CHAPTER 5

Think & Discuss *(p. 261)*

1. Answers may vary.

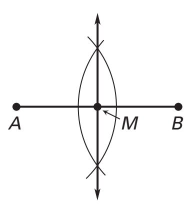
Sample answer:

At position B, the goalkeeper is closer to both sides of the imaginary triangle formed by the goal posts and the opponent. The goal keeper has an equal space to defend to the left and right and so has a greater chance of catching or deflecting a ball shot to either side than she would have at position A or position C.

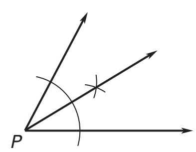
2. about 40°; an opponent could move closer to the goal to increase the shooting angle.

Skill Review *(p. 262)*

1. *Sample answer:*



2. *Sample answer:*



3. $M = \left(\frac{0 + (-2)}{2}, \frac{4 + 0}{2}\right) = \left(-\frac{2}{2}, \frac{4}{2}\right) = (-1, 2)$

4. $AB = \sqrt{(0 - 3)^2 + (4 - 0)^2}$

$= \sqrt{(-3)^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

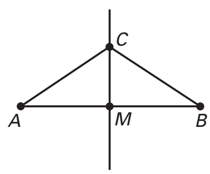
5. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-2 - 0} = \frac{-4}{-2} = 2$

6. The slope of the line perpendicular to $\overline{BC}$ is $-\frac{1}{2}$ because $-\frac{1}{2} \cdot 2 = -1$.

Lesson 5.1

Developing Concepts Activity 5.1 (p. 263)
Exploring the Concept

1.–3. *Sample answer*



4. *Sample answer:*

$MA = MB = 32$ mm

5. $m\angle CMA = 90^\circ$

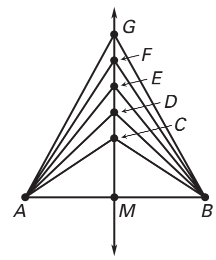
6. *Sample answer:*

$CA = CB = 37$ mm

Drawing Conclusions

1. $\overleftrightarrow{CM}$ is perpendicular to $\overline{AB}$ since $m\angle CAB = 90^\circ$ and $\overleftrightarrow{CM}$ intersects $\overline{AB}$ at its midpoint.

2. *Sample answers:*



3. *Sample answer:*

Point D	Point E	Point F	Point G
$DA = 43$ mm	$EA = 48$ mm	$FA = 58$ mm	$GA = 67$ mm
$DB = 43$ mm	$EB = 48$ mm	$FB = 58$ mm	$GB = 67$ mm

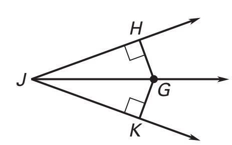
The distances from D to the endpoints A and B are equal, the distances from E to the endpoints A and B are equal, the distances from F to the endpoints A and B are equal, and the distances from G to the endpoints A and B are equal.

4. Any point on the perpendicular bisector is the same distance from either of the endpoints of the segment.

5.1 Guided Practice (p. 267)

1. If D is on the *perpendicular bisector* of $\overline{AB}$, then D is *equidistant* from A and B.

2. Point G must be on the bisector of $\angle HJK$ by Theorem 5.4.



3. $\overline{AD} \cong \overline{BD}$

4. $\angle ADC$ and $\angle BDC$ are both right angles and are congruent.

5. $\overline{AC} \cong \overline{BC}$ because C is on the perpendicular bisector of $\overline{AB}$.

6. $m\angle LPM = m\angle NPM$

7. The distance from M to $\overrightarrow{PL}$ is equal to the distance from M to $\overrightarrow{PN}$.

5.1 Practice and Applications (pp. 268–271)

8. No; C is not on the perpendicular bisector of $\overline{AB}$ because AC and BC are not equal.

9. No; the diagram does not show that $CA = CB$.

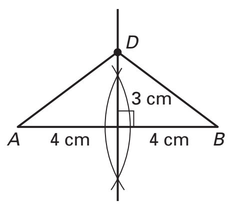
10. No; along with the information given, we would also need $AP = PB$.

11. No; since P is not equidistant from the sides of $\angle A$, P is not on the bisector of $\angle A$.

12. No; since we do not know for sure that one of the distances given is a perpendicular distance.

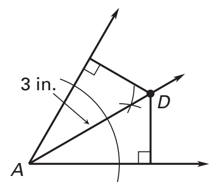
13. No; the diagram does not show that the segments with equal length are perpendicular segments.

14.



$AD = BD = 5$ cm

15.



D is $1\frac{1}{2}$ inches from each side of $\angle A$.

16. $VT = 8$ **17.** $SR = 17$

18. Point U must be on $\overleftrightarrow{SV}$, the perpendicular bisector of $\overline{RT}$.

19. $NQ = 2$

20. Point M must be on $\overrightarrow{JN}$, the bisector of $\angle HJK$. **21.** B

22. $m\angle XTV + m\angle TVX = 90°$

$m\angle XTV + 30° = 90°$

$m\angle XTV = 60°$

A

23. $m\angle VWU + m\angle UVW = 90°$

$m\angle VWU + 50° = 90°$

$m\angle VWU = 40°$

$m\angle VWU = m\angle VWX = 40°$

C

24. F **25.** D **26.** E

27. **Given:** P is on m.

Prove: $\overleftrightarrow{CP} \perp \overline{AB}$.

Statements	Reasons
1. P is on line m.	**1.** Given
2. $\overline{PA} \cong \overline{PB}$ $\overline{CA} \cong \overline{CB}$	**2.** By construction
3. $\overline{CP} \cong \overline{CP}$	**3.** Reflexive Property of Congruence
4. $\triangle CPA \cong \triangle CPB$	**4.** SSS Congruence Postulate
5. $\angle CPA \cong \angle CPB$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overleftrightarrow{CP} \perp \overline{AB}$	**6.** Theorem 3.1

28.

Statements	Reasons
1. $\overleftrightarrow{CP} \perp \overline{AB}$, $\overleftrightarrow{CP}$ bisects $\overline{AB}$.	**1.** Given
2. $\overline{AP} \cong \overline{BP}$	**2.** Definition of segment bisector
3. $\angle CPA$ and $\angle CPB$ are right angles.	**3.** Definition of perpendicular lines
4. $\overline{CP} \cong \overline{CP}$	**4.** Reflexive Property of Congruence
5. $\triangle APC \cong \triangle BPC$	**5.** SAS Congruence Postulate
6. $\overline{CA} \cong \overline{CB}$	**6.** Corresponding parts of congruent triangles are congruent.
7. $CA = CB$	**7.** Definition of congruent segments
8. C is equidistant from A and B.	**8.** Definition of equidistant

29.

Statements	Reasons
1. Construct $\overleftrightarrow{CP} \perp \overline{AB}$ intersecting $\overline{AB}$ at a point P.	**1.** Perpendicular Postulate
2. $\angle CPA$ and $\angle CPB$ are right angles.	**2.** Definition of perpendicular lines
3. $\triangle CPA$ and $\triangle CPB$ are right triangles.	**3.** Definition of right triangles
4. $CA = CB$, or $\overline{CA} \cong \overline{CB}$	**4.** Given; Definition of congruence
5. $\overline{CP} \cong \overline{CP}$	**5.** Reflexive Property of Congruence
6. $\triangle CPA \cong \triangle CPB$	**6.** HL Congruence Theorem
7. $\overline{AP} \cong \overline{BP}$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$ and C is on the perpendicular bisector of $\overline{AB}$.	**8.** Definition of perpendicular bisector

30.

