$$

The points where the lines intersect are the vertices of the triangle: (1, 13), (5, −1) and (−9, −15).



62. Plot the midpoints of the triangle on a graph. Connect the midpoints to form the midsegments. $\overline{LN}$ will be parallel to the side containing point M. $\overline{LM}$ will be parallel to the side containing point N. $\overline{MN}$ will be parallel to the side containing point L. Find the slope $\overline{LN}$ and draw a line through point M which is parallel to $\overline{LN}$.

$$\text{slope of } \overline{LN} = \frac{7 - 4}{0 - 2} = -\frac{3}{2}$$

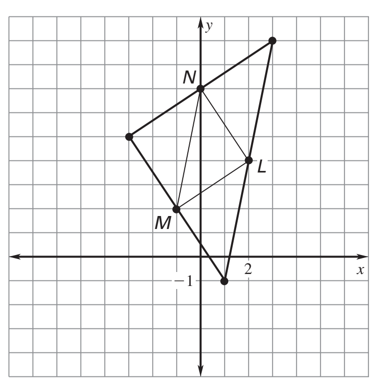
Find the slope of $\overline{LM}$ and draw a line through point N which is parallel to $\overline{LM}$.

$$\text{slope } \overline{LM} = \frac{2 - 4}{-1 - 2} = \frac{2}{3}$$

Find the slope of $\overline{MN}$ and draw a line through point L which is parallel to $\overline{MN}$.

$$\text{slope of } \overline{MN} = \frac{7 - 2}{0 - (-1)} = 5$$

The points where the lines intersect are the vertices of the triangle: (3, 9), (1, −1) and (−3, 5).



63. Plot the midpoints of the triangle on a graph. Connect the midpoints to form the midsegments. $\overline{LN}$ will be parallel to the side containing point M. $\overline{LM}$ will be parallel to the side containing point N. $\overline{MN}$ will be parallel to the side containing point L. Find the slope of $\overline{LN}$ and draw a line through point M which is parallel to $\overline{LN}$.

$$\text{slope of } \overline{LN} = \frac{-5-3}{3-(-1)} = -2$$

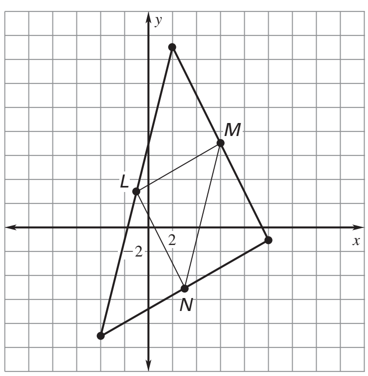
Find the slope of $\overline{LM}$ and draw a line through point N which is parallel to $\overline{LM}$.

$$\text{slope of } \overline{LM} = \frac{7-3}{6-(-1)} = \frac{4}{7}$$

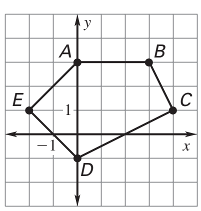
Find the slope of $\overline{MN}$ and draw a line through point L which is parallel to $\overline{MN}$.

$$\text{slope of } \overline{MN} = \frac{-5-7}{3-6} = 4$$

The points where the lines intersect are the vertices of the triangle: (2, 15), (−4, −9) and (10, −1).



64.



$$\begin{aligned} AC &= \sqrt{(1-3)^2 + (4-0)^2} \\ &= \sqrt{(-2)^2 + 4^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(0-0)^2 + (-1-3)^2} \\ &= \sqrt{0^2 + (-4)^2} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\begin{aligned} BE &= \sqrt{(-2-3)^2 + (1-3)^2} \\ &= \sqrt{(-5)^2 + (-2)^2} \\ &= \sqrt{25+4} = \sqrt{29} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(0-3)^2 + (-1-3)^2} \\ &= \sqrt{(-3)^2 + (-4)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} CE &= \sqrt{(-2-4)^2 + (1-1)^2} \\ &= \sqrt{(-6)^2 + 0^2} \\ &= \sqrt{36} = 6 \end{aligned}$$

Technology Activity 6.2 (p. 329)

1. Yes, $ABFC$ is always a parallelogram because the slopes of the opposite sides are always equal.
2. Opposite sides are congruent.
3. The opposite sides are always congruent.
4. Opposite sides are congruent and have the same slope.
5. Opposite angles are congruent. If one angle increases by $x°$, the opposite angle also increases by $x°$. Consecutive angles are supplementary. If one angle increases by $x°$, its consecutive angle decreases by $x°$.
6. Opposite angles of a parallelogram are congruent.

Extension

The diagonals of a parallelogram bisect each other.

Lesson 6.2

6.2 Guided Practice (p. 333)

1. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. No; only one pair of opposite sides is parallel.
3. Yes; both pairs of opposite sides are parallel.
4. $\overline{LM}$; opposite sides of a parallelogram are congruent.
5. $\overline{KN}$; diagonals of a parallelogram bisect each other.
6. $\angle KJM$; opposite angles of a parallelogram are congruent.
7. $\angle LMJ$; opposite angles of a parallelogram are congruent.
8. $\overline{LN}$; diagonals of a parallelogram bisect each other.
9. $\overline{JM}$; opposite sides of a parallelogram are congruent.
10. $\angle KNJ$; vertical angles are congruent.
11. $\angle KMJ$; alternate interior angles are congruent.
12. 13; opposite sides of a parallelogram are congruent.
13. 7; diagonals of a parallelogram bisect each other.
14. 8; opposite sides of a parallelogram are congruent.
15. 8.2; diagonals of a parallelogram bisect each other.
16. 80°; consecutive angles of a parallelogram are supplementary.
17. 80°; consecutive angles of a parallelogram are supplementary.

18. 100°; opposite angles of a parallelogram are congruent.

19. 29°; opposite sides of a parallelogram are parallel, so $\overline{LM} \parallel \overline{QN}$ and alternate interior angles LMQ and MQN are congruent.

6.2 Practice and Applications (pp. 334–337)

20. 10; diagonals of a parallelogram bisect each other.

21. 11; opposite sides of a parallelogram are congruent.

22. 12; opposite sides of a parallelogram are congruent.

23. 60°; consecutive angles of a parallelogram are supplementary.

24. 60°; consecutive angles of a parallelogram are supplementary

25. 120°; opposite angles of a parallelogram are congruent.

26. $x = 14, y = 10$

27. $a° = 180° - 101°$
$a = 79$
$b = 101$

28. $r = 6$
$s = 3.5$

29. $p = 5$
$q - 3 = 6$
$q = 9$

30. $2m° = 70°$
$m = 35$
$n = 110$

31. $k + 4 = 11$
$k = 7$
$m = 8$

32. $2x + 4 = 8$
$2x = 4$
$x = 2$
$3y = 9$
$y = 3$

33. $2u + 2 = 5u - 10$
$12 = 3u$
$4 = u$
$6 = \frac{v}{3}$
$18 = v$

34. $2z + 1 = 4z - 5$ $\quad 4w = w + 3$
$6 = 2z$ $\quad 3w = 3$
$3 = z$ $\quad w = 1$

35. $(b - 10)° + (b + 10)° = 180°$
$b - 10 + b + 10 = 180$
$2b = 180$
$b = 90$
$c° = (b - 10)°$ $\quad d° = (b + 10)°$
$c = 90 - 10$ $\quad d = 90 + 10$
$= 80$ $\quad = 100$

36. $2f - 5 = 5f - 17$ $\quad g = f + 2$
$12 = 3f$ $\quad g = 4 + 2$
$4 = f$ $\quad = 6$

37. $(3t - 15)° = (2t + 10)°$
$3t - 15 = 2t + 10$
$t = 25$
$35° + (3t - 15)° = 180°$
$3s + 3t - 15 = 180$
$3s + 3(25) - 15 = 180$ $\quad 3s = 4r$
$60 + 35 = 180$ $\quad 3(40) = 4r$
$3s = 120$ $\quad 120 = 4r$
$s = 40$ $\quad 30 = r$

38. **a.** $\overline{AD} \cong \overline{BC}$ or $\overline{AB} \cong \overline{DC}$
b. $\overline{AB} \cong \overline{DC}$ or $\overline{AD} \cong \overline{BC}$
c. $\overline{DB} \cong \overline{DB}$
d. SSS
e. corresponding
f. $\overline{AC}$

39. **1.** $JKLM$ is a ▱.
2. Opposite ⦣ of a ▱ are ≅.
3. 360°
4. Substitution property of equality
5. $m\angle J$; $m\angle K$
6. Division property of equality
7. Definition of supplementary angles

40. $(c, 0)$ **41.** $(a + c, b)$ **42.** $\left(\frac{a + c}{2}, \frac{b}{2}\right)$

43. $\left(\frac{a + c}{2}, \frac{b}{2}\right)$

44. They share a common midpoint. Then, since $\overline{PR}$ intersects $\overline{OS}$ at its midpoint, $\overline{PR}$ bisects $\overline{OS}$ by the definition of segment bisector. Similarly, $\overline{OS}$ bisects $\overline{PR}$.

45. Opposite sides of a parallelogram are parallel. If two parallel lines are cut by a transversal, then alternate interior angles are congruent. Therefore $\angle 3 \cong \angle 5$ and $m\angle 3 = m\angle 5$. Consecutive angles of a parallelogram are supplementary, so $m\angle 5 + m\angle 6 = 180°$. By the substitution property of equality, $m\angle 3 + m\angle 6 = 180°$. Therefore $\angle 3$ is supplementary to $\angle 6$.

46. $\angle 4$ and $\angle 3$ are supplementary by Theorem 6.4, so $m\angle 4 + m\angle 3 = 180°$. By the Alternate Interior Angles Theorem, $\angle 3 \cong \angle 5$, so $m\angle 3 = m\angle 5$. By substitution $m\angle 4 + m\angle 5 = 180°$, thus $\angle 4$ is supplementary to $\angle 5$.

47. $\angle 4$ **48.** $\angle 5$; $\angle 8$

49. Corresponding ⦣ Postulate (If 2 ∥ lines are cut by a transversal, then corresponding ⦣ are ≅.)

50. No; the corresponding longer sides of the two ▱s are not ≅.

51. $m\angle B = 180° - 120° = 60°$ **52.** It increases.

53. It increases. **54.** It increases.

55.

Statements	Reasons
1. *ABCD* is a ▱.	1. Given.
2. $\overline{AB} \cong \overline{CD}$	2. If a quadrilateral is a ▱, then its opposite sides are ≅.
3. *CEFD* is a ▱.	3. Given
4. $\overline{CD} \cong \overline{FE}$	4. If a quadrilateral is a ▱, then its opposite sides are ≅.
5. $\overline{AB} \cong \overline{FE}$	5. Transitive Property of ≅

56.

Statements	Reasons
1. *PQRS* and *TUVS* are ▱s.	1. Given.
2. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	2. If a quadrilateral is a ▱, then its opposite angles are ≅.
3. $\angle 1 \cong \angle 3$	3. Transitive Property of ≅

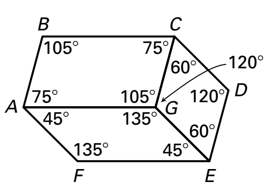
57.

Statements	Reasons
1. *WXYZ* is a ▱.	1. Given.
2. $\overline{WZ} \parallel \overline{XY}$, $\overline{WZ} \cong \overline{XY}$	2. Definition of a ▱
3. $\angle ZWM \cong \angle XYM$, $\angle MZW \cong \angle MXY$	3. If two ∥ lines are cut by a transversal, then alternate interior ∠s are ≅.
4. $\triangle WMZ \cong \triangle YMX$	4. ASA Congruence Postulate

58.

Statements	Reasons
1. *ABCD, EBGH,* and *HJKD* are ▱s.	1. Given.
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 1 \cong \angle 4$	2. If a quadrilateral is a ▱, then its opposite ∠s are ≅.
3. $\angle 2 \cong \angle 3$	3. Transitive Property of ≅

59.



Opposite ∠s of a ▱ are ≅.

Consecutive ∠s of a ▱ are supplementary.

The sum of the measures of the ∠s with vertex *G* is 360°.

60. B

$$(2s + 30)^\circ + (3s + 50)^\circ = 180^\circ$$
$$5s + 80 = 180$$
$$5s = 100$$
$$s = 20$$

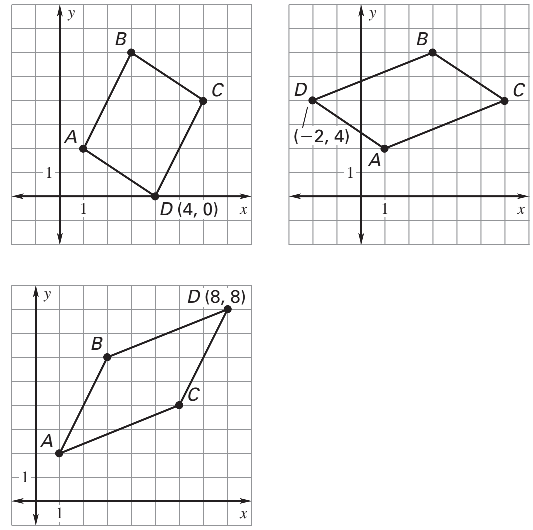
61. B

62. *Sample answer:* (4, 0) (See graph in answer to Ex. 64.)

63. *Sample answer:* Calculate slopes of the segments to show opposite sides are parallel.

64. three; (4, 0), (−2, 4), (8, 8) (See graphs below.)

Graphs for Exercises 62 and 64.



6.2 Mixed Review (p. 337)

65.
$$AB = \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{(4)^2 + (8)^2} = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$$

66.
$$AB = \sqrt{(2-(-4))^2 + (-1-2)^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

67.
$$AB = \sqrt{(-1-(-8))^2 + (-3-(-4))^2} = \sqrt{(7)^2 + (1)^2} = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$$

68.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{6-2} = \frac{8}{4} = 2$$

69.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{2-(-4)} = \frac{-3}{6} = -\frac{1}{2}$$

70.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-4)}{-1-(-8)} = \frac{1}{7}$$

71. Yes; if two coplanar lines are ⊥ to the same line, then they are ∥ to each other.

72. $m\angle B = 180° - (65° + 35°) = 80°$. If one $\angle$ of a $\triangle$ is larger than another $\angle$, then the side opposite the larger $\angle$ is longer than the side opposite the smaller $\angle$. So $\overline{AB}$ is the shortest side and $\overline{AC}$ is the longest side.

73. $m\angle D = 180° - (90° + 55°) = 35°$. If one $\angle$ of a $\triangle$ is larger than another $\angle$, then the side opposite the larger $\angle$ is longer than the side opposite the smaller $\angle$. So $\overline{EF}$ is the shortest side and $\overline{DF}$ is the longest side.

74. $m\angle H = 180° - (45° + 60°) = 75°$. If one $\angle$ of a $\triangle$ is larger than another $\angle$, then the side opposite the larger $\angle$ is longer than the side opposite the smaller $\angle$. So $\overline{GH}$ is the shortest side and $\overline{GJ}$ is the longest side.

Lesson 6.3

Activity 6.3 (p. 338)

Yes, it is always a parallelogram.

6.3 Guided Practice (p. 342)

1. No; a parallelogram is a quadrilateral.

2. Yes; if both pairs of opposite $\angle$s in a quad. are $\cong$, then the quad. is a ▱.

3. Yes; if an $\angle$ of a quad. is supplementary to both of its consecutive $\angle$s, then the quad. is a ▱.

4. Yes; if both pairs of opposite sides of a quad. are $\parallel$, then the quad. is a ▱.

5. Prove $\triangle BCA \cong \triangle DAC$. Then $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$ because $\cong$ parts of $\cong$ $\triangle$s are $\cong$. Since both pairs of opposite sides are $\cong$, then $ABCD$ is a ▱.

6. Since $\angle ACB \cong \angle CAD$ and $\angle BAC \cong \angle DAC$, and these are pairs of alt. interior $\angle$s, then $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$. Since both pairs of opposite sides of $ABCD$ are $\parallel$, $ABCD$ is a ▱.