Statements	Reasons
1. $\overline{GJ}$ is the perpendicular bisector of $\overline{HK}$.	**1.** Given
2. $\overline{GJ} \perp HK$, $HJ = JK$	**2.** Definition of perpendicular bisector of a segment
3. $GH = GK$, $MH = MK$	**3.** Perpendicular Bisector Theorem
4. $\overline{GH} \cong \overline{GK}$, $\overline{MH} \cong \overline{MK}$	**4.** Definition of congruent segments
5. $\overline{GM} \cong \overline{GM}$	**5.** Reflexive Property of Congruence
6. $\triangle GMH \cong \triangle GMK$	**6.** SSS Congruence Postulate

31. The post is the perpendicular bisector of the segment between the ends of the wires.

32.

Statements	Reasons
1. D is in the interior of $\angle ABC$.	**1.** Given
2. D is equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$.	**2.** Given
3. $DA = DC$	**3.** Definition of equidistant
4. $\overline{DA} \perp \overrightarrow{BA}$, $\overline{DC} \perp \overrightarrow{BC}$	**4.** Definition of distance from a point to a line
5. $\angle DAB$ and $\angle DCB$ are right angles.	**5.** If two lines are $\perp$, then they form 4 right angles.
6. $\triangle DAB$ and $\triangle DCB$ are right triangles.	**6.** Definition of right triangle
7. $\overline{BD} \cong \overline{BD}$	**7.** Reflexive Property of Congruence
8. $\triangle DAB \cong \triangle DCB$	**8.** HL Congruence Theorem
9. $\angle ABD \cong \angle CBD$	**9.** Corresponding parts of congruent triangles are congruent.
10. $\overrightarrow{BD}$ bisects $\angle ABC$ and point D is on the bisector of $\angle ABC$.	**10.** Definition of angle bisector

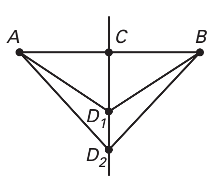
33. Line l is the perpendicular bisector of $\overline{AB}$.

34. $\overrightarrow{PG}$ should bisect $\angle APB$ to give the goalie equal distances to travel on each side of $\overrightarrow{PG}$.

35. $m\angle APB$ increases as the puck gets closer to the goal. This change makes it more difficult for the goalie because the goalie has a greater area to defend since the distances from goalie to the sides of $\angle APB$ (the shooting angle) increase.

36. Answers may vary.

Sample answer:

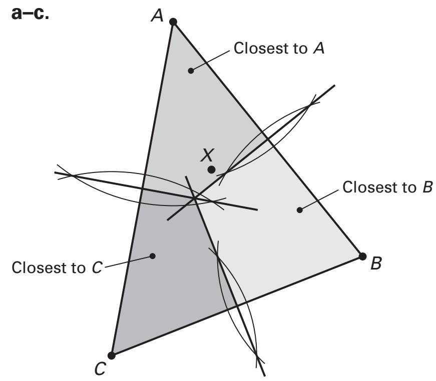


$D_1A = D_1B = 40$ mm

$D_2A = D_2B = 48$ mm

This demonstrates the Perpendicular Bisector Theorem because D is on the perpendicular bisector of $\overline{AB}$ and D is equidistant from A and B.

37. a–c.



The perpendicular bisectors meet in one point.

d. The fire station at A should respond because it is closest to the house at X.

38. slope of $\overline{WS} = m = \frac{5-4}{3-6} = \frac{1}{-3} = -\frac{1}{3}$

slope of $\overrightarrow{YX} = m = \frac{8-2}{4-2} = \frac{6}{2} = 3$

$\overline{WS} \perp \overrightarrow{YX}$ because the product of their slopes is -1.

$\left(-\frac{1}{3} \cdot 3 = -1\right)$.

slope of $\overline{WT} = m = \frac{1-4}{5-6} = \frac{-3}{-1} = 3$

slope of $\overline{YZ} = m = \frac{0-2}{8-2} = \frac{-2}{6} = -\frac{1}{3}$

$\overline{WT} \perp \overrightarrow{YZ}$ because the product of their slopes is -1.

$\left(3 \cdot \left(-\frac{1}{3}\right) = -1\right)$.

39. $WS = \sqrt{(3-6)^2 + (5-4)^2}$

$= \sqrt{(-3)^2 + 1^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$WT = \sqrt{(5-6)^2 + (1-4)^2}$

$= \sqrt{(-1)^2 + (-3)^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$

40. $\overrightarrow{YW}$ bisects $\angle XYZ$ because the perpendicular distances from W to $\overrightarrow{YS}$ and $\overrightarrow{YT}$ are equal. We know these are perpendicular distances because in problem 38 it was shown that $\overline{WS} \perp \overrightarrow{YX}$ and $\overline{WT} \perp \overrightarrow{YZ}$.

5.1 Mixed Review (p. 271)

41. $d = 12$ cm

$2r = 12$

$r = 6$ cm

42. $C = 2\pi r$

$\approx 2 \cdot 3.14 \cdot 6$

≈ 37.68 cm

43. $A = \pi r^2$

$\approx 3.14(6)^2$

$\approx 3.14 \cdot 36$

$\approx 113.04 \text{ cm}^2$

44. $m = \dfrac{10 - 5}{-2 - (-1)}$

$= \dfrac{5}{-1}$

$= -5$

45. $m = \dfrac{5 - (-3)}{-6 - 4} = \dfrac{8}{-10} = -\dfrac{4}{5}$

46. $m = \dfrac{5 - 5}{9 - 4} = \dfrac{0}{5} = 0$ **47.** $m = \dfrac{0 - 8}{-7 - 0} = \dfrac{-8}{-7} = \dfrac{8}{7}$

48. $m = \dfrac{12 - 11}{-10 - 3} = -\dfrac{1}{13}$

49. $m = \dfrac{-8 - (-8)}{8 - (-3)} = \dfrac{0}{11} = 0$

50. $x° + 31° = 90°$

$x = 59$

51. $(2x + 6)° = 40° + x°$

$2x = 34 + x$

$x = 34$

52. $(10x + 22)° = 70° + 4x°$

$10x = 48 + 4x$

$6x = 48$

$x = 8$

Lesson 5.2

Developing Concepts Activity (p. 272)

1.–4. Yes, all three bisectors intersect at the same point.

Conjecture: For any acute scalene triangle, the three perpendicular bisectors of the three sides will intersect at the same point.

$AP = BP = CP = 38$ mm

The three segments that are formed by connecting the vertices of the triangle to the point of intersection of the perpendicular bisectors of the sides have the same length.

5.2 Guided Practice (p. 275)

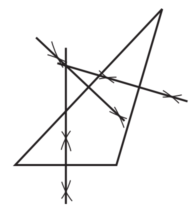
1. If three or more points intersect at the same point, the lines are *concurrent*.

2. The word *incenter* is made up of the words *in* and *center*. The incenter is the "center" of the circle that is "in" the triangle. The word *circumcenter* is made up of the parts *circum* and *center*. *Circum* can be short for *circumference*, which is the distance around the circle, and can help us to remember that the circumcenter is the "center" of the circle that is "around" the triangle.

3. $GC = GA = 7$ **4.** $MK = MJ = 5$

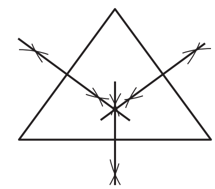
5.2 Practice and Applications (pp. 275–278)

5.



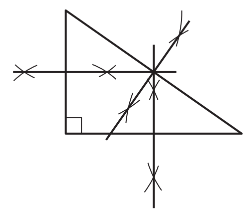
The perpendicular bisectors intersect outside the obtuse triangle.

6.



The perpendicular bisectors intersect at a point inside an acute triangle.

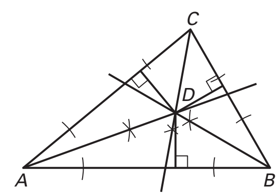
7.



The perpendicular bisectors intersect at a point on the right triangle.

8. and **9.**

Sample answer:



The segments are congruent. This confirms Theorem 5.6.

10. always **11.** always **12.** never **13.** sometimes

14. $DR = SD = 9$ **15.** $WB = WC = 20$

16. $(JC)^2 + (CK)^2 = (KJ)^2$ by the Pythagorean Theorem.

$4^2 + (CK)^2 = 5^2$

$16 + (CK)^2 = 25$

$(CK)^2 = 9$

$CK = 3$

$KB = CK = 3$

17. Let the midpoint of $\overline{PM}$ be called point R. Then

$PR = RM = \dfrac{PM}{2} = \dfrac{48}{2} = 24.$

$(PR)^2 + (QR)^2 = (PQ)^2$ by the Pythagorean Theorem.

$(24)^2 + (7)^2 = (PQ)^2$

$576 + 49 = (PQ)^2$

$625 = (PQ)^2$

$25 = PQ$

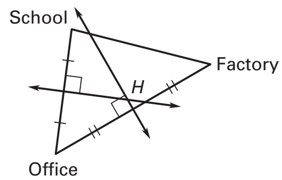
$QN = PQ = 25.$

18. The student's conclusion is false because D is not the point of intersection of the angle bisectors. D is the point of intersection of the perpendicular bisectors of the sides of the triangle. So $DA = DC = DB$.

19. The student's conclusion is false because the angle bisectors of a triangle intersect in a point that is equidistant from the sides of the triangle, but MQ and MN are not necessarily distances to the sides; M is equidistant from $\overline{JK}$, $\overline{LK}$, and $\overline{LJ}$.

20. To find the point that is equidistant from each location, draw the triangle and construct the perpendicular bisector of two sides. The point of intersection is the point that is equidistant from each location.

21. Point H is the point of intersection of the perpendicular bisectors. So H is equidistant from each location. H would be the best location for the new home.

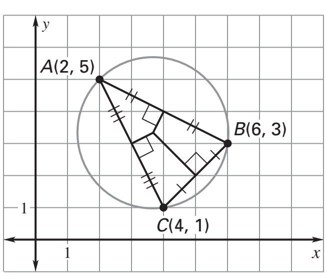


22.

Statements	Reasons
1. $\triangle ABC$, the bisectors of $\angle A$, $\angle B$, and $\angle C$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$	**1.** Given
2. $DE = DG$	**2.** $\overrightarrow{AD}$ bisects $\angle BAC$, so D is equidistant from the sides of $\angle BAC$.
3. $DE = DF$	**3.** $\overrightarrow{BD}$ bisects $\angle ABC$, so D is equidistant from the sides of $\angle ABC$.
4. $DF = DG$	**4.** Transitive property of equality
5. D is on the bisector of $\angle C$.	**5.** Converse of the Angle Bisector Theorem
6. D is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.	**6.** Givens and Steps 2, 3, and 4

23. She could construct the perpendicular bisectors to find the point that is equidistant from the vertices of the triangle. By doing so, she would see that the perpendicular bisectors do intersect at a point on the hypotenuse. Since the point on the hypotenuse would be the point of intersection of the perpendicular bisectors, then it would be equidistant from the vertices.

24.–25.



The radius is approximately $2\frac{1}{2}$ ft.

26. $2\frac{1}{2}$ ft $= 30$ inches

30 in. $\div$ 8 in. per year $= 3.75$ years

The mycelium is approximately 3.75 years old.

27.
$$m\angle ABC + m\angle BCA + m\angle CAB = 180°$$
$$100° + m\angle BCA + m\angle CAB = 180°$$
$$m\angle BCA + m\angle CAB = 80°$$
$$\tfrac{1}{2}(m\angle BCA + m\angle CAB) = \tfrac{1}{2} \cdot 80°$$
$$\tfrac{1}{2}(m\angle BCA) + \tfrac{1}{2}(m\angle CAB) = 40°$$
$$m\angle DCA + m\angle CAD = 40°$$
$$m\angle ADC + m\angle DCA + m\angle CAD = 180°$$
$$m\angle ADC + 40° = 180°$$
$$m\angle ADC = 140°$$

E

28. $XW = WZ = 13$

$(XT)^2 + (TW)^2 = (XW)^2$ (Pythagorean Theorem)

$(XT)^2 + 12^2 = 13^2$

$(XT)^2 + 144 = 169$

$(XT)^2 = 25$

$XT = 5$

$XY = 2(XT)$

because T is the midpoint of $\overline{XY}$.

$XY = 2 \cdot 5$

$XY = 10$

C

29. The midpoint of $\overline{AB}$ is

$$\left(\frac{0 + 12}{2}, \frac{0 + 6}{2}\right) = \left(\frac{12}{2}, \frac{6}{2}\right) = (6, 3).$$

The midpoint of $\overline{BC}$ is

$$\left(\frac{12 + 18}{2}, \frac{6 + 0}{2}\right) = \left(\frac{30}{2}, \frac{6}{2}\right) = (15, 3).$$

The midpoint of $\overline{AC}$ is

$$\left(\frac{0 + 18}{2}, \frac{0 + 0}{2}\right) = \left(\frac{18}{2}, \frac{0}{2}\right) = (9, 0).$$

The slope of $\overline{AB} = \frac{6 - 0}{12 - 0} = \frac{6}{12} = \frac{1}{2}$.

The perpendicular bisector has slope -2 because

$-2 \cdot \frac{1}{2} = -1$.

$y - 3 = -2(x - 6)$

$y - 3 = -2x + 12$

$y = -2x + 15$ is an equation of the perpendicular bisector of $\overline{AB}$.

The slope of $\overline{BC} = \frac{0 - 6}{18 - 12} = \frac{-6}{6} = -1$.

—CONTINUED—

29. —CONTINUED—

The perpendicular bisector of $\overline{BC}$ has slope 1 because $-1 \cdot 1 = -1$.

$y - 3 = 1 \cdot (x - 15)$

$y - 3 = x - 15$

$y = x - 12$ is an equation of the perpendicular bisector of $\overline{BC}$.

The slope of $\overline{AC} = \frac{0 - 0}{18 - 0} = \frac{0}{18} = 0$, so $\overline{AC}$ is horizontal.

So the perpendicular bisector is the vertical line $x = 9$.

30. The lines $y = -2x + 15$ and $x = 9$ intersect at the point $(9, -3)$ because $y = -2(9) + 15 = -18 + 15 = -3$. To show $(9, -3)$ is also on $y = x - 12$, substitute $x = 9$ and $y = -3$ in the equation.

$y = x - 12$

$-3 = 9 - 12$

$-3 = -3$

Since $-3 = -3$ is true, the point $(9, -3)$ is on the line $y = x - 12$.