7. Since corresponding angles C and D are $\cong$, $\overline{BC} \parallel \overline{AD}$ by the Corresp. $\angle$s Converse. Since alternate interior angles A and D are $\cong$, $\overline{BA} \parallel \overline{CD}$ by the Alt. Int. $\angle$s Converse. Then $ABCD$ is ▱ by the definition of a ▱.

8. Use slopes to show that both pairs of opp. sides are $\parallel$; use the Distance Formula to show that both pairs of opp. sides are $\cong$; use slope and the Distance Formula to show that one pair of opp. sides are both $\parallel$ and $\cong$; use the Midpoint Formula to show that the diagonals bisect each other.

6.3 Practice and Applications (pp. 342–345)

9. Yes; if both pairs of opposite sides of a quad. are $\cong$, then the quad. is a ▱.

10. Yes; if the diagonals of a quad. bisect each other, then the quad. is a ▱.

11. No; by the Vertical $\angle$s Thm., this would be true for any quad.

12. Yes; if one $\angle$ of a quad. is supp. to both of its consecutive $\angle$s, then the quad. is a ▱.

13. No; one pair of opposite sides which are $\cong$ to each other and to a diagonal isn't sufficient to prove that the quad. is a ▱.

14. Yes; if one pair of opposite sides of a quad. are both $\cong$ and $\parallel$ then the quad. is a ▱.

15. *Sample answer:* Corresp. parts of $\cong$ $\triangle$s are $\cong$, so $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$. Since both pairs of opposite sides of $ABCD$ are $\cong$, $ABCD$ is a ▱.

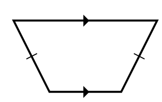
16. *Sample answer:* Since corresp. parts of $\cong$ $\triangle$s are $\cong$, $\angle ABX \cong \angle CDX$ and $\overline{AB} \cong \overline{CD}$. By the Alternate Interior $\angle$s Theorem, $\overline{AB} \parallel \overline{CD}$. One pair of opposite sides are both $\parallel$ and $\cong$, so $ABCD$ is a ▱.

17. $x = 70$

18.
$$2x° + x° = 180°$$
$$3x = 180$$
$$x = 60$$

19.
$$(x - 10)° + (x + 10)° = 180°$$
$$2x = 180$$
$$x = 90$$

20. *Sample answer:*



21.
$$AB = \sqrt{(3 - (-1))^2 + (5 - 6)^2} = \sqrt{4^2 + (-1)^2} = \sqrt{17}$$
$$BC = \sqrt{(5 - 3)^2 + (-3 - 5)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$
$$CD = \sqrt{(1 - 5)^2 + (-2 - (-3))^2} = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$$
$$AD = \sqrt{(1 - (-1))^2 + (-2 - 6)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$
$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

22. Midpoint of $\overline{AC} = \left(\frac{-1 + 5}{2}, \frac{6 + (-3)}{2}\right) = \left(2, \frac{3}{2}\right)$

Midpoint of $\overline{BD} = \left(\frac{3 + 1}{2}, \frac{5 + (-2)}{2}\right) = \left(2, \frac{3}{2}\right)$

Diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

23. Slope of $\overline{AB} = \frac{5-6}{3-(-1)} = -\frac{1}{4}$

Slope of $\overline{BC} = \frac{-3-5}{5-3} = -4$

Slope of $\overline{CD} = \frac{-2-(-3)}{1-5} = -\frac{1}{4}$

Slope of $\overline{AD} = \frac{-2-6}{1-(-1)} = -4$

$\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

24. Slope of $\overline{AB} = \frac{5-6}{3-(-1)} = -\frac{1}{4}$

Slope of $\overline{BC} = \frac{-3-5}{5-3} = -4$

Slope of $\overline{CD} = \frac{-2-(-3)}{1-5} = -\frac{1}{4}$

Slope of $\overline{DA} = \frac{-2-6}{1-(-1)} = -4$

$$AB = \sqrt{(3-(-1))^2 + (5-6)^2} = \sqrt{4^2 + (-1)^2} = \sqrt{17}$$

$$BC = \sqrt{(5-3)^2 + (-3-5)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$

$$CD = \sqrt{(1-5)^2 + (-2-(-3))^2} = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$$

$$DA = \sqrt{(1-(-1))^2 + (-2-6)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$$

$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$ or $\overline{BC} \parallel \overline{DA}$ and $\overline{BC} \cong \overline{DA}$.

25. *Sample answer:*

Midpoint of $\overline{JL} = \left(\frac{2+(-6)}{2}, \frac{-3+2}{2}\right) = \left(-2, -\frac{1}{2}\right)$

Midpoint of $\overline{KM} = \left(\frac{-1+(-3)}{2}, \frac{3+(-4)}{2}\right) = \left(-2, -\frac{1}{2}\right)$

The diagonals of *JKLM* bisect each other, so *JKLM* is a ▱ by Thm. 6.9.

26. *Sample answer:*

Slope of $\overline{PQ} = \frac{4-5}{8-2} = -\frac{1}{6}$

Slope of $\overline{QR} = \frac{-4-4}{9-8} = -8$

Slope of $\overline{RS} = \frac{-3-(-4)}{3-9} = -\frac{1}{6}$

Slope of $\overline{PS} = \frac{-3-5}{3-2} = -8$

Pairs of opposite sides have the same slope, so they are both ∥ and *PQRS* is a ▱.

27. $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. Since the opposite pairs of sides are ≅, *ABCD* is a ▱ and $\overline{AB} \parallel \overline{CD}$.

28. Corresp. ∠s are ≅, so the longer sides are ∥. Then both pairs of opp. sides are ∥ and the oblique I is a ▱ by definition.

29. If the diagonals of a quad. bisect each other, then the quad. is a ▱.

30. Check drawings. *Sample answers: By Theorem 6.6:* Draw two segments, $\overline{AB}$ and $\overline{BC}$ intersecting at *B*. Draw two arcs, one with center *A* and radius *BC* and one with center *C* and radius *AB*, intersecting at *D*. Draw $\overline{AD}$ and $\overline{CD}$. *By Theorem 6.8:* Draw two segments, $\overline{AB}$ and $\overline{BC}$ intersecting at *B*. Construct a line through *A* ∥ to $\overline{BC}$ and a line through *C* ∥ to $\overline{AB}$, intersecting at *D*. Draw $\overline{AD}$ and $\overline{CD}$. *By Theorem 6.10:* Draw two segments, $\overline{AB}$ and $\overline{BC}$ intersecting at *B*. Construct a line through *A* ∥ to $\overline{BC}$ and construct $\overline{AD}$ on the line ≅ to $\overline{BC}$. Draw $\overline{DC}$.

31. Make $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$. This will form a ▱ in which case $\overline{BC}$ will remain ∥ to $\overline{AD}$ while the binoculars are being raised or lowered.

32. The sum of the measures of the interior ∠s of a quad. is 360°, so $m\angle R + m\angle S + m\angle T + m\angle U = 360°$. It is given that $\angle R \cong \angle T$ and $\angle S \cong \angle U$, so $m\angle R = m\angle T$ and $m\angle S = m\angle U$ and, by the substitution property of equality, $m\angle T + m\angle S + m\angle T + m\angle S = 360°$. Then $2(m\angle S) + 2(m\angle T) = 360°$ and $m\angle S + m\angle T = 180°$. By the Consecutive Interior Angles Converse, $\overline{ST} \parallel \overline{RU}$. Similarly, $\overline{SR} \parallel \overline{TU}$ and *RSTU* is a ▱ by the def. of a ▱.

33.

Statements	Reasons
1. ∠*P* is supplementary to ∠*Q* and ∠*S*.	**1.** Given.
2. $\overline{QR} \parallel \overline{PS}$ and $\overline{QP} \parallel \overline{RS}$	**2.** Consecutive Interior ∠s Conv.
3. *PQRS* is a ▱.	**3.** Definition of a ▱

34. $(0, -a)$; the diagonals bisect each other at the origin. The length of $\overline{OM}$ is *a* so the length of $\overline{OP}$ must also be *a*, however, it is below the *x*-axis.

35. $(-b, -c)$; the diagonals of a ▱ bisect each other, so $(0, 0)$ is the midpoint of $\overline{QN}$. Let $Q = (x, y)$. By the Midpoint Formula, $(0, 0) = \left(\frac{x+b}{2}, \frac{y+c}{2}\right)$, so $x = -b$ and $y = -c$.

36. Slope of $\overline{MQ} = \frac{-c-a}{-b-0} = \frac{c+a}{b}$

Slope of $\overline{NP} = \frac{c-(-a)}{b-0} = \frac{c+a}{b}$

Slope of $\overline{MN} = \frac{c-a}{b-0} = \frac{c-a}{b}$

Slope of $\overline{QP} = \frac{-a-(-c)}{0-(-b)} = \frac{c-a}{b}$

Slope of $\overline{MQ}$ = slope of $\overline{NP} = \frac{c+a}{b}$ and slope of $\overline{MN}$ = slope of $\overline{QP} = \frac{c-a}{b}$.

37. a. $m\angle AEF + m\angle EAF + m\angle AFE = 180°$

$63° + 90° + m\angle AFE \approx 180°$

$m\angle AFF \approx 27°$

b. $m\angle FGD + m\angle GDF + m\angle DFG = 180°$

$m\angle FGD + 90° + 27° \approx 180°$

$m\angle FGD \approx 63°$

c. $m\angle GHC \approx 27°$; $m\angle EHB \approx 27°$

d. $m\angle E \approx 54°$, $m\angle F \approx 126°$, $m\angle G \approx 54°$, $m\angle H \approx 126°$; *EFGH* is a parallelogram because both pairs of opposite ∠s are ≅.

38. *PTRU* and *PQRS* share a common diagonal, $\overline{PR}$. *PTRU* and *QTSU* share a common diagonal, $\overline{TU}$. Since $\overline{PR}$ bisects $\overline{SQ}$ and $\overline{SQ}$ bisects $\overline{TU}$, $\overline{PR}$ must also bisect $\overline{TU}$. *PTRU is* a parallelogram by Thm 6.9.

6.3 Mixed Review (p. 345)

39. If $x^2 + 2 = 2$, then $x = 0$. If $x = 0$, then $x^2 + 2 = 2$.

40. If $4x + 7 = x + 37$, then $x = 10$. If $x = 10$, then $4x + 7 = x + 37$.

41. If a quadrilateral is a parallelogram, then each pair of opposite sides are parallel. If each pair of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.

42. A point is on the perpendicular bisector of a segment if and only if the point is equidistant from the endpoints of the segment.

43. A point is on the bisector of an angle if and only if the point is equidistant from the two sides of the angle.

44. The slope of the line through point $(1, -2)$ is $\frac{1}{4}$.

$y = mx + b$

$-z = \frac{1}{4}(1) + b$

$b = -\frac{9}{4}$

$y = \frac{1}{4}x - \frac{9}{4}$

45. $x° = 180° - (52° + 68°)$

$= 180 - 120$

$= 60$

46. $x° + (2x - 14)° + 50° = 180°$

$3x + 36 = 180$

$3x = 144$

$x = 48$

47. $x° + 85° = (2x + 50)°$

$-x = -35$

$x = 35$

Quiz 1 (p. 346)

1. convex, equilateral, equiangular, regular

2. $2x° + 2x° + 110° + 110° = 360°$

$4x + 220 = 360$

$4x = 140$

$x = 35$

The sum of the measures of the interior angles of a quadrilateral is 360°.

3.

Statements	Reasons
1. *ABCG* and *CDEF* are ▱s.	**1.** Given.
2. $\angle A \cong \angle BCG$, $\angle DCF \cong \angle E$	**2.** If a quad. is a ▱, then the opposite ∠s are ≅.
3. $\angle BCG \cong \angle DCF$	**3.** Vertical Angles Theorem
4. $\angle A \cong \angle E$	**4.** Transitive Property of Congruence.

4. *Sample answer:* Use slopes to show that both pairs of opposite sides are parallel, use the Distance Formula to show that both pairs of opposite sides are congruent, use slope and the Distance Formula to show that one pair of opposite sides are both parallel and congruent, use the Midpoint Formula to show that the diagonals bisect each other.

Math and History (p. 346)

1. $A = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(2800 + 1800)(500)$

$= 1{,}150{,}000 \text{ ft}^2$

6.4 Guided Practice (p. 351)

1. rhombus

2. If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. If each diagonal of a parallogram bisects a pair of opposite angles, then the parallelogram is a rhombus. Diagonal $\overline{PR}$ bisects $\angle SPQ$ and $\angle SRQ$, and $\overline{SQ}$ bisects $\angle PSR$ and $\angle PQR$.

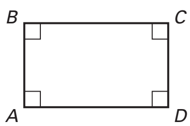
3. always **4.** sometimes **5.** sometimes **6.** always

7. C, D **8.** B, D **9.** B, D **10.** A, B, C, D

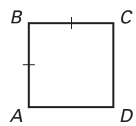
11. $2x° = 90°$

$x = 45$

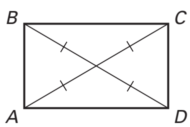
6.4 Practice and Applications (pp. 351–355)

12. 

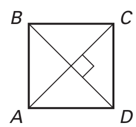
Always; a rectangle has four right angles and right angles are congruent.

13. 

Sometimes; if rectangle *ABCD* is also a rhombus (a square), then $\overline{AB} \cong \overline{BC}$.

14. 

Always; the diagonals of a rectangle are congruent.

15. 

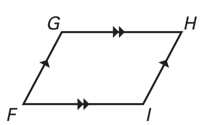
Sometimes; if rectangle *ABCD* is also a rhombus (a square), then the diagonals of *ABCD* are $\perp$.

16. rectangle, square **17.** square **18.** rhombus, square

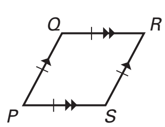
19. parallelogram, rectangle, rhombus, square

20. parallelogram, rectangle, rhombus, square

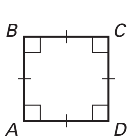
21. rhombus, square

22. 

$\overline{FG} \cong \overline{HI}$, $\overline{FI} \cong \overline{GH}$, $\overline{FG} \parallel \overline{HI}$, $\overline{GH} \parallel \overline{FI}$, $\angle G \cong \angle I$, $\angle F \cong \angle H$, $\overline{FH}$ and $\overline{GI}$ bisect each other.

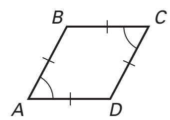
23. 

$\overline{PQ} \parallel \overline{RS}$, $\overline{PS} \parallel \overline{QR}$, $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$, $\angle P \cong \angle R$, $\angle S \cong \angle Q$, $\overline{PR}$ and $\overline{QS}$ bisect each other, $\overline{QS} \perp \overline{RP}$, $\overline{QS}$ bisects $\angle PSR$ and $\angle PQR$. $\overline{PR}$ bisects $\angle SPQ$ and $\angle SRQ$.

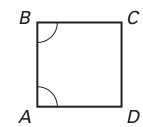
24. 

Pairs of opposite sides are $\parallel$, all four sides are congruent, all four angles are right angles; the diagonals are congruent, bisect each other, and bisect opposite angles.

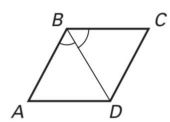
25. rectangle **26.** square

27. 

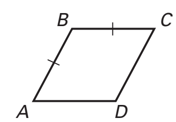
Always; if a quad. is a ▱, then the opp. ∠s are ≅.

28. 

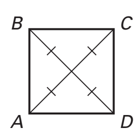
Sometimes; if a rhombus is also a rectangle (a square) then all four angles are congruent.

29. 

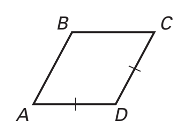
Always; each diagonal of a rhombus bisects a pair of opposite ∠s.

30. 

Always; a rhombus is a parallelogram with four congruent sides.

31. 

Sometimes; if a rhombus is also a rectangle (square) then its diagonals are congruent.