31. Let P be the point $(9, -3)$.

$$AP = \sqrt{(9 - 0)^2 + (-3 - 0)^2} = \sqrt{9^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$$

$$BP = \sqrt{(9 - 12)^2 + (-3 - 6)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{9 + 81} = \sqrt{90} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$$

$$CP = \sqrt{(9 - 18)^2 + (-3 - 0)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$$

Since $AP = BP = CP = 3\sqrt{10}$, P is equidistant from A, B, and C.

5.2 Mixed Review (p. 278)

32. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 9 \cdot 5 = \frac{45}{2} = 22.5$ square units

33. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 22 \cdot 7 = \frac{154}{2} = 77$ square units

34. j has slope $-\frac{1}{3}$ because $-\frac{1}{3} \cdot 3 = -1$.

$y - 4 = -\frac{1}{3}(x - 1)$

$y - 4 = -\frac{1}{3}x + \frac{1}{3}$

$y = -\frac{1}{3}x + \frac{13}{3}$

An equation of j is $y = -\frac{1}{3}x + \frac{13}{3}$.

35. j has slope $\frac{1}{2}$ because $-2 \cdot \frac{1}{2} = -1$.

$y - 6 = \frac{1}{2}(x - 7)$

$y - 6 = \frac{1}{2}x - \frac{7}{2}$

$y = \frac{1}{2}x + \frac{5}{2}$

An equation of j is $y = \frac{1}{2}x + \frac{5}{2}$.

36. j has slope $\frac{3}{2}$ because $-\frac{2}{3} \cdot \frac{3}{2} = -1$.

$y - 8 = \frac{3}{2}(x - 2)$

$y - 8 = \frac{3}{2}x - 3$

$y = \frac{3}{2}x + 5$

An equation of j is $y = \frac{3}{2}x + 5$.

37. j has slope $-\frac{11}{10}$ because $-\frac{11}{10} \cdot \frac{10}{11} = -1$.

$y - (-9) = -\frac{11}{10}(x - (-2))$

$y + 9 = -\frac{11}{10}(x + 2)$

$y + 9 = -\frac{11}{10}x - \frac{11}{5}$

$y = -\frac{11}{10}x - \frac{56}{5}$

An equation of j is $y = -\frac{11}{10}x - \frac{56}{5}$.

38. There is enough information to prove $\triangle ABC \cong \triangle DEC$ by using the SAS Congruence Postulate.

39. There is not enough information given to prove $\triangle GJF \cong \triangle GJH$. One pair of congruent sides, one side congruent to itself, and one pair of congruent angles are given. But the angles must be the included angles and they are not.

40. There is enough information given to prove $\triangle PMN \cong \triangle KML$. One pair of congruent legs and one pair of congruent hypotenuses are given. The HL Congruence Theorem can be used to prove $\triangle PMN \cong \triangle KML$.

Lesson 5.3

5.3 Guided Practice (p. 282)

1. The *centroid* of a triangle is the point where the three *medians* intersect.

2. The legs, $\overline{KM}$ and $\overline{LM}$, of right $\triangle KLM$ are also altitudes of $\triangle KLM$ because $\overline{KM}$ is the perpendicular segment from K to $\overline{LM}$ and $\overline{LM}$ is the perpendicular segment from L to $\overline{KM}$.

3. $\overline{DG} \cong \overline{FG}$ indicates that G is the midpoint of $\overline{DF}$. Therefore, $\overline{EG}$ is a median of $\triangle DEF$.

4. $\overline{EG} \perp \overline{DF}$ indicates that $\overline{EG}$ is an altitude of $\triangle DEF$.

5. $\angle DEG \cong \angle FEG$ indicates the $\angle DEF$ is bisected. So $\overline{EG}$ is an angle bisector of $\triangle DEF$.

6. $\overline{EG} \perp \overline{DF}$ and $\overline{DG} \cong \overline{FG}$ indicates G is the midpoint of $\overline{DF}$ and $\overline{EG}$ is a perpendicular bisector of $\triangle DEF$, but it is also a median, an altitude, and an angle bisector since $\triangle DGE \cong \triangle FGE$ by the SAS Congruence Postulate so $\angle DEG \cong \angle FEG$.

7. $\triangle DGE \cong \triangle FGE$ indicates $\angle DEG \cong \angle FEG$, $\angle DGE \cong \angle FGE$, and $\overline{DG} \cong \overline{FG}$. In this case $\overline{EG}$ is a perpendicular bisector, an angle bisector, a median, and an altitude of $\triangle DEF$.

5.3 Practice and Applications (pp. 282–284)

8. $FH = DH = 9$

9.
$$EP = \tfrac{2}{3}EH$$
$$8 = \tfrac{2}{3}EH$$
$$\tfrac{3}{2} \cdot 8 = EH$$
$$12 = EH$$

10.
$$PH + PE = EH$$
$$PH + 8 = 12$$
$$PH = 4$$

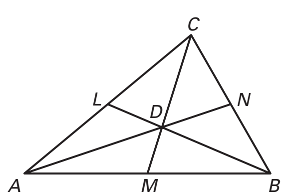
11.
$$(EH)^2 + (HF)^2 = (EF)^2$$
$$12^2 + 9^2 = (EF)^2$$
$$144 + 81 = (EF)^2$$
$$225 = (EF)^2$$
$$15 = EF$$

Perimeter of $\Delta DEF = DE + EF + FD$
$= 2(DG) + 15 + 2(DH)$
$= 2 \cdot 7.5 + 15 + 2 \cdot 9$
$= 15 + 15 + 18$
$= 48$ units

12. $\dfrac{PH}{EH} = \dfrac{4}{12} = \dfrac{1}{3}$

$\dfrac{PH}{EP} = \dfrac{4}{8} = \dfrac{1}{2}$

13.–14. *Sample answer:*



$\triangle ABC$ is an acute triangle.

15. Yes, they all met at the same point. It is labeled as point D.

16. *Sample answer:*

$AD = 64$ mm
$AN = 96$
$AD = \tfrac{2}{3}(AN)$
$64 = \tfrac{2}{3} \cdot 96$
$64 = 64$
$BD = 54$ mm
$BL = 81$ mm
$BD = \tfrac{2}{3}(BL)$
$54 = \tfrac{2}{3} \cdot 81$
$54 = 54$
$CD = 40$ mm
$CM = 60$ mm
$CD = \tfrac{2}{3}(CM)$
$40 = \tfrac{2}{3} \cdot 60$
$40 = 40$

The distance from the centroid to a vertex is two thirds of the distance from that vertex to the midpoint of the opposite side.

17. $Q = \left(\dfrac{-1 + 11}{2}, \dfrac{-2 + 2}{2}\right) = \left(\dfrac{10}{2}, \dfrac{0}{2}\right) = (5, 0)$

18.
$$PQ = \sqrt{(5-5)^2 + (0-6)^2}$$
$$= \sqrt{0^2 + (-6)^2}$$
$$= \sqrt{0 + 36}$$
$$= \sqrt{36}$$
$$= 6$$

19.
$$PT = \tfrac{2}{3}PQ$$
$$PT = \tfrac{2}{3} \cdot 6$$
$$PT = 4$$

The coordinates of T are $(5, 6 - 4) = (5, 2)$.

20. $R = \left(\dfrac{-1 + 5}{2}, \dfrac{-2 + 6}{2}\right) = \left(\dfrac{4}{2}, \dfrac{4}{2}\right) = (2, 2)$

$NT = 11 - 5 = 6$

$NR = 11 - 2 = 9$

$\dfrac{NT}{NR} = \dfrac{6}{9} = \dfrac{2}{3}$

21. $M = \left(\dfrac{5 + 3}{2}, \dfrac{2 + 6}{2}\right) = \left(\dfrac{8}{2}, \dfrac{8}{2}\right) = (4, 4)$

22. $JP = \sqrt{(5 - 7)^2 + (6 - 10)^2}$

$= \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= \sqrt{4} \cdot \sqrt{5}$

$= 2\sqrt{5}$

$JM = \sqrt{(4 - 7)^2 + (4 - 10)}$

$= \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= \sqrt{9} \cdot \sqrt{5}$

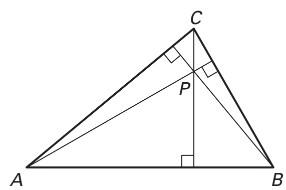
$= 3\sqrt{5}$

23. $JP = \frac{2}{3}JM$

$2\sqrt{5} = \frac{2}{3} \cdot 3\sqrt{5}$

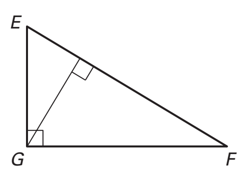
$2\sqrt{5} = 2\sqrt{5}$

24. *Sample answer:*



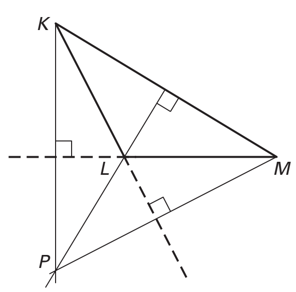
P is the orthocenter of $\triangle ABC$.

25. *Sample answer:*



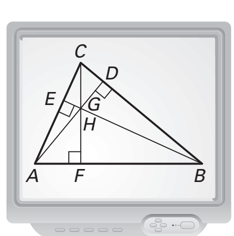
The orthocenter of $\triangle EFG$ is point *G*.

26. *Sample answer:*



P is the orthocenter of $\triangle KLM$.

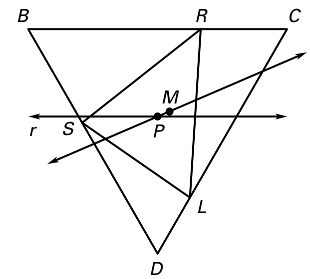
27.



28. *G* and *H* are the same point.

29. When $\overline{GH}$ is measured, it is found that $GH = 0$. Since $GH = 0$, then *G* and *H* must be the same point; therefore the lines containing the three altitudes intersect at one point.

30.–32.



33. The measure of the angle between *r* and $\overline{MP}$ is approximately 20°.

34. a. $(AD)^2 + (CD)^2 = (AC)^2$

$12^2 + (CD)^2 = 15^2$

$144 + (CD)^2 = 225$

$(CD)^2 = 81$

$CD = 9$

b. $A = \frac{1}{2}bh = \frac{1}{2} \cdot 20 \cdot 9 = 90$ square units

c. Check drawings.

d. $A = \frac{1}{2}bh$

$90 = \frac{1}{2} \cdot CA \cdot BE$

$90 = \frac{1}{2} \cdot 15 \cdot BE$

$90 = 7.5 \cdot BE$

$12 = BE$

e. $A = \frac{1}{2}bh$

$2A = bh$

$\frac{2A}{b} = h$

The length of the altitude is equal to twice the area divided by the base.

35.

Statements	Reasons
1. $\triangle ABC$ is isosceles; $\overline{BD}$ is a median to base $\overline{AC}$.	**1.** Given
2. D is the midpoint of $\overline{AC}$.	**2.** Definition of median
3. $\overline{AD} \cong \overline{CD}$	**3.** Definition of midpoint
4. $\overline{AB} \cong \overline{CB}$	**4.** Definition of isosceles triangle
5. $\overline{BD} \cong \overline{BD}$	**5.** Reflexive Property of Congruence
6. $\triangle BDC \cong \triangle BDA$	**6.** SSS Congruence Postulate
7. $\angle BDC \cong \angle BDA$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\angle BDC$ and $\angle BDA$ are a linear pair.	**8.** Definition of linear pair
9. $\overline{BD} \perp \overline{AC}$	**9.** If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
10. $\overline{BD}$ is also an altitude.	**10.** Definition of altitude

36. No, medians to the *legs* of an isosceles triangle are not perpendicular to the legs (unless the triangle is actually equilateral).

37. Yes, the medians of an equilateral triangle are also altitudes because the proof for the isosceles triangle could be used for the equilateral triangle.

Yes, the medians would be contained in the angle bisectors. By looking at the proof in Exercise 35, it can be seen that the median was also the angle bisector since the two triangles are congruent.

Yes the medians would be contained in the perpendicular bisectors because it was shown in Exercise 35 that the median was perpendicular to the side at the midpoint.

38. The median of an equilateral triangle is also a perpendicular bisector of a side, an altitude, and an angle bisector.

5.3 Mixed Review (p. 284)

39. The parallel line would also have slope -1.

$$y - 7 = -1(x - 1)$$
$$y - 7 = -x + 1$$
$$y = -x + 8$$

An equation of the line through P that is parallel to $y = -x + 3$ is $y = -x + 8$.

40. The parallel line would also have slope -2.

$$y - (-8) = -2(x - (-3))$$
$$y + 8 = -2(x + 3)$$
$$y + 8 = -2x - 6$$
$$y = -2x - 14$$

An equation of the line through P that is parallel to $y = -2x - 3$ is $y = -2x - 14$.

41. The parallel line would also have slope 3.

$$y - (-9) = 3(x - 4)$$
$$y + 9 = 3x - 12$$
$$y = 3x - 21$$

An equation of the line through P that is parallel to $y = 3x + 5$ is $y = 3x - 21$.

42. The parallel line would also have slope $-\frac{1}{2}$.

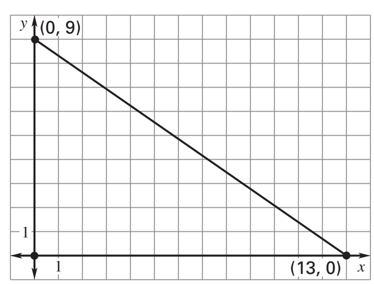
$$y - (-2) = -\tfrac{1}{2}(x - 4)$$
$$y + 2 = -\tfrac{1}{2}x + 2$$
$$y = -\tfrac{1}{2}x$$

An equation of the line through P that is parallel to $y = -\frac{1}{2}x - 1$ is $y = -\frac{1}{2}x$.

43. $\angle E \cong \angle H$ because you need the angles which do not have $\overline{DF}$ or $\overline{GJ}$ as a side.

44. $\angle F \cong \angle J$ because you need the angles which have $\overline{EF}$ or $\overline{HJ}$ as a side.