32. 

Always; a rhombus is a parallelogram with four congruent sides.

33. $5x° = 90°$

$x = 18$

34. $x° = 180° - 130°$

$x = 50$

35. $(x + 40)° + (2x - 10)° = 180°$

$3x + 30 = 180$

$3x = 150$

$x = 50$

36. $2x = 10$

$x = 5$

37. $3x = x + 2$

$2x = 2$

$x = 1$

38. $8x - 13 = 7x + 11$

$x = 24$

39. $2\sqrt{2}$ **40.** $90°$ **41.** $45°$

42. $P = HJ + KJ + KH = 2 + 2 + 2\sqrt{2} = 4 + 2\sqrt{2}$

43. $24 = 5x - 1 + 13 - x$

$12 = 4x$

$3 = x$

$WY = XZ = 13 - 3 = 10$

44. *Sample answer:* You would need to know that $\angle A$, $\angle B$, $\angle C$, or $\angle D$ is a right angle.

45. Assume temporarily that $\overline{MN} \parallel \overline{PQ}$, $\angle 1 \not\cong \angle 2$, and that $\overline{MQ} \parallel \overline{PN}$. By the definition of a ▱, *MNPQ* is a ▱. This contradicts the given information that $\angle 1 \not\cong \angle 2$. It follows that $\overline{MQ}$ is not parallel to $\overline{NP}$.

46.

Statements	Reasons
1. $RSTU$ is a ▱, $\overline{SU} \perp \overline{RT}$.	**1.** Given
2. $RSTU$ is a rhombus.	**2.** A ▱ is a rhombus if its diagonals are $\perp$.
3. $\angle STR \cong \angle UTR$	**3.** Each diagonal of a rhombus bisects a pair of opp. $\angle$s.

47. If a ▱ is a rectangle, then its diagonals are congruent. If the diagonals of a ▱ are congruent, then the ▱ is a rectangle. $\overline{JL} \cong \overline{KM}$

48. If a quadrilateral is a rhombus, then it has four congruent sides. (A rhombus by definition has four congruent sides.)

If a quadrilateral has four congruent sides, then it is a rhombus. (Both pairs of opposite sides are congruent, so the quadrilateral is a ▱. By definition a ▱ with four congruent sides is a rhombus.)

49. If a quadrilateral is a rectangle, then it has four right angles. (def. of a rectangle)

If a quadrilateral has four right angles, then it is a rectangle. (Both pairs of opp. $\angle$s are $\cong$, so the quad. is a ▱. By definition, a ▱ with 4 right $\angle$s is a rectangle.)

50. If a quadrilateral is a square, then it is a rhombus and a rectangle. (A square has four $\cong$ sides, which makes it a rhombus and four rt. $\angle$s, which makes it a rectangle.)

If a quadrilateral is a rhombus and a rectangle, then it is a square. (By definition, a rhombus has four $\cong$ sides and a rectangle has four rt. $\angle$s. The only quad. that has four $\cong$ sides and four rt. $\angle$s is a square.)

51. *Sample answer:*

Statements	Reasons
1. $PQRT$ is a rhombus.	**1.** Given
2. $\overline{PQ} \cong \overline{PT} \cong \overline{QR} \cong \overline{RT}$	**2.** A quad. is a rhombus if and only if it has 4 $\cong$ sides.
3. $\overline{PR} \cong \overline{PR}$, $\overline{QT} \cong \overline{QT}$	**3.** Reflexive Prop. of Congruence
4. $\triangle PRQ \cong \triangle PRT$; $\triangle PTQ \cong \triangle RTQ$	**4.** SSS Cong. Postulate
5. $\angle TPR \cong \angle QPR$, $\angle TRP \cong \angle QRP$; $\angle PTQ \cong \angle RTQ$, $\angle PQT \cong \angle RQT$	**5.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
6. $\overline{PR}$ bisects $\angle TPQ$ and $\angle QRT$, $\overline{QT}$ bisects $\angle PTR$ and $\angle RQP$.	**6.** Def. of $\angle$ bisector

52.

Statements	Reasons
1. $FGHJ$ is a ▱, $\overline{FH}$ bisects $\angle JFG$ and $\angle GHJ$, $\overline{JG}$ bisects $\angle FJH$ and $\angle HGF$.	**1.** Given
2. $\angle HFG \cong \angle HFJ$, $\angle FHG \cong \angle FHJ$; $\angle FGJ \cong \angle HGJ$, $\angle FJG \cong \angle HJG$	**2.** Def. of $\angle$ bisector
3. $\overline{FH} \cong \overline{FH}$, $\overline{GJ} \cong \overline{GJ}$	**3.** Reflexive Prop. of Congruence
4. $\triangle FHJ \cong \triangle FHG$, $\triangle FGJ \cong \triangle HGJ$	**4.** ASA Cong. Post.
5. $\overline{JH} \cong \overline{GH}$, $\overline{FG} \cong \overline{GH}$, $\overline{JH} \cong \overline{FJ}$, $\overline{FG} \cong \overline{FJ}$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{JH} \cong \overline{FG} \cong \overline{GH} \cong \overline{FJ}$	**6.** Transitive Property of Congruence
7. $FGHJ$ is a rhombus.	**7.** Def. of a rhombus

53. *Sample answer:* Draw a line f and $\overline{GH}$ where f is not $\perp$ to $\overline{GH}$. f should intersect $\overline{GH}$ at H. Construct $\overline{HJ}$ on f so that $\overline{HJ} \cong \overline{GH}$. With centers G and J, construct two arcs with radius GH which intersect at K. Draw $\overline{GK}$ and $\overline{JK}$. Since all four sides of $GHJK$ are $\cong$, $GHJK$ is a rhombus. It is not a square or rectangle because $\overline{GH}$ and $\overline{HJ}$ are not $\perp$.

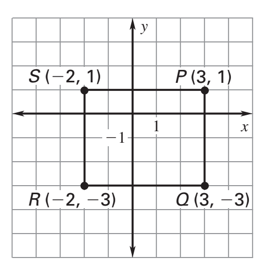
54. *Sample answer:* Draw a segment $\overline{AB}$. Construct lines $\perp$ to $\overline{AB}$ at A and B. These lines will be $\parallel$. Choose D on the $\perp$ at A such that $AD \neq AB$. Construct $\overline{BC}$ on the $\perp$ at B such that $\overline{BC} \cong \overline{AD}$. Draw $\overline{DC}$. Since $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$, $ABCD$ is a ▱. $\angle D$ is supplementary to $\angle A$ (Consecutive Int. $\angle$s Thm.) so $\angle D$ is a right angle. Similarly, $\angle C$ is a right angle. Then $ABCD$ is a rectangle. It is not a square because $AD \neq AB$.

55. Rectangle;

$$PR = \sqrt{[3-(-2)]^2 + [1-(-3)]^2} = \sqrt{5^2 + 4^2} = \sqrt{41}$$

$$QS = \sqrt{[3-(-2)]^2 + (-3-1)^2} = \sqrt{5^2 + (-4)^2} = \sqrt{41}$$

If the diagonals of a ▱ are $\cong$, then the ▱ is a rectangle.

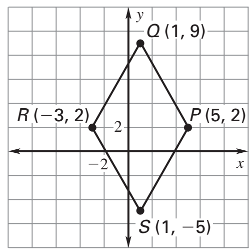


56. Rhombus; slope of $\overline{PR} = \frac{2-2}{5-(-3)} = \frac{0}{8} = 0$

slope of $\overline{QS} = \frac{9-(-5)}{1-1} = \frac{14}{0}$, which is undefined

$\overline{PR} \perp \overline{QS}$

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

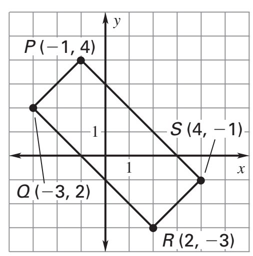


57. Rectangle;

$PR = \sqrt{(-1-2)^2 + (4-(-3))^2}$
$= \sqrt{(-3)^2 + 7^2}$
$= \sqrt{58}$

$QS = \sqrt{(-3-4)^2 + [2-(-1)]^2}$
$= \sqrt{(-7)^2 + 3^2}$
$= \sqrt{58}$

If the diagonals of a parallelogram are ≅, then the ▱ is a rectangle.



58. Square;

$PQ = \sqrt{(5-2)^2 + (2-5)^2}$
$= \sqrt{3^2 + (-3)^2}$
$= 3\sqrt{2}$

$QR = \sqrt{[2-(-1)]^2 + (5-2)^2}$
$= \sqrt{3^2 + 3^2}$
$= 3\sqrt{2}$

$RS = \sqrt{(-1-2)^2 + [2-(-1)]^2}$
$= \sqrt{(-3)^2 + 3^2}$
$= 3\sqrt{2}$

$SP = \sqrt{(2-5)^2 + (-1-2)^2}$
$= \sqrt{b(-3)^2 + (-3)^2}$
$= 3\sqrt{2}$

So, $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$.

slope of $\overline{PQ} = \frac{5-2}{2-5} = -1$

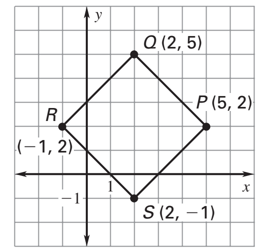
slope of $\overline{QR} = \frac{5-2}{2-(-1)} = 1$

slope of $\overline{PS} = \frac{2+1}{5-2} = \frac{3}{3} = 1$

slope of $\overline{RS} = \frac{2+1}{-1-2} = \frac{3}{-3} = -1$

So, $\overline{PQ} \perp \overline{QR}$, $\overline{PS} \perp \overline{RS}$.

A quadrilateral is a square if it has 4 congruent sides and 4 right angles.



59. (b, a); $\overline{KM} \cong \overline{ON}$, so $KM = b$ and $\overline{KO} \cong \overline{MN}$, so $MN = a$.

60. $OM = \sqrt{(b-0)^2 + (a-0)^2}$
$= \sqrt{b^2 + a^2}$

$KN = \sqrt{(b-0)^2 + (0-a)^2}$
$= \sqrt{b^2 + a^2}$

$\overline{OM} \cong \overline{KN}$

61. *Sample answer: ABDC* is a ▱ since cross braces $\overline{AD}$ and $\overline{BC}$ bisect each other. $\overline{AD} \cong \overline{BC}$ so *ABDC* is a rectangle. Since a rectangle has four right angles, $m\angle CAB = m\angle DBA = 90°$. Then, $m\angle BAC = m\angle BAE$ and $m\angle ABD = m\angle ABF$. By substitution $m\angle BAE = m\angle ABF = 90°$. So tabletop $\overline{AB}$ is perpendicular to legs $\overline{AE}$ and $\overline{BF}$ by the def. of perpendicular.

62. *Sample answer:* Since *ABDC* is a rectangle, $\overline{AC} \parallel \overline{BD}$. *A*, *C*, and *E* are collinear as are *B*, *D*, and *F* so $\overline{AE} \parallel \overline{BF}$. Since $\overline{AE} \cong \overline{BF}$, *ABFD* is a ▱ and $\overline{AB} \parallel \overline{EF}$.

63. Rhombus; $\overline{AE} \cong \overline{CE} \cong \overline{AF} \cong \overline{CF}$; *AECF* remains a rhombus; *AECF* remains a rhombus.

64. When *A* or *C* is dragged, $m\angle FAC$ and $m\angle EAC$ remain equal, as do $m\angle AEF$ and $m\angle CEF$.

65. Each diagonal of a rhombus bisects a pair of opp. ⦣s. (Theorem 6.12)

66. D $\quad 7x - 3 = 4x + 9$
$3x = 12$
$x = 4$

67. B $\quad (9x + 9)° = 90° \qquad 2(xy)° = 90°$
$9x = 81 \qquad 2(9)(y) = 90$
$x = 9 \qquad y = 5$

68. In a parallelogram, opposite ⦣s are ≅ and consecutive ⦣s are supp. Therefore, all four ⦣s must measure 90° in which case, the ▱ would be a rectangle.

69. The diagonals of a ▱ bisect each other. So, $OA = OC$ and $OD = OB$. $AO + OC = AC$ and $DO + OB = DB$, then $AO + OC = DO + OB$ by substitution. $AO + AO = DO + DO$ and $OC + OC = OB + OB$, again by substitution, giving $AO = DO$ and $OC = OB$. By the Transitive Property of Congruence, $OA = OB = OC = OD$.

70. $OB = \sqrt{(0-b)^2 + (0-0)^2} = \sqrt{b^2} = b$

Since $OB = OA$, $OA = b$ and let $A = (a, y)$.

$$b = \sqrt{(a-0)^2 + (y-0)^2}$$
$$b = \sqrt{a^2 + y^2}$$
$$b^2 = a^2 + y^2$$
$$b^2 - a^2 = y^2$$
$$\sqrt{b^2 - a^2} = y \text{ (The } y\text{-coordinate of } A \text{ is positive.)}$$
$$A\left(a, \sqrt{b^2 - a^2}\right)$$

71. Since $OB = OD$, $OD = b$, and $\overline{OD}$ lies on the x-axis, $D = (-b, 0)$. Since $OA = OC$, $OC = b$, and C is in quadrant III, the coordinates of C must be negative. So, $C = \left(-a, -\sqrt{b^2 - a^2}\right)$.

72. Slope of $\overline{AB}$: $\dfrac{\sqrt{b^2 - a^2} - 0}{a - b} = \dfrac{\sqrt{b^2 - a^2}}{a - b}$

Slope of $\overline{BC}$: $\dfrac{0 - \left(-\sqrt{b^2 - a^2}\right)}{b - (-a)} = \dfrac{\sqrt{b^2 - a^2}}{b + a}$

$$\left(\frac{\sqrt{b^2 - a^2}}{a - b}\right)\left(\frac{\sqrt{b^2 - a^2}}{b + a}\right) = \frac{b^2 - a^2}{(a-b)(a+b)}$$
$$= \frac{(b+a)(b-a)}{(a-b)(a+b)}$$
$$= \frac{-(a-b)}{a-b} = -1$$

$\overline{AB} \perp \overline{BC}$

Slope of $\overline{CD}$: $\dfrac{-\sqrt{b^2 - a^2} - 0}{-a - (-b)} = \dfrac{-\sqrt{b^2 - a^2}}{-a + b}$

Slope of $\overline{DA}$: $\dfrac{\sqrt{b^2 - a^2} - 0}{a - (-b)} = \dfrac{\sqrt{b^2 - a^2}}{a + b}$

$$\left(\frac{-\sqrt{b^2 - a^2}}{-a + b}\right)\left(\frac{\sqrt{b^2 - a^2}}{a + b}\right) = -\frac{(b^2 - a^2)}{(-a+b)(a+b)}$$
$$= -\frac{(b+a)(b-a)}{(-a+b)(a+b)}$$
$$= -1$$

$\overline{CD} \perp \overline{DA}$

$ABCD$ is a parallelogram with four right angles, that is, $ABCD$ is a rectangle.

6.4 Mixed Review (p. 355)

73. yes **74.** no **75.** no **76.** no **77.** yes **78.** yes

79. $AP = \frac{2}{3}AD$

$1 = \frac{2}{3}AD$

$\frac{3}{2} = AD$

$PD = AD - AP$

$PD = \frac{3}{2} - 1$

$= \frac{1}{2}$

80. $PC = \frac{2}{3}CE$

$6.6 = \frac{2}{3}CE$

$9.9 = CE$

$EP + PC = CE$

$EP + 6.6 = 9.9$

$EP = 3.3$

81. $PB = \frac{2}{3}FB$

$6 = \frac{2}{3}FB$

$9 = FB$

82. $PD = \frac{1}{3}AD$

$= \frac{1}{3}(39)$

$= 13$

83. Assume temporarily that $ABCD$ is a quad. with 4 acute ∠s, that is, $m\angle A < 90°$, $m\angle B < 90°$, $m\angle C < 90°$, and $m\angle D < 90°$. Then $m\angle A + m\angle B + m\angle C + m\angle D < 360°$. This contradicts the Interior Angles of a Quad. Theorem. Therefore, no quad. has 4 acute ∠s.