45. *Sample answer:*



$$h = \sqrt{(13 - 0)^2 + (0 - 9)^2}$$
$$= \sqrt{13^2 + 9^2}$$
$$= \sqrt{169 + 81}$$
$$= \sqrt{250}$$
$$= \sqrt{25} \cdot \sqrt{10}$$
$$= 5\sqrt{10}$$

Quiz 1 (p. 285)

1. $4x + 9 = 3x + 25$

$$4x = 3x + 16$$
$$x = 16$$

2. $3y = y + 24$

$$2y = 24$$
$$y = 12$$

3. $6^2 + 8^2 = (VT)^2$

$36 + 64 = (VT)^2$

$100 = (VT)^2$

$10 = VT$

4. $VT = VS = 10$ because the perpendicular bisectors intersect at a point equidistant from the vertices of the triangle.

5. The balancing point would be at point G because that is the centroid of the triangle.

5.3 Math and History (p. 285)

1. You need to go to the post office (P), then the market (M), then the library (L) or in reverse order.

2. The goalie's position on the angle bisector optimizes the chance of blocking a scoring shot because the distance the goalie would have to travel to protect either side of the goal would be the same.

Technology Activity 5.3 (p. 286)

Investigate

1. $m\angle BAF = m\angle CAF$

So $\overrightarrow{AF}$ is the angle bisector of $\angle BAC$.

2. F was the point of intersection of the angle bisectors of angles $\angle ABC$ and $\angle BCA$ by construction. Since $m\angle BAF = m\angle CAF$, $\overrightarrow{AF}$ is the angle bisector of $\angle BAC$. Because angle bisector $\overrightarrow{AF}$ passes through the intersection point, F, of $\overrightarrow{BD}$ and $\overrightarrow{CE}$, the three angle bisectors are concurrent.

3. $AG = BG$. This makes $\overline{CG}$ a median also.

4. F was the point of intersection of two medians by construction. Since $AG = BG$, G is the midpoint of $\overline{AB}$ and $\overline{CG}$ is the third median of the triangle. Since F is on $\overline{CG}$ by construction, F is on all three medians and the medians are concurrent.

5. *Sample measures are given:*

$AD = 72$ mm

$AF = 108$ mm

$\frac{AD}{AF} = \frac{72}{108} = \frac{2}{3}$

Yes, $AD = \frac{2}{3}AF$

6. No, the quotient $\frac{AD}{AF}$ does not change.

Extension

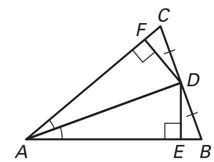
For any triangle in which the angle bisector is contained in the same line as the median, the line will also contain an altitude and perpendicular bisector of the triangle.

—CONTINUED—

This will occur for the bisector of the vertex angle of an isosceles triangle and for the bisector of each angle of an equilateral triangle.

Given: $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{AD}$ is a median.

Prove: $\overline{AD}$ is an altitude and $\overline{AD}$ is a perpendicular bisector of $\overline{BC}$.



In this drawing $\overrightarrow{AD}$ is the angle bisector of $\angle BAC$ and $\overline{AD}$ is the median of the triangle. So $\overline{DC} \cong \overline{DB}$. Since $\overrightarrow{AD}$ is the angle bisector, $DE = DF$ or $\overline{DE} \cong \overline{DF}$. $\overline{AD} \cong \overline{AD}$.

Then $\triangle ADE \cong \triangle ADF$ by the HL Congruence Theorem. Therefore, $\angle ADE \cong \angle ADF$ or $m\angle ADE = m\angle ADF$ because corresponding parts of congruent triangles are congruent. Also $\triangle CFD \cong \triangle BED$ by the HL Congruence Theorem. So, $\angle CDF \cong \angle BDE$ or $m\angle CDF = m\angle BDE$ because corresponding parts of congruent triangles are congruent. Then $m\angle CDF + m\angle ADF = m\angle BDE + m\angle ADE$. So $m\angle ADC = m\angle ADB$ which are linear pairs. Therefore, $\angle ADC \cong \angle ADB$ and $\overline{AD} \perp \overline{CD}$. This would mean that $\overline{AD}$ would be an altitude and a perpendicular bisector, also.

Lesson 5.4

5.4 Guided Practice (p. 290)

1. In $\triangle ABC$, if M is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{AC}$, and P is the midpoint of $\overline{BC}$, then $\overline{MN}$, $\overline{NP}$, and $\overline{PN}$ are *midsegments* of triangle $\triangle ABC$.

2. It is convenient to position one of the sides of the triangle along the x-axis because some of the coordinates of two of the points will be zero and one side will be horizontal and have a slope of 0.

3. $\overline{JH} \parallel \overline{DF}$ **4.** $\overline{GH} \parallel \overline{DE}$

5. $EF = 2 \cdot EH$

$= 2 \cdot 10.6$

$= 21.2$

6. $GH = \frac{1}{2} \cdot DE$

$= \frac{1}{2} \cdot 24$

$= 12$

7. $DF = 2 \cdot DG$

$= 2 \cdot 8$

$= 16$

8. $JH = \frac{1}{2} \cdot DF$

$= \frac{1}{2} \cdot 16$

$= 8$

9. $GJ = \frac{1}{2} \cdot EF$

$= \frac{1}{2} \cdot 21.2$

$= 10.6$

Perimeter $= GJ + JH + GH$

$= 10.6 + 8 + 12$

$= 30.6$

10. $A = \left(\frac{0+2}{2}, \frac{0+8}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

$B = \left(\frac{2+10}{2}, \frac{8+4}{2}\right) = \left(\frac{12}{2}, \frac{12}{2}\right) = (6, 6)$

11. $AB = \sqrt{(6-1)^2 + (6-4)^2}$

$= \sqrt{5^2 + 2^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29} \approx 5.4$

$AB \approx 5.4 \cdot 10 \text{ yd} \approx 54 \text{ yd}$

12. $\overline{LM} \parallel \overline{BC}$ **13.** $\overline{AB} \parallel \overline{MN}$

14. $LN = \frac{1}{2} \cdot AC = \frac{1}{2} \cdot 20 = 10$

15. $AB = 2 \cdot MN = 2 \cdot 7 = 14$

16. $BC = 2 \cdot NC = 2 \cdot 9 = 18$

$LM = \frac{1}{2} \cdot BC = NC = 9$

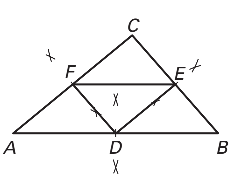
17.
$$\begin{aligned} LM &= \tfrac{1}{2} \cdot BC \\ 3x + 7 &= \tfrac{1}{2}(7x + 6) \\ 3x + 7 &= \tfrac{7}{2}x + 3 \\ 3x &= \tfrac{7}{2}x - 4 \\ -\tfrac{1}{2}x &= -4 \\ x &= 8 \end{aligned}$$
$$\begin{aligned} LM &= 3x + 7 \\ &= 3 \cdot 8 + 7 \\ &= 24 + 7 \\ &= 31 \end{aligned}$$

18.
$$\begin{aligned} MN &= \tfrac{1}{2} \cdot AB \\ x - 1 &= \tfrac{1}{2}(6x - 18) \\ x - 1 &= 3x - 9 \\ x &= 3x - 8 \\ -2x &= -8 \\ x &= 4 \end{aligned}$$
$$\begin{aligned} AB &= 6x - 18 \\ &= 6 \cdot 4 - 18 \\ &= 24 - 18 \\ &= 6 \end{aligned}$$

19. $\angle BLN$, $\angle A$, and $\angle NMC$ are congruent by the Corresponding Angles Postulate, as are $\angle BNL$, $\angle C$, and $\angle LMA$. $\angle LMA \cong \angle NLM$ and $\angle NMC \cong \angle LNM$ by the Alternate Interior Angles Theorem.

So, by the Transitive Property of Congruence, $\angle BLN$, $\angle A$, $\angle NMC$, and $\angle LNM$ are congruent, as are $\angle BNL$, $\angle C$, $\angle LMA$, and $\angle NLM$. Then $\angle B$, $\angle ALM$, $\angle LMN$, and $\angle MNC$ are all congruent by the Third Angles Theorem and the Transitive Property of Congruence.

20. *Sample answer:*



Use the construction of the perpendicular bisectors of the sides to find the midpoints *D, E,* and *F*.

$\overline{DE}$, $\overline{EF}$, and $\overline{DF}$ are the midsegments of $\triangle ABC$.

21. $D = \left(\frac{0+5}{2}, \frac{2+(-2)}{2}\right) = \left(\frac{5}{2}, \frac{0}{2}\right) = \left(\frac{5}{2}, 0\right)$

$E = \left(\frac{5+10}{2}, \frac{-2+6}{2}\right) = \left(\frac{15}{2}, \frac{4}{2}\right) = \left(\frac{15}{2}, 2\right)$

$F = \left(\frac{0+10}{2}, \frac{2+6}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$

22. Slope of $\overline{BC} = \frac{6-(-2)}{10-5} = \frac{8}{5}$

Slope of $\overline{DF} = \frac{4-0}{5-\frac{5}{2}} = \frac{4}{\frac{5}{2}} = 4 \cdot \frac{2}{5} = \frac{8}{5}$

Since the slopes of $\overline{BC}$ and $\overline{DF}$ are equal, $\overline{BC} \parallel \overline{DF}$.

$DF = \sqrt{\left(5 - \frac{5}{2}\right)^2 + (4-0)^2}$

$= \sqrt{2.5^2 + 4^2}$

$= \sqrt{6.25 + 16}$

$= \sqrt{22.25}$

$BC = \sqrt{(10-5)^2 + (6-(-2))^2}$

$= \sqrt{5^2 + 8^2}$

$= \sqrt{25 + 64}$

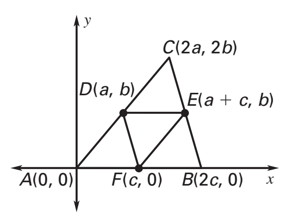
$= \sqrt{89}$

$DF = \sqrt{22.25} = \sqrt{\frac{89}{4}} = \frac{\sqrt{89}}{\sqrt{4}} = \frac{\sqrt{89}}{2} = \frac{1}{2} \cdot \sqrt{89}$

$DF = \frac{1}{2}BC$

So $\overline{BC} \parallel \overline{DF}$ and $DF = \frac{1}{2}BC$.

23.



$F = \left(\frac{0+2c}{2}, \frac{0+0}{2}\right) = \left(\frac{2c}{2}, \frac{0}{2}\right) = (c, 0)$

24. Slope of $\overline{DF} = \frac{b-0}{a-c}$

$= \frac{b}{a-c}$

Slope of $\overline{CB} = \frac{2b-0}{2a-2c}$

$= \frac{2b}{2a-2c}$

$= \frac{2b}{2(a-c)}$

$= \frac{b}{a-c}$

—CONTINUED—

24. —CONTINUED—

Since the slopes of $\overline{DF}$ and $\overline{CB}$ are equal, $\overline{DF} \parallel \overline{CB}$.

Slope of $\overline{EF} = \dfrac{b - 0}{a + c - c} = \dfrac{b}{a}$

Slope of $\overline{CA} = \dfrac{2b - 0}{2a - 0} = \dfrac{2b}{2a} = \dfrac{b}{a}$

Since the slopes of $\overline{EF}$ and $\overline{CA}$ are equal, $\overline{EF} \parallel \overline{CA}$.

25. $DF = \sqrt{(a - c)^2 + (b - 0)^2}$

$= \sqrt{(a - c)^2 + b^2}$

$EF = \sqrt{(a + c - c)^2 + (b - 0)^2}$

$= \sqrt{a^2 + b^2}$

$CB = \sqrt{(2a - 2c)^2 + (2b - 0)^2}$

$= \sqrt{2^2(a - c)^2 + (2b)^2}$

$= \sqrt{4(a - c)^2 + 4b^2}$

$= \sqrt{4[(a - c)^2 + b^2]}$

$= \sqrt{4}\sqrt{(a - c)^2 + b^2}$

$= 2\sqrt{(a - c)^2 + b^2}$

$CA = \sqrt{(2a - 0)^2 + (2b - 0)^2}$

$= \sqrt{(2a)^2 + (2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= \sqrt{4(a^2 + b^2)}$

$= \sqrt{4}\sqrt{a^2 + b^2}$

$= 2\sqrt{a^2 + b^2}$

$DF = \sqrt{(a - c)^2 + b^2}$

$= \frac{1}{2}\left(2\sqrt{(a - c)^2 + b^2}\right)$

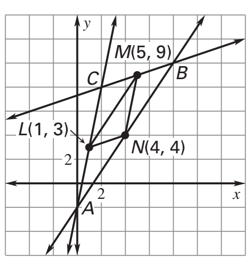
$= \frac{1}{2}CB$

$EF = \sqrt{a^2 + b^2}$

$= \frac{1}{2} \cdot 2\sqrt{a^2 + b^2}$

$= \frac{1}{2}CA$

26.



Slope of $\overline{LN} = \dfrac{4 - 3}{4 - 1} = \dfrac{1}{3}$

Draw a line through M with slope $\dfrac{1}{3}$.

Slope of $\overline{MN} = \dfrac{9 - 4}{5 - 4} = \dfrac{5}{1} = 5$

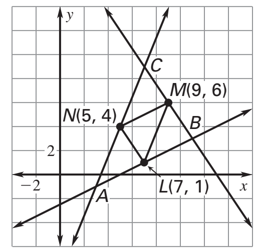
Draw a line through L with slope 5.

Slope of $\overline{LM} = \dfrac{9 - 3}{5 - 1} = \dfrac{6}{4} = \dfrac{3}{2}$

Draw a line through N with slope $\dfrac{3}{2}$.

The lines intersect at $A(0, -2)$, $B(8, 10)$, and $C(2, 8)$.

27.



Slope of $\overline{LN} = \dfrac{4 - 1}{5 - 7} = \dfrac{3}{-2} = -\dfrac{3}{2}$

Draw a line through M with slope $-\dfrac{3}{2}$.

Slope of $\overline{MN} = \dfrac{6 - 4}{9 - 5} = \dfrac{2}{4} = \dfrac{1}{2}$

Draw a line through L with slope $\dfrac{1}{2}$.

Slope of $\overline{LM} = \dfrac{6 - 1}{9 - 7} = \dfrac{5}{2}$

Draw a line through N with slope $\dfrac{5}{2}$.

The lines intersect at $A(3, -1)$, $B(11, 3)$, and $C(7, 9)$.

28. $GF = \frac{1}{2}BD$

$8 = \frac{1}{2}BD$

$16 = BD$

$BC = BG + GC$

$= GC + GC$ because $\overline{BG} \cong \overline{GC}$.