Lesson 6.5

6.5 Guided Practice (p. 359)

1. $\overline{CD}$ and $\overline{AB}$

2. A kite is a quadrilateral that has two pairs of consecutive congruent sides, but opp. sides are not ≅. In a rhombus, both pairs of opp. sides are ≅, so a rhombus can't be a kite.

3. isosceles trapezoid **4.** kite **5.** trapezoid

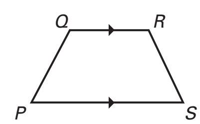
6. Use coordinates to show that $\overline{AC} \cong \overline{BD}$ or $\overline{AD} \cong \overline{BC}$.

7. $\frac{1}{2}(11 + 7) = \frac{1}{2}(18) = 9$

8. $\frac{1}{2}(7 + 3) = \frac{1}{2}(10) = 5$ **9.** $\frac{1}{2}(12 + 7) = \frac{1}{2}(19) = \frac{19}{2}$

6.5 Practice and Applications (pp. 359–362)

Figure for Exs. 10–15:



10. bases **11.** legs **12.** consecutive sides

13. diagonals **14.** opposite angles **15.** base angles

16. $m\angle M = m\angle J = 44°$, $m\angle K = 180° - 44° = 136°$; $m\angle L = m\angle K = 136°$

17. $m\angle J = 180° - 78° = 102°$; $m\angle L = 180° - 132° = 48°$

18. $m\angle J = m\angle M = 82°$, $m\angle L = 180° - 82° = 98°$; $m\angle K = m\angle L = 98°$

19. $MN = \frac{1}{2}(7 + 9) = \frac{1}{2}(16) = 8$

20. $MN = \frac{1}{2}(16 + 14) = \frac{1}{2}(30) = 15$

21. $MN = \frac{1}{2}(15 + 9) = \frac{1}{2}(24) = 12$

22. $7 = \frac{1}{2}(x + 9)$

$14 = x + 9$

$5 = x$

23. $7 = \frac{1}{2}(x + 4)$

$14 = x + 4$

$10 = x$

24. $8 = \frac{1}{2}(11 + x)$

$16 = 11 + x$

$5 = x$

25. Yes; X is equidistant from the vertices of the dodecagon, so $\overline{XA} \cong \overline{XB}$ and $\angle XAB \cong \angle XBA$ by the Base Angles Theorem. Since trapezoid $ABPQ$ has a pair of congruent base angles, $ABPQ$ is isosceles.

26. $m\angle AXB = \frac{360°}{12} = 30°$

27. In $\triangle ABX$, $m\angle AXB = 30°$, and $\angle A \cong \angle B$, so

$m\angle A = m\angle B = 75°$, $m\angle P = 180° - 75° = 105°$;
$m\angle Q = m\angle P = 105°$

28. $AD = AB = \sqrt{2^2 + 3^2}$
$= \sqrt{13}$
≈ 3.61

$BC = DC = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

29. $EH = GH = \sqrt{5^2 + 7^2}$
$= \sqrt{74}$
≈ 8.60

$EF = GF = \sqrt{5^2 + 4^2}$
$= \sqrt{41}$
≈ 6.40

30. $JK = JM = \sqrt{12^2 + 8^2}$
$= \sqrt{208}$
≈ 14.42

$KL = LM = \sqrt{8^2 + 5^2}$
$= \sqrt{89}$
≈ 9.43

31. $m\angle E = m\angle G$

$2m\angle G = 360° - (120° + 50°)$

$2m\angle G = 190°$

$m\angle G = 95°$

32. $m\angle G = 360° - (110° + 90° + 90°)$
$= 70°$

33. $m\angle F = m\angle H = 100°$

$m\angle G = 360° - (100° + 100° + 70°)$
$= 90°$

34. A trapezoid has exactly one pair of parallel opposite sides. Because a ▱ has two pairs of parallel opposite sides, *ABCD* cannot be a trapezoid.

35. any two positive numbers (except 5 and 5) whose sum is 10

36. $AB = \sqrt{(-3 - 4)^2 + (3 - 5)^2}$
$= \sqrt{(-7)^2 + (-2)^2}$
$= \sqrt{53}$

$BC = \sqrt{(-3 - (-6))^2 + (3 - (-13))^2}$
$= \sqrt{3^2 + 16^2}$
$= \sqrt{265}$

$CD = \sqrt{(-6 - 6)^2 + (-13 - (-2))^2}$
$= \sqrt{(-12)^2 + (-11)^2}$
$= \sqrt{265}$

$DA = \sqrt{(6 - 4)^2 + (-2 - 5)^2}$
$= \sqrt{2^2 + (-7)^2}$
$= \sqrt{53}$

$\overline{AB} \cong \overline{DA}$, so $\overline{AB}$ and $\overline{DA}$ are consecutive ≅ sides.

$\overline{DC} \cong \overline{BC}$, so $\overline{DC}$ and $\overline{BC}$ are consecutive ≅ sides.

So, the points are vertices of a kite.

37. Slope of $\overline{AB} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$

Slope of $\overline{BC} = \frac{4 - 4}{5 - 0} = \frac{0}{5} = 0$

Slope of $\overline{CD} = \frac{0 - 4}{8 - 5} = -\frac{4}{3}$

Slope of $\overline{AD} = \frac{0 - 0}{8 - (-2)} = \frac{0}{10} = 0$

$AB = \sqrt{(-2 - 0)^2 + (0 - 4)^2}$
$= \sqrt{(-2)^2 + (-4)^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

$CD = \sqrt{(5 - 8)^2 + (4 - 0)^2}$
$= \sqrt{(-3)^2 + 4^2}$
$= \sqrt{25} = 5$

ABCD is a trapezoid; slope of $\overline{BC} =$ slope of $\overline{AD} = 0$, so $\overline{BC} \parallel \overline{AD}$; slope of $\overline{AB} = 2$ and slope of $\overline{CD} = -\frac{4}{3}$, so $\overline{AB}$ is not ∥ to $\overline{CD}$. *ABCD* is not isosceles; $AB = 2\sqrt{5}$ and $CD = 5$.

38. Slope of $\overline{EF} = \frac{2 - 9}{4 - 1} = -\frac{7}{3}$

Slope of $\overline{FG} = \frac{2 - 2}{5 - 4} = \frac{0}{1} = 0$

Slope of $\overline{GH} = \frac{9 - 2}{8 - 5} = \frac{7}{3}$

Slope of $\overline{EH} = \frac{9 - 9}{8 - 1} = \frac{0}{7} = 0$

$EF = \sqrt{(4 - 1)^2 + (2 - 9)^2}$
$= \sqrt{3^2 + (-7)^2}$
$= \sqrt{58}$

$GH = \sqrt{(8 - 5)^2 + (9 - 2)^2}$
$= \sqrt{3^2 + 7^2}$
$= \sqrt{58}$

ABCD is a trapezoid; slope of $\overline{FG} =$ slope of $\overline{EH} = 0$, so $\overline{FG} \parallel \overline{EH}$; slope of $\overline{EF} = -\frac{7}{3}$ and slope of $\overline{GH} = \frac{7}{3}$, so $\overline{EF}$ is not ∥ to $\overline{GH}$. *ABCD* is isosceles; $EF = GH = \sqrt{58}$.

39. $BE = \frac{1}{2}(CD + AF)$
$= \frac{1}{2}(10 + 22)$
$= \frac{1}{2}(32)$
$= 16$ in.

40. It is given that *ABCD* is an isosceles trapezoid with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. If $\overline{AE}$ is drawn so that *ABCE* is a parallelogram, since opposite sides of a parallelogram are congruent, $\overline{BC} \cong \overline{AE}$ and by the Transitive Property of Congruence, $\overline{AE} \cong \overline{AD}$. By the Base Angles Theorem $\angle D \cong \angle AED$. Because $\overline{AE} \parallel \overline{BC}$, $\angle AED \cong \angle C$. Then $\angle D \cong \angle C$ by the Transitive Property of Congruence. By the Consecutive Interior Angles Theorem,

—CONTINUED—

40. **—CONTINUED—**
$m\angle B + m\angle C = 180°$ and $m\angle DAB + m\angle D = 180°$. By the Substitution property of equality, $m\angle B + m\angle C = m\angle DAB + m\angle D$; $m\angle B + m\angle C = m\angle DAB + m\angle C$. Finally $m\angle B = m\angle DAB$ by the Subtraction property of equality. So, $\angle B \cong \angle DAB$ by definition of congruence.

41. $TQRS$ is an isosceles trapezoid so $\angle QTS \cong \angle RST$ because base ∠s of an isosceles trapezoid are ≅. $\overline{QT} \cong \overline{RS}$ and by the Reflexive Prop. of Cong., $\overline{TS} \cong \overline{TS}$, $\triangle QTS \cong \triangle RST$ by the SAS Congruence Postulate. Then $\overline{TR} \cong \overline{SQ}$ because corresp. parts of ≅ triangles are ≅.

42. $BG = \frac{1}{2}CD$ and $\overline{BG} \parallel \overline{CD}$ by the Midsegment Thm. and $EG = \frac{1}{2}AF$ and $\overline{EG} \parallel \overline{AF}$ by the Midsegment Thm. $BE = BG + GE$, so by substitution, $BE = \frac{1}{2}CD + \frac{1}{2}AF = \frac{1}{2}(CD + AF)$. $\overline{BE}$ is parallel to both $\overline{CD}$ and $\overline{AF}$ because $\overline{BG}$, $\overline{GE}$, and $\overline{BE}$ all lie on the same line.

43. If $AC \neq BC$ then $ACBD$ is a kite; $AC = AD$ and $BC = BD$, so the quadrilateral has two pairs of congruent sides, but opposite sides are not congruent. (If $AC = BC$ then $ABDC$ is a rhombus.) $ABDC$ remains a kite in all 3 cases.

44. The ∠s are ≅; the measures of the ∠s change, but the ∠s remain ≅.

45. If a quad. is a kite, then exactly one pair of opp. ∠s are ≅. (Theorem 6.19)

46.

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$	**1.** Given
2. $\overline{BD} \cong \overline{BD}$	**2.** Reflexive Prop. of Congruence
3. $\triangle BCD \cong \triangle BAD$	**3.** SSS Congruence Post.
4. $\angle CBX \cong \angle ABX$	**4.** Corresp. parts of ≅ △s are ≅.
5. $\overline{BX} \cong \overline{BX}$	**5.** Reflexive Prop. of Congruence
6. $\triangle CBX \cong \triangle ABX$	**6.** SAS Congruence Post.
7. $\angle CXB \cong \angle AXB$	**7.** Corresp. parts of ≅ △s are ≅.
8. $\angle CXB$ and $\angle AXB$ are a linear pair.	**8.** Def. of linear pair
9. $\overline{AC} \perp \overline{BD}$	**9.** If two lines intersect to form a linear pair of congruent angles, then the lines are ⊥.

47. Draw $\overline{BD}$. (Through any 2 points, there is exactly 1 line.) Since $\overline{BC} \cong \overline{BA}$ and $\overline{CD} \cong \overline{AD}$, $\triangle BCD \cong \triangle BAD$ by the SSS Congruence Postulate. $\angle A \cong \angle C$ because corresp. parts of ≅ △s are ≅. Assume temporarily that $\angle B \cong \angle D$, then both pairs of opp. ∠s of $ABCD$ would be ≅ making $ABCD$ a parallelogram. This contradicts the def. of a kite. Therefore $\angle B \not\cong \angle D$.

48. Yes; The legs are ≅ and it has one pair of ∥ sides.

49. Yes; The diagonals are ≅ and it has one pair of ∥ sides.

50. Yes; $\angle A \cong \angle B$ and $\angle D \cong \angle C$. Then $m\angle A = m\angle B$ and $m\angle D = m\angle C$. By the Interior ∠s of a Quad. Theorem, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$. By substitution and the properties of equality, $m\angle A + m\angle D = 180°$. By the Consecutive ∠s Converse, $\overline{AB} \parallel \overline{DC}$. $ABCD$ is not a parallelogram because it is given that $\angle A \not\cong \angle C$ so opposite ∠s are not ≅. So $ABCD$ is a trapezoid and since it has a pair of congruent base angles, then it is an isosceles trapezoid.

51. $15 = \frac{1}{2}(3x + 2 + 2x - 2)$ **52.** C

$30 = 5x$

$6 = x$

E

53. In trapezoid $PQRS$, $\overline{QR} \parallel \overline{PS}$. Draw a perpendicular segment from Q to $\overline{PS}$ and label the point of intersection M. Draw a perpendicular segment from R to $\overline{PS}$ and label the point of intersection N. $\angle QMP \cong \angle RNS$ because perpendicular lines form right angles and all right angles are congruent. In a plane, if two lines are perpendicular to the same line, then they are parallel to each other, so $\overline{QM} \parallel \overline{RN}$. $\overline{MN}$ lies on $\overline{PS}$, so $\overline{QR} \parallel \overline{MN}$. Therefore, $QRNM$ is a parallelogram, because both pairs of opposite sides are parallel, and $\overline{QM} \cong \overline{RN}$. We know that $\triangle QMS$ and $\triangle RNP$ are right triangles, and it is given that $\overline{PR} \cong \overline{SQ}$, so $\triangle QMS \cong \triangle RNP$ by the HL Congruence Theorem. $\angle QSP \cong \angle RPS$ because corresponding parts of congruent triangles are congruent. $\overline{PS} \cong \overline{PS}$ by the Reflexive Property of Congruence. $\triangle QPS \cong \triangle RSP$ by the SAS Congruence Postulate. Therefore, $\overline{QP} \cong \overline{RS}$ because corresponding parts of congruent triangles are congruent.

6.5 Mixed Review (p. 363)

54. If a triangle is scalene, then it has no congruent sides.

55. If a quadrilateral is a kite, then it has perpendicular diagonals.

56. If a polygon is a pentagon, then it has five sides.

57. 5.6 **58.** 10 **59.** 7 **60.** 11.2 **61.** 80° **62.** 100°

63. Yes; *Sample answer:*

Slope of $\overline{AB} = \frac{8 - 8}{5 + 2} = \frac{0}{7} = 0$

Slope of $\overline{BC} = \frac{8 - 0}{5 - 2} = \frac{8}{3}$

Slope of $\overline{CD} = \frac{0 - 0}{2 + 5} = \frac{0}{7} = 0$

Slope of $\overline{AD} = \frac{8 - 0}{-2 - (-5)} = \frac{8}{3}$

Both pairs of opposite sides are parallel so $ABCD$ is a parallelogram.

64. Yes; *Sample answer:*

$PQ = \sqrt{(4 - 9)^2 + (-3 - (-1))^2}$

$= \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$

$QR = \sqrt{(9 - 8)^2 + (-1 - (-6))^2}$

$= \sqrt{1^2 + 5^2} = \sqrt{26}$

$RS = \sqrt{(8 - 3)^2 + (-6 - (-8))^2}$

$= \sqrt{5^2 + 2^2} = \sqrt{29}$

$PS = \sqrt{(4 - 3)^2 + (-3 - (-8))^2}$

$= \sqrt{1^2 + 5^2} = \sqrt{26}$

Both pairs of opposite sides are congruent, so *PQRS* is a parallelogram.

Quiz 2 (p. 363)

1. *Sample answer:* Opposite sides of *EBFJ* are ≅, so *EBFJ* is a ▱. Opposite angles of a ▱ are ≅, so $\angle BFJ \cong \angle BEJ$. $\angle KFJ \cong \angle JEH$ by the Congruent Supplements Thm. Since $\overline{HE} \cong \overline{JE} \cong \overline{JF} \cong \overline{KF}$, $\triangle JFK \cong \triangle HEJ$ by the SAS Congruence Postulate and since corresponding parts of congruent triangles are congruent, $\overline{HJ} \cong \overline{JK}$.

2. $PQ = \sqrt{(2 - (-4))^2 + (5 - 5)^2} = \sqrt{36} = 6$

$RS = \sqrt{(2 - (-4))^2 + (-7 - (-7))^2} = \sqrt{36} = 6$

$PR = \sqrt{(2 - 2)^2 + (5 - (-7))^2} = \sqrt{144} = 12$

$QS = \sqrt{(-4 - (-4))^2 + (-7 - 5)^2} = \sqrt{144} = 12$

$QR = \sqrt{(-4 - 2)^2 + (5 - (-7))^2}$

$= \sqrt{(-6)^2 + 12^2}$

$= \sqrt{180}$

$= 6\sqrt{5}$

$PS = \sqrt{(2 - (-4))^2 + (5 - (-7))^2}$

$= \sqrt{6^2 + 12^2}$

$= \sqrt{180}$

$= 6\sqrt{5}$

Opposite sides are congruent and diagonals are congruent, so *PQSR* is a rectangle.

3. $AB = \sqrt{(-3 - 0)^2 + (6 - 9)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(0 - 3)^2 + (9 - 6)^2}$

$= \sqrt{(-3)^2 + 3^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$CD = \sqrt{(3 - 0)^2 + (6 - (-10))^2}$

$= \sqrt{3^2 + 16^2}$

$= \sqrt{265}$

$AD = \sqrt{(-3 - 0)^2 + (6 - (-10))^2}$

$= \sqrt{(-3)^2 + 16^2}$

$= \sqrt{265}$

Two pairs of consecutive sides are congruent, so *ABCD* is a kite.

4. $JK = \sqrt{(-5 - (-4))^2 + (6 - (-2))^2}$

$= \sqrt{(-1)^2 + 8^2}$

$= \sqrt{65}$

$KL = \sqrt{(-4 - 4)^2 + (-2 - (-1))^2}$

$= \sqrt{(-8)^2 + (-1)^2}$

$= \sqrt{65}$

$LM = \sqrt{(4 - 3)^2 + (-1 - 7)^2}$

$= \sqrt{12 + (-8)^2}$

$= \sqrt{65}$

$MJ = \sqrt{(-5 - 3)^2 + (6 - 7)^2}$

$= \sqrt{(-8)^2 + (-1)^2}$

$= \sqrt{65}$

$MK = \sqrt{(-4 - 3)^2 + (-2 - 7)^2}$

$= \sqrt{(-7)^2 + (-9)^2}$

$= \sqrt{130}$

$JL = \sqrt{(-5 - 4)^2 + (6 - (-1))^2}$

$= \sqrt{(-9)^2 + 7^2}$

$= \sqrt{130}$

All sides are congruent and diagonals are congruent, so *JKLM* is a square.

5. Slope of $\overline{PS} = \dfrac{-3 - 9}{-5 - 7} = \dfrac{-12}{-12} = 1$

Slope of $\overline{SR} = \dfrac{9 - 3}{7 - 6} = \dfrac{6}{1} = 6$

Slope of $\overline{RQ} = \dfrac{3 - (-2)}{6 - 1} = \dfrac{5}{5} = 1$

Slope of $\overline{QP} = \dfrac{-2 - (-3)}{1 - (-5)} = \dfrac{1}{6}$

$\overline{PS} \parallel \overline{RQ}$

Exactly one pair of opposite sides are parallel, so *PQRS* is a trapezoid.

6. It is given that $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{DC}$ and $\angle D \cong \angle C$. Draw $\overline{AE} \parallel \overline{BC}$, so $ABCE$ is a parallelogram. $\angle AED \cong \angle C$ by the Corr. △ Postulate. $\angle AED \cong \angle D$ by the Trans. Prop. of Cong. $\overline{AD} \cong \overline{AE}$ by the Converse of Base △ Thm. $\overline{AE} \cong \overline{BC}$ because opp. sides of a parallelogram are congruent. By the Trans. Prop. of Congruence, $\overline{AD} \cong \overline{BC}$.

Lesson 6.6

6.6 Guided Practice (p. 367)

1. Draw $\overline{DB}$. By the Midsegment Theorem for Triangles, $\overline{EF} \parallel \overline{DB}$ and $\overline{HG} \parallel \overline{DB}$. Two lines $\parallel$ to the same line are $\parallel$ to each other, so $\overline{EF} \parallel \overline{HG}$.

	Property	▱	*Rectangle*	*Rhombus*	*Square*	*Kite*	*Trapezoid*
2.	Both pairs of opp. sides are $\parallel$.	X	X	X	X		
3.	Exactly 1 pair of opp. sides are $\parallel$.						X
4.	Diagonals are $\perp$.			X	X	X	
5.	Diagonals are $\cong$.		X		X		
6.	Diagonals bisect each other.	X	X	X	X		

7. parallelogram, rectangle, rhombus, square

6.6 Practice and Applications (pp. 367–370)

	Property	▱	*Rectangle*	*Rhombus*	*Square*	*Kite*	*Trapezoid*
8.	Both pairs of opp. sides are $\cong$.	X	X	X	X		
9.	Exactly 1 pair of opp. sides are $\cong$.						
10.	All sides are $\cong$.			X	X		
11.	Both pairs of opp. △ are $\cong$.	X	X	X	X		
12.	Exactly 1 pair of opp. △ are $\cong$.					X	
13.	All △ are $\cong$.		X		X		

14. trapezoid **15.** isosceles trapezoid **16.** trapezoid

17. square **18.** kite

19. ▱, rectangle, square, rhombus, kite **20.** rectangle, square, isosceles trapezoid

21. rhombus, square **22.** ▱, rectangle, square, rhombus, isosceles trapezoid

23. rectangle, square **24.** square

25. Show that the quad. has two pairs of consecutive congruent sides, but opposite sides are not congruent (def. of a kite).

26. Show that the quad. has four rt. △ with four $\cong$ sides (def. of a square).

27. Show that the quad. has four rt. △; show that the quad. is a ▱ and that the diagonals are $\cong$.

28. Show that only one pair of opp. sides is $\parallel$ (def. of a trapezoid).

29. Show that exactly 2 sides are parallel and that the nonparallel sides are congruent (def. of isosceles trap.); show that the quad. is a trapezoid and that the pair of base △ are $\cong$; show that the quad. is a trapezoid and that its diagonals are $\cong$.

30. $\angle A$ and $\angle D$ or $\angle B$ and $\angle C$; $\overline{BC} \parallel \overline{AD}$ by Consec. Int △ Converse, so if $\angle A \cong \angle D$ (or $\angle B \cong \angle C$), $\overline{AB}$ and $\overline{DC}$ are not $\parallel$ and $ABCD$ is a trapezoid. Since base △ are $\cong$, $ABCD$ is an isosceles trapezoid.

31. $\overline{BE}$ and $\overline{DE}$; if the diagonals of a quad. bisect each other, the quad. is a ▱.

32. $\overline{AB}$ and $\overline{CD}$; if $\overline{AB} \cong \overline{CD}$, then one pair of opposite sides are both $\parallel$ and $\cong$, and $ABCD$ is a ▱. Since the diagonals are perpendicular, $ABCD$ is a rhombus.

33. $\overline{AC}$ and $\overline{BD}$; because the diagonals of $ABCD$ bisect each other, $ABCD$ is a parallelogram. If the diagonals of a ▱ are $\cong$, then the ▱ is a rectangle.

34. $\overline{BC}$ and $\overline{DC}$ or $\angle BAC$ and $\angle DAC$; use either SAS or ASA to prove that $\triangle ABC \cong \triangle ADC$, which implies that $\overline{AB} \cong \overline{AD}$. A quad. that has two pairs of consecutive congruent sides is a kite.

35. Any two consecutive sides; $ABCD$ is a rectangle by the Rectangle Corollary. If, for example, $\overline{AB} \cong \overline{AD}$, then since $ABCD$ is a ▱, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Then by the Trans. Prop. of $\cong$, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $ABCD$ is a rhombus. A quad. that is both a rectangle and a rhombus is a square.

36.
$$PQ = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$
$$QR = \sqrt{(0-5)^2 + (2-5)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$$
$$RS = \sqrt{(5-2)^2 + (5-0)^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$$
$$PS = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$

Kite; $\overline{PQ} \cong \overline{PS}$ and $\overline{QR} \cong \overline{RS}$, but pairs of opposite sides are not necessarily congruent, so it isn't a rhombus.

37. Slope of $\overline{SR} = \frac{8-8}{2-4} = \frac{0}{-2} = 0$

Slope of $\overline{RQ} = \frac{8-1}{4-5} = \frac{7}{-1} = -7$

Slope of $\overline{QP} = \frac{1-1}{5-1} = \frac{0}{4} = 0$

Slope of $\overline{PS} = \frac{8-1}{2-1} = \frac{7}{1} = 7$

$SR = \sqrt{(4-2)^2 + (8-8)^2}$
$= \sqrt{4}$
$= 2$

$RQ = \sqrt{(5-4)^2 + (1-8)^2}$
$= \sqrt{1^2 + (-7)^2}$
$= \sqrt{50}$
$= 5\sqrt{2}$

$QP = \sqrt{(1-5)^2 + (1-1)^2}$
$= \sqrt{16} = 4$

$PS = \sqrt{(1+2)^2 + (1-8)^2}$
$= \sqrt{(-1)^2 + (-7)^2}$
$= \sqrt{50}$
$= 5\sqrt{2}$

Isosceles trapezoid; $\overline{SR} \parallel \overline{QP}$, and $\overline{PS}$ and $\overline{RQ}$ are congruent but not parallel.

38. $PQ = \sqrt{(2-7)^2 + (1-1)^2}$
$= \sqrt{25}$
$= 5$

$QR = \sqrt{(7-7)^2 + (1-7)^2}$
$= \sqrt{36}$
$= 6$

$RS = \sqrt{(7-2)^2 + (7-5)^2}$
$= \sqrt{5^2 + 2^2}$
$= \sqrt{29}$

$SP = \sqrt{(2-2)^2 + (5-1)^2}$
$= \sqrt{16}$
$= 4$

Slope of $\overline{PQ} = \frac{1-1}{7-2} = \frac{0}{5} = 0$

Slope of $\overline{QR} = \frac{7-1}{7-7} = \frac{6}{0}$, which is undefined

Slope of $\overline{RS} = \frac{7-5}{7-2} = \frac{2}{5}$

Slope of $\overline{SP} = \frac{5-1}{2-2} = \frac{4}{0}$, which is undefined

Trapezoid; $\overline{QR}$ and $\overline{SP}$ are the only pair of parallel sides, so not a ▱. No opposite sides are ≅, so not isosceles.

39. Slope of $\overline{PQ} = \frac{8-7}{4-0} = \frac{1}{4}$

Slope of $\overline{QR} = \frac{8-2}{4-5} = \frac{6}{-1} = -6$

Slope of $\overline{RS} = \frac{2-1}{5-1} = \frac{1}{4}$

Slope of $\overline{SP} = \frac{7-1}{0-1} = \frac{6}{-1} = -6$

Parallelogram; *Sample answer:* $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{SP}$.

40. Slope of $\overline{PQ} = \frac{9-7}{5-1} = \frac{2}{4} = \frac{1}{2}$

Slope of $\overline{QR} = \frac{3-9}{8-5} = \frac{-6}{3} = -2$

Slope of $\overline{RS} = \frac{3-1}{8-4} = \frac{2}{4} = \frac{1}{2}$

Slope of $\overline{SP} = \frac{7-1}{1-4} = \frac{6}{-3} = -2$

$PR = \sqrt{(8-1)^2 + (3-7)^2}$
$= \sqrt{7^2 + (-4)^2}$
$= \sqrt{65}$

$QS = \sqrt{(5-4)^2 + (9-1)^2}$
$= \sqrt{1^2 + 8^2}$
$= \sqrt{65}$

Rectangle; *Sample answer: PQRS* is a ▱, and the diagonals $\overline{PR}$ and $\overline{QS}$ are congruent.

41. $PQ = \sqrt{(5-9)^2 + (1-6)^2}$
$= \sqrt{(-4)^2 + (-5)^2}$
$= \sqrt{41}$

$QR = \sqrt{(9-5)^2 + (6-11)^2}$
$= \sqrt{4^2 + (-5)^2}$
$= \sqrt{41}$

$RS = \sqrt{(5-1)^2 + (11-6)^2}$
$= \sqrt{4^2 + 5^2}$
$= \sqrt{41}$

$SP = \sqrt{(5-1)^2 + (1-6)^2}$
$= \sqrt{4^2 + (-5)^2}$
$= \sqrt{41}$

Rhombus; *Sample answer:* $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$.

42. trapezoid **43.** isosceles trapezoid

44. *APBQ* is a kite. Its diagonals are $\overline{AB}$ and $\overline{PQ}$ which are, therefore, $\perp$.

45. ▱; if the diagonals of a quad. bisect each other, the quad. is a parallelogram. Since the diagonals are not perpendicular, the ▱ is not a rhombus and since the diagonals are not congruent, the ▱ is not a rectangle.

46. Rhombus; if the diagonals of a quad. bisect each other, the quad is a ▱. Because the diagonals are perpendicular, the ▱ is a rhombus. Since $\overline{AC} \ncong \overline{BD}$, the ▱ is not a rectangle, so it is not a square.

47. Kite; $\overline{AC} \perp \overline{BD}$ and $\overline{AC}$ bisects $\overline{BD}$, so you can use congruent $\triangle$s to prove that $\overline{AB} \cong \overline{AD}$ and that $\overline{CB} \cong \overline{CD}$. $\overline{BD}$ does not bisect $\overline{AC}$, so *ABCD* is not a ▱. Opp. sides are not $\cong$, so *ABCD* is a kite.

48. *EFLM* is a parallelogram; *EFGH* is a ▱, so $\overline{EF} \parallel \overline{HG}$ and $\overline{EF} \cong \overline{HG}$, *GHJK* is a ▱, so $\overline{HG} \parallel \overline{JK}$ and $\overline{GH} \cong \overline{JK}$; *JKLM* is a ▱, so $\overline{JK} \parallel \overline{LM}$ and $\overline{KJ} \cong \overline{LM}$. Therefore $\overline{EF} \parallel \overline{LM}$ by repeated application of Thm. 3.12 and $\overline{EF} \cong \overline{LM}$ by the Trans. Prop. of $\cong$.

49. Draw a line through $C \parallel$ to $\overline{DE}$. Draw a line through *E* parallel to $\overline{CD}$. Label the intersection *F*. *CDEF* is a ▱ by def. of a ▱. $\angle CDE$ and $\angle DEF$ are rt. $\angle$s because consec. $\angle$s of a ▱ are supplementary. $\angle DCF$ and $\angle CFE$ are rt. $\angle$s because opp. $\angle$s of a ▱ are $\cong$ and *CDEF* is a rectangle. The diagonals of a parallelogram bisect each other, so $DM = \frac{1}{2}DF$ and $CM = \frac{1}{2}CE$. The diagonals of a rectangle are $\cong$, so $DF = CE$, $\frac{1}{2}DF = \frac{1}{2}CE$, and $DM = CM$. By the definition of congruence, $\overline{DM} \cong \overline{CM}$.

50. *ABCD* is a quad. with diagonals $\overline{BD} \cong \overline{CA}$. The diagonals intersect at *N* with $\overline{BN} \cong \overline{ND} \cong \overline{CN} \cong \overline{NA}$. $\angle ABN \cong \angle NBC \cong \angle BCN \cong \angle NCD \cong \angle CDN \cong \angle NDA \cong \angle DAN \cong \angle NAB$ by the Base $\angle$s Thm. Let $m\angle ABN = x°$ then $8x° = 360°$ because the sum of the int. $\angle$s of a quad. is 360°. Therefore $x° = 45°$. By the Angle Add. Postulate, $m\angle ABN + m\angle BNC = 2x°$ or 90° and $m\angle BCD = m\angle BCN + m\angle NCD = 2x°$ or 90°.
By the same reasoning $m\angle BAD = 90°$ and $m\angle CDA = 90°$. Then *ABCD* is a rectangle.

51.

Statements	Reasons
1. *PQRS* is a square; *E, F, G,* and *H* are midpoints of the sides of the square.	**1.** Given
2. $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$; $\angle P$, $\angle Q$, $\angle R$, and $\angle S$ are right angles.	**2.** Definition of square
3. $\angle P \cong \angle Q \cong \angle R \cong \angle S$	**3.** Right Angle Congruence Theorem
4. $\overline{PF} \cong \overline{FQ} \cong \overline{QG} \cong \overline{GR} \cong \overline{RH} \cong \overline{HS} \cong \overline{SE} \cong \overline{EP}$	**4.** Definition of midpoint and square
5. $\triangle EPF$, $\triangle FQG$, $\triangle GRH$, $\triangle HSE$ are isosceles triangles.	**5.** Definition of isosceles triangle
6. $\triangle EPF \cong \triangle FQG \cong \triangle GRH \cong \triangle HSE$	**6.** SAS Congruence Postulate
7. $\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{HE}$	**7.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
8. *EFGH* is a rhombus.	**8.** Rhombus Corollary
9. $\angle FEP \cong \angle EFP \cong \angle GFQ \cong \angle FGQ \cong \angle RGH \cong \angle RHG \cong \angle SHE \cong \angle SEH$	**9.** Base Angles Theorem; corresponding parts of $\cong$ $\triangle$s are $\cong$.
10. $m\angle FEP = m\angle EFP = m\angle GFQ = m\angle FGQ = m\angle RGH = m\angle RHG = m\angle SHE = m\angle SEH = 45°$	**10.** Def of congruent angles and Triangle Sum Theorem ($x° + x° + 90° = 180°$, so $x° = 45°$)
11. $m\angle FEP + m\angle HES + m\angle HEF = 180°$, $m\angle EFP + m\angle GFQ + m\angle GFE = 180°$, $m\angle QGF + m\angle RGH + m\angle HGF = 180°$, $m\angle RHG + m\angle SHE + m\angle GHE = 180°$	**11.** Angle Addition Postulate
12. $m\angle HEF = m\angle EFG = m\angle HGF = m\angle GHE = 90°$	**12.** Subtraction and substitution properties of equality
13. *EFGH* is a square.	**13.** A rhombus with four right angles is a square.

52. *Sample answer:*

Statements	Reasons
1. $\overline{JK} \cong \overline{LM}$, *E, F, G,* and *H* are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.	**1.** Given
2. $JK = LM$	**2.** Def. of $\cong$
3. $\overline{EH}$, $\overline{EF}$, $\overline{FG}$, and $\overline{GH}$ are midsegments.	**3.** Def. of midsegment
4. $EH = \frac{1}{2}LM$, $EF = \frac{1}{2}JK$, $FG = \frac{1}{2}LM$, $GH = \frac{1}{2}JK$	**4.** Midsegment Thm.
5. $EH = EF = FG = GH$	**5.** Substitution prop. of eq. and trans. prop. of eq.
6. $\overline{EH} \cong \overline{EF} \cong \overline{FG} \cong \overline{GH}$	**6.** Def. of $\cong$
7. *EFGH* is a rhombus.	**7.** Rhombus Corollary

53. a. Isosceles; by the Angle Addition Post., $m\angle KLM = m\angle KLJ + m\angle JLM$ and $m\angle KJN = m\angle KJL + m\angle LJN$. Since $JKLMN$ is a regular pentagon, $m\angle KLM = m\angle KJN$ and, so, $m\angle KLJ + m\angle JLM = m\angle KJL + m\angle LJN$. By the Base $\angle$s Thm., $\angle KLJ \cong \angle KJL$, so $m\angle KLJ = m\angle KJL$ and by the subtraction prop. of $=$, $m\angle JLM = m\angle LJN$ and $\angle LJN \cong \angle JLM$.

b. $m\angle LMN = m\angle JNM$ and $m\angle LJN = m\angle JLM$; Since the sum of measures of the interior $\angle$s of a quad. is 360°, $m\angle LMN + m\angle JNM + m\angle LJN + m\angle JLM = 360°$. Then, by the substitution prop. of $=$, $2m\angle JLM + 2m\angle LMN = 360°$ and $m\angle JLM + m\angle LMN = 180°$. Since $\angle MLJ$ and $\angle LMN$ are supp., $\overline{MN} \parallel \overline{LJ}$ by the Consec. Int. $\angle$s Converse.

c. parallelogram

d. Yes; because $JPMN$ is a parallelogram, opposite sides must be congruent. We know that $\overline{JN} \cong \overline{MN}$ ($JKLMN$ is a regular pentagon), so $\overline{JN} \cong \overline{PM}$ and $\overline{JP} \cong \overline{MN}$. By the Transitive Property of Congruence, $\overline{JN} \cong \overline{PM} \cong \overline{JP} \cong \overline{MN}$, so $JPMN$ is a rhombus.

54. Isosceles trapezoid; use the SAS Cong. Post. to show that $\triangle AND \cong \triangle BNC$, so $\overline{AD} \cong \overline{BC}$. Then use the Vertical $\angle$s Thm., the Triangle Sum Thm., and the Base $\angle$s Thm. to show that $\angle ACD \cong \angle CAB$, so that $\overline{AB} \parallel \overline{CD}$. Finally, since $\overline{AC}$ and $\overline{BD}$ do not bisect each other, $ABCD$ is not a ▱, so $ABCD$ is an isosceles trapezoid.

6.6 Mixed Review (p. 370)

55. $A = s^2 = 4^2 = 16$ sq. units

56. $A = s^2 = 7^2 = 49$ sq. units

57. $A = lw = (5)(3) = 15$ sq. units

58. $A = lw = (9)(6) = 54$ sq. units

59. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 12 \cdot 5 = 30$ sq. units

60. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 12 \cdot 8 = 48$ sq. units

61. $(32x + 15)° + 133° + 80° + (44x - 1)° = 360°$

$$76x + 227 = 360$$
$$76x = 133$$
$$x = 1.75$$

62. $m\angle A = (32x + 15)° = [32(1.75) + 15]° = 71°$

63. midsegment $= \frac{1}{2}(AB + DC) = \frac{1}{2}(4 + 10) = 7$

64. midsegment $= \frac{1}{2}(AD + BC) = \frac{1}{2}(4 + 8) = 6$

65. midsegment $= \frac{1}{2}(AD + BC) = \frac{1}{2}(2 + 8) = 5$

Lesson 6.7

Developing Concepts Activity 6.7 (p. 371)

1. The areas, bases and heights are the same.

2. The area of each △ is $\frac{1}{2}$ the area of the ▱. The bases and heights are the same for both the △ and the ▱.

3. The area of each trapezoid is $\frac{1}{2}$ the area of the parallelogram. The base of the parallelogram is equal to the sum of the bases of the trapezoids. The heights are the same.

Extension

parallelogram: $A = bh$; triangle: $A = \frac{1}{2}bh$; trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

6.7 Guided Practice (p. 376)

1. The midsegment of a trapezoid is the segment that connects the midpoints of the legs.

2. 6 **3.** A **4.** E **5.** C **6.** B **7.** D

8. $A = \frac{1}{2}bh = \frac{1}{2}(7)(4) = 14$ sq. units

9. $A = s^2 = 5^2 = 25$ sq. units

10. $A = bh = (9)(4) = 36$ sq. units

11. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(10)(8) = 40$ sq. units

12. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(12)(12) = 72$ sq. units

13. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(6)(8 + 4) = 36$ sq. units

6.7 Practice and Applications (pp. 376–379)

14. $A = \frac{1}{2}bh = \frac{1}{2}(5)(7) = 17.5$ sq. units

15. $A = s^2 = 7^2 = 49$ sq. units

16. $A = bh = (5)(9) = 45$ sq. units

17. $A = bh = (15)(8) = 120$ sq. units

18. $A = bh = (22)(21) = 462$ sq. units

19. $A = \frac{1}{2}bh = \frac{1}{2}(5)(4) = 10$ sq. units

20. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(6 + 10)(8)$
$= 64$ sq. units

21. $A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(38)(19)$
$= 361$ sq. units

22. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(24 + 7)(24)$
$= 372$ sq. units

23. $A = bh$
$= 16 \cdot 15$
$= 240$ sq. units

24. $A = \frac{1}{2}(b_1 + b_2)$
$= \frac{1}{2}(8 + 16)(14)$
$= 168$ sq. units

25. $A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(14)(10)$
$= 70$ sq. units

26. $bh = A$
$7x = 63$
$x = 9$ cm

27. $\frac{1}{2}d_1d_2 = A$
$\frac{1}{2}(8)(x) = 48$
$4x = 48$
$x = 12$ ft

28. $\frac{1}{2}d_1d_2 = A$
$\frac{1}{2}(2x)(16) = 48$
$16x = 48$
$x = 3$ in.

29. $A = \frac{1}{2}bh$
$\frac{2A}{h} = b$

30. $A = \frac{1}{2}d_1d_2$

$\frac{2A}{d_2} = d_1$

31. $A = \frac{1}{2}(b_1 + b_2)h$

$\frac{2A}{h} = b_1 + b_2$

$\frac{2A}{h} - b_2 = b_1$

32. $A = bh$

$= (3)(4)$

$= 12$ sq. units

33. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(2)(4)$

$= 4$ sq. units

34. $A = s^2$

$S = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$A = \left(\sqrt{5}\right)^2$

$= 5$ square units

35. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(3)(2)$

$= 3\text{ ft}^2$

36. $A = bh + \frac{1}{2}bh$

$= (48)(32) + \frac{1}{2}(48)(12)$

$= 1824\text{ in.}^2$

37. $A = 2\left[\frac{1}{2}(b_1 + b_2)h\right]$

$= (16 + 30)12$

$= 552\text{ in.}^2$

38. $A = bh + \frac{1}{2}(b_1 + b_2)h$

$= (20)(16) + \frac{1}{2}(9 + 20)(5)$

$= 392.5\text{ in.}^2$

39. No; although the bases and the heights of two such ▱s are the same, the angles of the two parallelograms may not be congruent.

40. Yes. By definition, a rectangle has four right angles. So the angles of one rectangle will always be congruent to the ∠s of any other rectangle. Any two rectangles with area 24 ft² and base 6 ft have a height of 4 ft and are congruent.

41. $10^2 - 6^2 = b^2$

$100 - 36 = b^2$

$64 = b^2$

$8 = b$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(8)(6)$

$= 24$ sq. units

42. $13^2 - 12^2 = b^2$

$169 - 144 = b^2$

$25 = b^2$

$5 = b$

$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(20)(10)$

$= 100$ sq. units

43. $h^2 = 20^2 - 16^2$

$h^2 = 400 - 256$

$h^2 = 144$

$h = 12$

$A = bh$

$= (16)(12)$

$= 192$ sq. units

44. The area would be doubled if the length of one of the diagonals was doubled. The area of the kite would be quadrupled if the lengths of both diagonals were doubled.

45. $A = (2)(5) = 10\text{ ft}^2$

$10\text{ ft}^2 \times \frac{144\text{ in.}^2}{1\text{ ft}^2} = 1440\text{ in.}^2$

$\frac{1440\text{ in.}^2}{3\text{ in.}^2} \approx 480$ carnations

46. $A = \frac{1}{2}(5 + 3)(2) = 8\text{ ft}^2$

$8\text{ ft}^2 \times \frac{144\text{ in.}^2}{1\text{ ft}^2} = 1152\text{ in.}^2$

$\frac{1152\text{ in.}^2}{2\text{ in.}^2} \approx 576$ daisies

47. $A = \frac{1}{2}(3)(8) = 12\text{ ft}^2$

$12\text{ ft}^2 \times \frac{144\text{ in.}^2}{1\text{ ft}^2} = 1728\text{ in.}^2$

$\frac{1728\text{ in.}^2}{4\text{ in.}^2} \approx 432$ chrysanthemums

48. $A = 2\left[\frac{1}{2}(b_1 + b_2)h\right] + 2\left(\frac{1}{2}bh\right)$

$= (1908 + 1644)(158) + (252)(163)$

$= 561{,}216 + 41{,}076$

$= 602{,}292\text{ in.}^2$ or about 4182.6 ft^2

49. $\frac{602{,}292\text{ in.}^2}{100\text{ in.}^2} \approx 6023$ shakes

50. $s = \sqrt{3^2 + 6^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Area of blue $= s^2 = \left(3\sqrt{5}\right)^2 = 45$ sq. units

Area of yellow $= 4\left(\frac{1}{2}bh\right)$

$= 2(3)(6)$

$= 36$ sq. units

51. Area of blue $= 2\left(\frac{1}{2}\text{bh}\right)$

$= (12)(8)$

$= 96$ sq. units

Area of yellow $= bh -$ Area of blue

$= (12)(16) - 96$

$= 96$ sq. units

52. $A_{\text{Total}} = \frac{1}{2}\left(11\sqrt{2}\right)\left(11\sqrt{2}\right)$

$= 121$ sq. units

Area of yellow $= \frac{1}{2}\left(4\sqrt{2}\right)\left(4\sqrt{2}\right) + \frac{1}{2}\left(7\sqrt{2}\right)\left(7\sqrt{2}\right)$

$= 16 + 49$

$= 65$ sq. units

Area of blue $= bh$

$= \left(4\sqrt{2}\right)\left(7\sqrt{2}\right)$

$= 56$ sq. units

53. Square, square; *Sample answer:* In quad. *EBFJ*, ∠*E*, ∠*J*, and ∠*F* are right angles by the Linear Pair Postulate and ∠*B* is a right angle by the Interior Angles of a Quadrilateral Theorem. Then *EBFJ* is a rectangle by the Rectangle Corollary. $\overline{EJ} \cong \overline{FJ}$ because they are corresp. parts of ≅ ▱s. Then, by the definition of a ▱ and the Transitive Property of Congruence, *EBFJ* is a rhombus and, therefore, a square. Similarly, *HJGD* is a square.

54. Square; each side has the same length, $b + h$, so $ABCD$ is a rhombus, and each $\angle$ is a right $\angle$, so $ABCD$ is a rectangle.

55. length of each side $= b + h$ and the area of $ABCD$ would be $(b + h)^2 = b^2 + 2bh + h^2$.

56. The area of $EBFJ = h^2$. The area of $HJGD = b^2$.

57. $b^2 + 2bh + h^2 = b^2 + h^2 + 2A$

$2bh = 2A$

$bh = A$

58. Show that the area of $KMNQ = (b_1 + b_2)h$. Show that Area of $KLPQ$ + Area of $LMNP$ = Area of $KMNQ$. Then, use the distributive and division properties of equality to show that the Area of $LPQK = \frac{1}{2}(b_1 + b_2)h$.

59. Show that the area of $AEGH = \frac{1}{2}h(b_1 + b_2)$. Since $EBCF$ and $GHDF$ are $\cong$, Area of $ABCD$ = Area of $AEFD$ + Area of $EBCF$ = Area of $AEFD$ + Area $GHDF$ = Area of $AEGH = \frac{1}{2}h(b_1 + b_2)$.

60. $A = \frac{1}{2}(13 + 8)(4) = 42 \text{ in.}^2$

D

61. $A = (5)(3) = 15 \text{ cm}^2$

B

62. Area of $PQRS$ = Area of $\triangle PQR$ + Area of $\triangle PRS$. Area of $\triangle PQR = \frac{1}{2}(PR)(QT)$. The area of $\triangle PRS = \frac{1}{2}(PR)(TS)$. So, the area of $PQRS = \frac{1}{2}(PR)(QT + TS)$, but PR is the length of one diagonal, d_1, and $QT + TS$ is the length of the other diagonal, d_2. Therefore, $A = \frac{1}{2}d_1 d_2$.

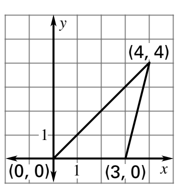
6.7 Mixed Review (p. 380)

63. obtuse; about 140°

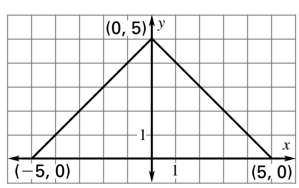
64. right; about 90°

65. acute; about 15°

66. *Sample answer:*



67. *Sample answer:*



68. $14 = \frac{2}{3}(x + 14)$

$21 = x + 14$

$7 = x$

69. $4 = \frac{2}{3}(4 + 2x)$

$6 = 4 + 2x$

$2 = 2x$

$1 = x$

70. $x + 6 = \frac{2}{3}(2x + 6)$

$x + 6 = \frac{4}{3}x + 4$

$2 = \frac{1}{3}x$

$6 = x$

Quiz 3 (p. 380)

1. $ON = \sqrt{(0 - 0)^2 + (0 - 1)^2}$

$= \sqrt{1} = 1$

$OP = \sqrt{(0 - 1)^2 + (0 - 0)^2}$

$= \sqrt{1} = 1$

$PM = \sqrt{(1 - 3)^2 + (0 - 3)^2}$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{13}$

$MN = \sqrt{(0 - 3)^2 + (1 - 3)^2}$

$= \sqrt{(-3)^2 + (-2)^2}$

$= \sqrt{13}$

Kite; $\overline{ON} \cong \overline{OP}$ and $\overline{PM} \cong \overline{MN}$, but opposite sides are not congruent.

2. Slope of $\overline{QR} = \frac{2 - 1}{2 - 2} = \frac{1}{0}$, which is undefined

Slope of $\overline{RS} = \frac{1 - 0}{2 - 4} = -\frac{1}{2}$

Slope of $\overline{ST} = \frac{0 - 4}{4 - 4} = -\frac{4}{0}$, which is undefined

Slope of $\overline{TQ} = \frac{4 - 2}{4 - 2} = \frac{2}{2} = 1$

$RS = \sqrt{(2 - 4)^2 + (1 - 0)^2}$

$= \sqrt{(-2)^2 + 1^2}$

$= \sqrt{5}$

$TQ = \sqrt{(4 - 2)^2 + (4 - 2)^2}$

$= \sqrt{2^2 + 2^2}$

$= \sqrt{8} = 2\sqrt{2}$

Trapezoid; $\overline{QR} \parallel \overline{ST}$, but $\overline{TQ}$ and $\overline{RS}$ are not parallel.

3. $WX = \sqrt{(2 - 4)^2 + (0 - 1)^2}$

$= \sqrt{(-2)^2 + (-1)^2}$

$= \sqrt{5}$

$XY = \sqrt{(4 - 2)^2 + (1 - 3)^2}$

$= \sqrt{2^2 + (-2)^2}$

$= \sqrt{8} = 2\sqrt{2}$

$YZ = \sqrt{(2 - 0)^2 + (3 - 2)^2}$

$= \sqrt{2^2 + 1^2}$

$= \sqrt{5}$

$ZW = \sqrt{(0 - 2)^2 + (2 - 0)^2}$

$= \sqrt{(-2)^2 + 2^2}$

$= \sqrt{8} = 2\sqrt{2}$

Slope of $\overline{WX} = \frac{1 - 0}{4 - 2} = \frac{1}{2}$

Slope of $\overline{XY} = \frac{1 - 3}{4 - 2} = \frac{-2}{2} = -1$

—CONTINUED—

3. —CONTINUED—

Slope of $\overline{YZ} = \frac{3 - 2}{2 - 0} = \frac{1}{2}$

Slope of $\overline{ZW} = \frac{2 - 0}{0 - 2} = \frac{2}{-2} = -1$

Parallelogram; *Sample answers:* $\overline{WX} \parallel \overline{YZ}$ and $\overline{XY} \parallel \overline{ZW}$, or $\overline{WX} \cong \overline{YZ}$ and $\overline{XY} \cong \overline{ZW}$.

4. $bh = A$

$12x = 60$

$x = 5$ in.

5. $\frac{1}{2}bh = A$

$\frac{1}{2}(x)(10) = 60$

$x = 12$ in.

6. $A = \frac{1}{2}d_1d_2$

$60 = \frac{1}{2}(15)(x)$

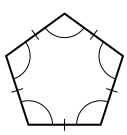
$8 \text{ in.} = x$

7. $A = \frac{1}{2}(8.3 + 11)(5.4)$

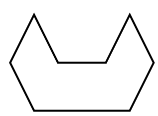
$= 52.11 \text{ cm}^2$

Chapter 6 Review (p. 382)

1. *Sample answer:*



2. *Sample answer:*



3. $67° + 115° + 63° + x° = 360°$

$x = 360 - (67 + 115 + 63)$

$= 115$

4. $5x° + 3x° + 90° + 90° = 360°$

$8x + 180 = 360$

$8x = 180$

$x = 22.5$

5. $6x° + 9x° + 75° + 90° = 360°$

$15x + 165 = 360$

$15x = 195$

$x = 13$

6. $FH = DH = 9.5$

$DF = DH + FH = 9.5 + 9.5 = 19$

7. $m\angle EFG = m\angle GDE = 65°$

$m\angle DEF = 180° - m\angle EFG = 180° - 65° = 115°$

8. $P = 2l + 2w$

$= 2(12) + 2(10)$

$= 44$ units

9. No; you are not given information about opposite sides.

10. Yes; opposite ∠s are ≅.

11. Yes; you can prove $\triangle PQT \cong \triangle SRT$ and opposite sides are ≅.

12. Yes; consecutive angles are supplementary.

13. rhombus, square

14. parallelogram, rectangle, rhombus, square

15. rhombus, square

16. midsegment $= \frac{1}{2}(6 + 16)$

$= 11$ units

17. $m\angle ABC = m\angle DAB = 112°$

$m\angle ADC = 180° - m\angle DAB = 180° - 112° = 68°$

$m\angle BCD = m\angle ADC = 68°$

18. It is given that $\overline{AD} \cong \overline{BC}$ and $\angle ADC \cong \angle BCD$. $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence. $\triangle ADC \cong \triangle BCD$, by the SAS Congruence Postulate. $\angle ACD \cong \angle BDC$ by corresponding parts of ≅ △s are ≅.

19. $PQ = \sqrt{(5 - 0)^2 + (6 - 3)^2}$

$= \sqrt{5^2 + 3^2}$

$= \sqrt{34}$

$RS = \sqrt{(2 - (-3))^2 + (11 - 8)^2}$

$= \sqrt{5^2 + 3^2}$

$= \sqrt{34}$

$PR = \sqrt{(2 - 0)^2 + (11 - 3)^2}$

$= \sqrt{2^2 + 8^2}$

$= \sqrt{68}$

$= 2\sqrt{17}$

$QR = \sqrt{(5 - 2)^2 + (6 - 11)^2}$

$= \sqrt{3^2 + (-5)^2}$

$= \sqrt{34}$

$PS = \sqrt{(0 - (-3))^2 + (3 - 8)^2}$

$= \sqrt{3^2 + (-5)^2}$

$= \sqrt{34}$

$QS = \sqrt{(5 - (-3))^2 + (6 - 8)^2}$

$= \sqrt{8^2 + (-2)^2}$

$= \sqrt{68}$

$= 2\sqrt{17}$

Square; *Sample answer:* $PQ = RS = QR = PS$ so $PQRS$ is a rhombus. Diagonals $\overline{PR}$ and $\overline{QS}$ are ≅, so $PQRS$ is a rectangle. A quad. that is both a rhombus and a rectangle is a square.

20. $PQ = \sqrt{(6 - 0)^2 + (8 - 0)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

$QR = \sqrt{(8 - 6)^2 + (5 - 8)^2}$

$= \sqrt{2^2 + (-3)^2}$

$= \sqrt{13}$

—CONTINUED—

20. **—CONTINUED—**

$RS = \sqrt{(8-4)^2 + (5-(-6))^2}$
$= \sqrt{4^2 + 11^2}$
$= \sqrt{137}$

$SP = \sqrt{(4-0)^2 + (-6-0)^2}$
$= \sqrt{16 + 36}$
$= \sqrt{52} = 2\sqrt{13}$

Slope of $\overline{PQ} = \frac{8-0}{6-0} = \frac{8}{6} = \frac{4}{3}$

Slope of $\overline{RS} = \frac{-6-5}{4-8} = \frac{-11}{-4} = \frac{11}{4}$

Slope of $\overline{PS} = \frac{-6-0}{4-0} = \frac{-6}{4} = -\frac{3}{2}$

Slope of $\overline{QR} = \frac{5-8}{8-6} = -\frac{3}{2}$

Trapezoid; $\overline{PS} \parallel \overline{QR}$, but $\overline{PQ}$ and $\overline{RS}$ are not parallel.

21. $PQ = \sqrt{(4-2)^2 + (-5-(-1))^2}$
$= \sqrt{2^2 + (-4)^2}$
$= \sqrt{20} = 2\sqrt{5}$

$QR = \sqrt{(4-0)^2 + (-5-(-3))^2}$
$= \sqrt{4^2 + (-2)^2}$
$= \sqrt{20} = 2\sqrt{5}$

$RS = \sqrt{(0-(-2))^2 + (-3-1)^2}$
$= \sqrt{2^2 + (-4)^2}$
$= \sqrt{20} = 2\sqrt{5}$

$SP = \sqrt{(-2-2)^2 + (1-(-1))^2}$
$= \sqrt{(-4)^2 + 2^2}$
$= \sqrt{20} = 2\sqrt{5}$

Slope of $\overline{PR} = \frac{-3-(-1)}{0-2} = \frac{-2}{-2} = 1$

Slope of $\overline{QS} = \frac{1-(-5)}{-2-4} = \frac{6}{-6} = -1$

$PR = \sqrt{(2-0)^2 + (-1-(-3))^2}$
$= \sqrt{2^2 + 2^2}$
$= \sqrt{8} = 2\sqrt{2}$

$QS = \sqrt{(4-(-2))^2 + (-5-1)^2}$
$= \sqrt{6^2 + 6^2}$
$= \sqrt{72} = 6\sqrt{2}$

Rhombus; $PQ = QR = RS = SP$ and the diagonals are $\perp$ but not $\cong$ so $PQRS$ is not a rectangle (square).

22. $PQ = \sqrt{(-5-(-3))^2 + (0-6)^2}$
$= \sqrt{(-2)^2 + (-6)^2}$
$= \sqrt{40}$
$= 2\sqrt{10}$

$QR = \sqrt{(-3-1)^2 + (6-6)^2}$
$= \sqrt{(-4)^2}$
$= 4$

$RS = \sqrt{(1-1)^2 + (6-2)^2}$
$= \sqrt{4^2}$
$= 4$

$SP = \sqrt{(1-(-5))^2 + (2-0)^2}$
$= \sqrt{6^2 + 2^2}$
$= \sqrt{40}$
$= 2\sqrt{10}$

Kite; two pairs of consecutive sides are congruent but opp. sides are not congruent.

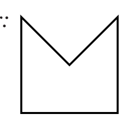
23. $A = \frac{1}{2}bh$
$= \frac{1}{2}\left(8\frac{1}{2}\right)(7)$
$= 29\frac{3}{4}$ in.^{2}

24. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(3 + 6)(3)$
$= 13.5$ ft^2

25. $A = \frac{1}{2}d_1 d_2$
$= \frac{1}{2}(4)(6)$
$= 12$ sq. units

Chapter 6 Test (p. 385)

1. *Sample answer:*



2. $x° + 100° + 70° + 75° = 360°$
$x + 245 = 360$
$x = 115$

3. $3x = 5x - 6$
$-2x = -6$
$x = 3$

$\frac{1}{2}y = 4$
$y = 8$

4. $x = 110$
$y = 180 - 110 = 70$

5. $2y = 7$
$y = \frac{7}{2}$

$x + 6 = 10$
$x = 4$

6. no **7.** yes **8.** no

9. yes **10.** sometimes **11.** never **12.** always

13. Trapezoid; exactly one pair of sides are parallel.

14. Rhombus; the diagonals bisect each other and are perpendicular.

15. Rectangle; one pair of opposite sides are both $\cong$ and $\parallel$, and since one angle is a right angle, the opposite angle and the two consecutive angles are also right angles.

16. Square; the diagonals bisect each other and are congruent so the quad. is a ▱, a rectangle, and a rhombus. So it is a square.

17. $WX = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$XY = \sqrt{(0-(-a))^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

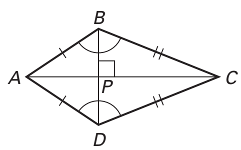
$YZ = \sqrt{(0-(-a))^2 + (-b-0)^2} = \sqrt{a^2 + b^2}$

$ZW = \sqrt{(0-a)^2 + (-b-0)^2} = \sqrt{a^2 + b^2}$

So, $WX = XY = YZ = ZW$.

Let O be the origin (where the diagonals meet). $OX = OZ = b$ and $OW = OY = a$, so the diagonals bisect each other. One diagonal is vertical and the other is horizontal, so they are perpendicular.

18.



$\overline{AC} \perp \overline{BD}$; $\triangle ABC \cong \triangle ADC$ (SSS Cong. Post.) so $\angle BAC \cong \angle DAC$. Then $\triangle BAP \cong \triangle DAP$ (SAS Cong. Post.). Corr. ∡s BPA and DPA are ≅. Since $\overline{AC}$ and $\overline{BD}$ form a linear pair of ≅ ∡s, $\overline{AC} \perp \overline{BD}$.

19. $x = \frac{1}{2}(6 + 15)$

$= 10.5$ in.

20. $A = 2\left[\frac{1}{2}(b_1 + b_2)h\right] + 2\left(\frac{1}{2}bh\right)$

$= (32 + 22)(17) + (20)(15)$

$= 918 + 300 = 1218 \text{ ft}^2$

Chapter 6 Standardized Test (pp. 386–387)

1. D

2. $(7x + 1)^\circ + (8x - 27)^\circ + 108^\circ + (5x + 18)^\circ = 360^\circ$

$20x + 100 = 360$

$20x = 260$

$x = 13$

C

3. $(8p - 17)^\circ = (7p - 8)^\circ$

$p = 9$

$(8p - 17)^\circ + (6q + 17)^\circ = 180^\circ$

$8(9) - 17 + 6q + 17 = 180$

$6q + 72 = 180$

$6q = 108$

$q = 18$

C

4. D

5. $MN = \sqrt{(3-1)^2 + (4-(-6))^2}$

$= \sqrt{2^2 + 10^2}$

$= \sqrt{104} = 2\sqrt{26}$

$NP = \sqrt{(1-6)^2 + (-6-(-7))^2}$

$= \sqrt{(-5)^2 + 1^2}$

$= \sqrt{26}$

$PQ = \sqrt{(8-6)^2 + (3-(-7))^2}$

$= \sqrt{2^2 + 10^2}$

$= \sqrt{104} = 2\sqrt{26}$

$QM = \sqrt{(8-3)^2 + (3-4)^2}$

$= \sqrt{5^2 + (-1)^2}$

$= \sqrt{26}$

Opposite sides are congruent so $MNPQ$ is a ▱. The diagonals are ≅, so $MNPQ$ is a rectangle.

C

6. A

7. $RS = \sqrt{(-5-(-3))^2 + (-7-(-9))^2}$

$= \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$ST = \sqrt{(-3-(-1))^2 + (-9-(-7))^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$TU = \sqrt{(-1-(-3))^2 + (-7-11)^2}$

$= \sqrt{2^2 + (-18)^2} = \sqrt{328} = 2\sqrt{82}$

$UR = \sqrt{(-3-(-5))^2 + (11-(-7))^2}$

$= \sqrt{2^2 + 18^2} = \sqrt{328} = 2\sqrt{82}$

Two pairs of consecutive sides are congruent but opposite sides are not congruent.

B

8. The figure in column A is a trapezoid with base lengths of 8 and 12 and height 9. The area is 90 sq. units. The figure in column B is a parallelogram with base 11 and height 9 so its area is 99 sq. units.

B

9. $21 = \frac{1}{2}\left[(x - 4) + \left(\frac{1}{2}x + 1\right)\right]$

$21 = \frac{1}{2}\left(\frac{3}{2}x - 3\right)$

$42 = \frac{3}{2}x - 3$

$45 = \frac{3}{2}x$

$30 = x$

D

10. The base = 13 units and height = 10 units. So, $A = \frac{1}{2}(10)(13) = 65$ sq. units.

D

11. Yes; $\overline{FB} \parallel \overline{DH}$, and $\overline{BD} \parallel \overline{FH}$ so $FBDH$ is a parallelogram and the diagonals of a parallelogram bisect each other. Therefore, $\overline{BE} \cong \overline{HE}$.

12. $m\angle DHF = m\angle FBD = 180^\circ - m\angle ABF - m\angle CBD$

$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$

$m\angle BDH = 180^\circ - m\angle DHF = 180^\circ - 100^\circ = 80^\circ$

13. $FBDH$ is a ▱ (Ex. 11), so $\overline{BF} \cong \overline{HD}$, $\overline{BE} \cong \overline{HE}$, and $\overline{EF} \cong \overline{ED}$ so $\triangle BEF \cong \triangle HED$ (SSS Cong. Post.).

14. $FBDH$ would be a rhombus because a ▱ in which the diagonals are $\perp$ to each other is a rhombus.

15. $YZ = \frac{1}{2}(QR + UT)$

$9 = \frac{1}{2}[(3x - 10) + (2x + 3)]$

$18 = 5x - 7$

$25 = 5x$

$5 = x$

16. $UT = PQ + QR + RS$

$6y + 1 = 2y + 5 + 2y$

$6y + 1 = 4y + 5$

$2y = 4$

$y = 2$

17. $PT = 2UW$

$3a - 2 = 2(14 - a)$

$3a - 2 = 28 - 2a$

$5a = 30$

$a = 6$

18. $UY = TZ$

$3b + 4 = 4b - 5$

$9 = b$

19. convex pentagon

20.

Statements	Reasons
1. $PSTU$ is a rectangle, $\overline{PQ} \cong \overline{SR}$.	**1.** Given
2. $\angle UPQ$ and $\angle TSR$ are rt. $\angle$s.	**2.** Def. of rectangle
3. $\angle UPQ \cong \angle TSR$	**3.** All rt. $\angle$s are $\cong$.
4. $\overline{PU} \cong \overline{ST}$	**4.** If a quad is a ▱, then opp. sides are $\cong$.
5. $\triangle PQU \cong \triangle SRT$	**5.** SAS Congruence Post.

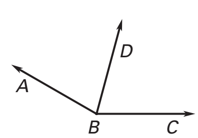
21. $ABCD$ is an isosceles trapezoid; (1) show that $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \cong \overline{BC}$, and $\overline{AD}$ and $\overline{BC}$ are not $\parallel$; or (2) show that $\overline{AB} \parallel \overline{CD}$, $\overline{AD}$ and $\overline{BC}$ are not $\parallel$, and $\overline{AC} \cong \overline{BD}$.

22. $A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(8 + 4)(12) = 72$ sq. units

Cumulative Practice, Chs. 1–6 (pp. 388–389)

1. $\frac{4}{99} = 0.\overline{04}$; $\frac{18}{99} = 0.\overline{18}$; $\frac{35}{99} = 0.\overline{35}$
So, $\frac{89}{99}$ would be $0.\overline{89}$.

2.



3. $m\angle RQP = m\angle SQP$ $(\angle 1 \cong \angle 2)$

$2m\angle RQP = 180° - m\angle QST$ (Consec. Int. $\angle$s Thm.)

$2m\angle RQP = 180° - 90°$ $(\overline{QS} \perp \overline{ST})$

$2m\angle RQP = 90°$

$m\angle RQP = 45° = m\angle PTS$ (Alt. Int. $\angle$s Thm.)

$m\angle URQ = m\angle RQP + m\angle QPR$ (Ext. $\angle$ Thm.)

$= 45° + 90°$

$= 135°$

4. *Sample answers:*

a. $\angle QPR$ and $\angle SPT$ **b.** $\angle SQT$ and $\angle QRS$

c. $\angle RQS$ and $\angle TSQ$ **d.** $\angle RQS$ and $\angle TSQ$

5. $\triangle QPR \cong \triangle QPS$ and $\triangle QPR \cong \triangle TPS$; $\angle 1 \cong \angle 2$, $\overline{QP} \cong \overline{QP}$, and $m\angle QPS = 90° = m\angle QPR$, so $\triangle QPR \cong \triangle QPS$ by the ASA Congruence Postulate. Therefore, since corresponding parts of $\cong$ $\triangle$s are $\cong$, $\overline{PR} \cong \overline{PS}$. $\angle 2 \cong \angle T$ and $\angle QRP \cong \angle TSP$ by the Alternate Interior Angles Theorem, so $\triangle QPR \cong \triangle TPS$ by the AAS Congruence Theorem.

6. Since $PR = PS$ (Ex. 5) by the Segment Addition Postulate, $RS = PS + PR$. By the Substitution property of equality, $RS = PS + PS = 2 \cdot PS$.

7. P is equidistant from $\overrightarrow{QS}$ and $\overrightarrow{QR}$ by the Angle Bisector Theorem.

8. concave pentagon

9. $(5x + 18)° + (14x - 3)° = (21x + 1)°$

$19x + 15 = 21x + 1$

$-2x = -14$

$14 = 2x$

$7 = x$

$[5(7) + 18]° = 53°$

$[14(7) - 3]° = 95°$

$[180 - (21(7) + 1)]° = 32°$

obtuse

10. no **11.** no **12.** yes; SAS Congruence Postulate

13. yes; HL Congruence Theorem

14. slope of $\overleftrightarrow{AB} = \frac{6 - 1}{3 - 11} = -\frac{5}{8}$

$y = mx + b$

$-4 = -\frac{5}{8}(3) + b$

$-4 = -\frac{15}{8} + b$

$-\frac{17}{8} = b$

$y = -\frac{5}{8}x - \frac{17}{8}$

15. $AB = \sqrt{(11 - 3)^2 + (1 - 6)^2}$

$= \sqrt{8^2 + (-5)^2}$

$= \sqrt{89}$

$BC = \sqrt{(3 - 3)^2 + (6 - (-4))^2}$

$= \sqrt{10^2}$

$= 10$

$CA = \sqrt{(11 - 3)^2 + (1 - (-4))^2}$

$= \sqrt{8^2 + 5^2}$

$= \sqrt{89}$

Since $\overline{AB} \cong \overline{CA}$, $\triangle ABC$ is an isosceles $\triangle$.

16. $\angle B \cong \angle C$ by the Base Angles Theorem.

17. slope of $\overline{AC} = \frac{-4 - 1}{3 - 11} = \frac{5}{8}$

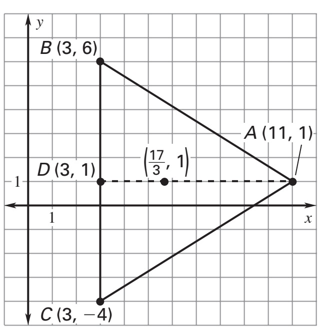
The slope of the $\perp$ bisector of $\overline{AC}$ is $-\frac{8}{5}$.

$$\text{midpoint of } \overline{AC} = \left(\frac{11 + 3}{2}, \frac{1 - 4}{2}\right) = \left(7, -\frac{3}{2}\right)$$

$$y = mx + b$$
$$-\frac{3}{2} = -\frac{8}{5}(7) + b$$
$$-\frac{3}{2} = -\frac{56}{5} + b$$
$$\frac{97}{10} = b$$
$$y = -\frac{8}{5}x + \frac{97}{10}$$

18. the circumcenter, or the intersection of the three $\perp$ bisectors of $\triangle ABC$

19. The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side. Use the median from vertex A to $\overline{BC}$. The midpoint of $\overline{BC}$ is $(3, 1)$. Call this D. $AD = 8$ since $11 - 3 = 8$. So the distance from A to the centroid $= \frac{2}{3}(AD) = \frac{2}{3}(8) = \frac{16}{3}$. The x-coordinate of the centroid is then $11 - \frac{16}{3} = \frac{33}{3} - \frac{16}{3} = \frac{17}{3}$. The coordinates of the centroid are $\left(\frac{17}{3}, 1\right)$.



20. length of midsegment $= \frac{1}{2}AC$
$$= \frac{1}{2}\sqrt{(11 - 3)^2 + (1 - (-4))^2}$$
$$= \frac{1}{2}\sqrt{89} = \frac{\sqrt{89}}{2}$$

21. If two angles are supplementary, then they form a linear pair; false; *Sample answer:* two consecutive angles of a parallelogram are supplementary, but they do not form a linear pair.

22. $4 < XZ < 20$

23. $m\angle X > m\angle Z$; the angle opposite the longer side is larger than the angle opposite the shorter side.

24. Use the converse of the Hinge Theorem in triangles ACD and WYZ. Since $AC > WY$, $m\angle D > m\angle Z$.

25. rhombus

26. $m\angle Z = 25°$; $m\angle W = m\angle Y = 180° - 25° = 155°$

27. slope of $\overline{PQ} = \frac{3 - 4}{8 - 0} = -\frac{1}{8}$

slope of $\overline{QR} = \frac{8 - 3}{9 - 8} = 5$

slope of $\overline{RS} = \frac{9 - 8}{1 - 9} = \frac{1}{8}$

slope of $\overline{SP} = \frac{9 - 4}{1 - 0} = 5$

Sample answer: PQRS is a parallelogram because both pairs of opposite sides are parallel.

28. Trapezoid; $m\angle H = 113°$. Since $\angle E$ and $\angle F$ are supplementary, $\overline{EH} \parallel \overline{FG}$, but the other two sides are not $\parallel$.

29. **a.** rhombus, kite, square

b. rectangle, isosceles trapezoid, square

30. Length of midsegment $= \frac{1}{2}(b_1 + b_2) = \frac{1}{2}(12 + 5)$
$$= \frac{17}{2} \text{ units}$$

Area $= \frac{1}{2}(b_1 + b_2)h$
$$= \frac{1}{2}(12 + 5)(6)$$
$$= 51 \text{ sq. units}$$

31. *Sample answer:* Show that $m\angle C = m\angle F = 65°$ by the Linear Pair Post. $m\angle ABC = 90° = m\angle DEF$, so $\triangle ABC \cong \triangle DEF$ by the AAS Congruence Theorem.

32. Rectangle; since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$. $\overline{AB}$ and $\overline{DE}$ are both perpendicular to $\overline{BE}$, so $\overline{AB} \parallel \overline{DE}$ by the Property of Perpendicular Lines. One pair of opposite sides of quad. $ADEB$ are both congruent and parallel, so $ADEB$ is a parallelogram. $\angle ABE$ and $\angle DEB$ are both right angles, so all four angles are right angles and quad. $ADEB$ must be a rectangle.

33. $180° - 42° = 138°$

$138° \div 2 = 69°$

34. The length of the middle shelf is one half the distance between the supports on the floor. So, the distance between supports on the floor is 60 in. The length of the top shelf is $\frac{1}{2}(30) = 15$ in.

35. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(30 + 15)(19\frac{1}{2})$
$= 438.75 \text{ in.}^2$

Algebra Review (pp. 390–391)

1. $\frac{20}{25} = \frac{4}{5}$ **2.** $\frac{7}{4}$ **3.** $\frac{27-2}{30-3} = \frac{25}{27}$ **4.** $\frac{22}{8} = \frac{11}{4}$

5. $\frac{824}{360} = \frac{103}{45}$ **6.** $\frac{220}{180} = \frac{11}{9}$ **7.** $\frac{8}{2} = \frac{4}{1}$ **8.** $\frac{20}{16} = \frac{5}{4}$ **9.** $\frac{16}{16} = \frac{1}{1}$

10. $2(x + 7) = 20$
$2x + 14 = 20$
$2x = 6$
$x = 3$

11. $8(x + 6) = 24$
$8x + 48 = 24$
$8x = -24$
$x = -3$

12. $6(x - 2) = 24$
$6x - 12 = 24$
$6x = 36$
$x = 6$

13. $-10(y + 8) = -40$
$-10y - 80 = -40$
$-10y = 40$
$y = -4$

14. $16(3 - d) = -4$
$48 - 16d = -4$
$-16d = -52$
$d = \frac{13}{4}$

15. $7(2 - x) = 5x$
$14 - 7x = 5x$
$14 = 12x$
$\frac{7}{6} = x$

16. $-4(x - 6) = 28$
$-4x + 24 = 28$
$-4x = 4$
$x = -1$

17. $-9(5 - 3x) = 9$
$-45 + 27x = 9$
$27x = 54$
$x = 2$

18. $\frac{1}{2}(10 - 9x) = \frac{3}{2}$
$5 - \frac{9}{2}x = \frac{3}{2}$
$-\frac{9}{2}x = -\frac{7}{2}$
$x = \frac{7}{9}$

19. $\frac{2}{3}(m + 4) - 8 = \frac{11}{3}$
$\frac{2}{3}m + \frac{8}{3} - 8 = \frac{11}{3}$
$\frac{2}{3}m - \frac{16}{3} = \frac{11}{3}$
$\frac{2}{3}m = 9$
$m = \frac{27}{2}$

20. $5(3a - 2) = 2(6a - 8)$
$15a - 10 = 12a - 16$
$3a = -6$
$a = -2$

21. $3(x - 1) + 3 = 4(x - 2)$
$3x - 3 + 3 = 4x - 8$
$3x = 4x - 8$
$8 = x$

22. $\frac{x}{20} = \frac{1}{5}$
$5x = 20$
$x = 4$

23. $\frac{2}{q} = \frac{4}{18}$
$4q = 36$
$q = 9$

24. $\frac{7}{100} = \frac{14}{y}$
$7y = 1400$
$y = 200$

25. $\frac{t}{27} = \frac{4}{9}$
$9t = 108$
$t = 12$

26. $\frac{5}{6} = \frac{4}{r}$
$5r = 24$
$r = \frac{24}{5}$

27. $\frac{w}{6} = \frac{7}{17}$
$17w = 42$
$w = \frac{42}{17}$

28. $\frac{27}{5} = \frac{3}{z}$
$27z = 15$
$z = \frac{15}{27} = \frac{5}{9}$

29. $\frac{y}{50} = \frac{3}{100}$
$100y = 150$
$y = \frac{3}{2}$

30. $\frac{6}{19} = \frac{m}{95}$
$19m = 570$
$m = 30$

31. $\frac{3}{8} = \frac{3}{2d}$
$6d = 24$
$d = 4$

32. $\frac{6}{5m} = \frac{6}{25}$
$150 = 30m$
$5 = m$

33. $\frac{19}{x} = \frac{9}{5}$
$9x = 95$
$x = \frac{95}{9}$

34. $\frac{3w + 6}{28} = \frac{3}{4}$
$12w + 24 = 84$
$12w = 60$
$w = 5$

35. $\frac{6}{45} = \frac{2z + 10}{15}$
$90 = 90z + 450$
$-360 = 90z$
$-4 = z$

36. $\frac{3a}{11} = \frac{54}{22}$
$66a = 594$
$a = 9$

37. $\frac{-3}{8} = \frac{21}{2(y + 1)}$
$-6y - 6 = 168$
$-6y = 174$
$y = -29$

38. $\frac{1}{18} = \frac{5}{-4(x - 1)}$
$-4x + 4 = 90$
$-4x = 86$
$x = -\frac{86}{4} = -\frac{43}{2}$

39. $\frac{3}{m+4} = \frac{9}{14}$

$42 = 9m + 36$

$6 = 9m$

$\frac{2}{3} = m$

40. $\frac{3}{p-6} = \frac{1}{p}$

$3p = p - 6$

$2p = -6$

$p = -3$

41. $\frac{r}{3r+1} = \frac{2}{3}$

$3r = 6r + 2$

$-3r = 2$

$r = -\frac{2}{3}$

42. $\frac{w}{4} = \frac{9}{w}$

$w^2 = 36$

$w = \pm 6$