$= 2(GC)$

$= 2 \cdot 5$

$= 10$

$P = CD + BD + BC = 14 + 16 + 10 = 40$ units

29. $TU = \frac{1}{2}PQ$

$= \frac{1}{2}(20)$

$= 10$

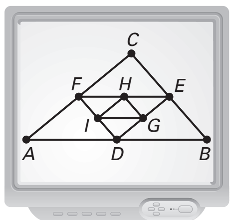
$ST = \frac{1}{2}(QR)$

$= QU$

$= 9$

$P = SU + TU + ST = 12 + 10 + 9 = 31$ units

30.



Perimeter of $\triangle ABC$ is 4 times the perimeter of $\triangle GHI$.

$$GH = \tfrac{1}{2}FD = \tfrac{1}{2}\left(\tfrac{1}{2}BC\right) = \tfrac{1}{4}BC$$

$$HI = \tfrac{1}{2}DE = \tfrac{1}{2}\left(\tfrac{1}{2}AC\right) = \tfrac{1}{4}AC$$

$$GI = \tfrac{1}{2}FE = \tfrac{1}{2}\left(\tfrac{1}{2}AB\right) = \tfrac{1}{4}AB$$

Perimeter of $\triangle GHI = GH + HI + GI$

$= \tfrac{1}{4}BC + \tfrac{1}{4}AC + \tfrac{1}{4}AB$

$= \tfrac{1}{4}(BC + AC + AB)$

Perimeter of $\triangle GHI = \tfrac{1}{4}$ of perimeter of $\triangle ABC$

31. The perimeter of the shaded triangle in Stage 1 is $\frac{1}{2}$ because the length of each side of the shaded triangle is $\frac{1}{2}$ the length of a side in the original triangle.

The total perimeter of the shaded triangle in Stage 2 is $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1\frac{1}{4}$.

The total perimeter of the shaded triangles in Stage 3 is $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 2\frac{3}{8}$.

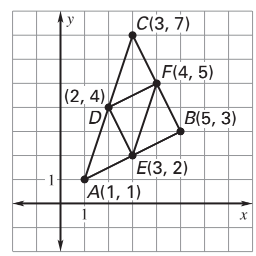
32. The bottoms of the legs will be 60 inches apart. Since the cross bar attaches at the midpoints of the legs, the cross bar is a midsegment of the triangle formed by the two legs and the ground. Since the length of the midsegment is half of the length of third side, the length of the cross bar, 30 inches, is half of the distance between the bottoms of the legs, 60 inches.

33. $\overline{DE}$ is a midsegment of $\triangle ABC$, so D is the midpoint of $\overline{AB}$ and $\overline{AD} \cong \overline{DB}$. By the Midsegment Theorem, $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$. But F is the midpoint of $\overline{BC}$, so $BF = \frac{1}{2}BC$. Then by the transitive property of equality and the definition of congruent segments, $\overline{DE} \cong \overline{BF}$. Corresponding angles $\angle ADE$ and $\angle ABC$ are congruent, so $\triangle ADE \cong \triangle DBF$ by the SAS Congruence Postulate.

34. In Exercise 33, it was shown that $\overline{AD} \cong \overline{DB}$ and $\overline{DE} \cong \overline{BF}$. So all that is left to show is that $\overline{AE} \cong \overline{DF}$. This can be done in the same manner that it was shown that $\overline{DE} \cong \overline{BF}$. By using the fact that $\overline{DE}$ is a midsegment of $\triangle ABC$, $DF = \frac{1}{2}AC$. By using the fact that E is the midpoint of $\overline{AC}$, we can get $AE = \frac{1}{2}AC$. Therefore, $AE = DF$ or $\overline{AE} \cong \overline{DF}$ and the triangles are congruent by the SSS Congruence Postulate.

35. Since $\overline{PQ}$ is closer to $\overline{RS}$ than $\overline{MN}$, it must be longer than $\overline{MN}$. Since $\overline{MN}$ is the midsegment, $MN = \frac{1}{2}RS$ or $MN = \frac{1}{2} \cdot 24 = 12$ feet. So $\overline{PQ}$ cannot be 10 or 12 feet long since it must be longer than $\overline{MN}$, which is 12 feet long. $\overline{PQ}$ could be 14 feet long but not 24 feet long since PQ cannot equal RS. So, $MN < PQ < RS$, or $12 < PQ < 24$.

36. a.



b. slope of $\overline{DE} = m_1 = \dfrac{4 - 2}{2 - 3} = \dfrac{2}{-1} = -2$

c. The line containing $\overline{CB}$ has slope -2 and passes through $F(4, 5)$.

$y - 5 = -2(x - 4)$

An equation of $\overleftrightarrow{CB}$ is $y - 5 = -2(x - 4)$.

d. slope of $\overline{EF} = m_2 = \dfrac{5 - 2}{4 - 3} = \dfrac{3}{1} = 3$

slope of $\overline{FD} = m_3 = \dfrac{5 - 4}{4 - 2} = \dfrac{1}{2}$

The line containing $\overline{AC}$ has slope 3 and passes through $D(2, 4)$.

$y - 4 = 3(x - 2)$

An equation of $\overleftrightarrow{AC}$ is $y - 4 = 3(x - 2)$.

The line containing $\overline{AB}$ has slope $\frac{1}{2}$ and passes through $E(3, 2)$.

$y - 2 = \frac{1}{2}(x - 3)$

An equation of $\overleftrightarrow{AB}$ is $y - 2 = \frac{1}{2}(x - 3)$.

e. $\overleftrightarrow{BC}$: $y - 5 = -2(x - 4)$, $y - 5 = -2x + 8$, $y = -2x + 13$

$\overleftrightarrow{AC}$: $y - 4 = 3(x - 2)$, $y - 4 = 3x - 6$, $y = 3x - 2$

$\overleftrightarrow{AB}$: $y - 2 = \frac{1}{2}(x - 3)$, $y - 2 = \frac{1}{2}x - \frac{3}{2}$, $y = \frac{1}{2}x + \frac{1}{2}$

—CONTINUED—

f. To find A, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

$\overleftrightarrow{AB}$ has the equation $y = \frac{1}{2}x + \frac{1}{2}$.

$\overleftrightarrow{AC}$ has the equation $y = 3x - 2$.

By substitution, $\frac{1}{2}x + \frac{1}{2} = 3x - 2$

$\frac{1}{2}x = 3x - \frac{5}{2}$

$-\frac{5}{2}x = -\frac{5}{2}$

$x = 1$

$y = 3x - 2$

$y = 3 \cdot 1 - 2$

$y = 3 - 2 = 1$

So, $A(1, 1)$.

To find B, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

$\overleftrightarrow{AB}$ has the equation $y = \frac{1}{2}x + \frac{1}{2}$.

$\overleftrightarrow{BC}$ has the equation $y = -2x + 13$.

By substitution, $\frac{1}{2}x + \frac{1}{2} = -2x + 13$

$\frac{1}{2}x = -2x + \frac{25}{5}$

$\frac{5}{2}x = \frac{25}{2}$

$x = 5$

$y = -2x + 13$

$y = -2 \cdot 5 + 13$

$y = -10 + 13$

$y = 3$

So, $B(5, 3)$.

To find C, find the point of intersection of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

$\overleftrightarrow{AC}$ has the equation $y = 3x - 2$.

$\overleftrightarrow{BC}$ has the equation $y = -2x + 13$.

By substitution, $3x - 2 = -2x + 13$

$3x = -2x + 15$

$5x = 15$

$x = 3$

$y = 3x - 2$

$y = 3 \cdot 3 - 2$

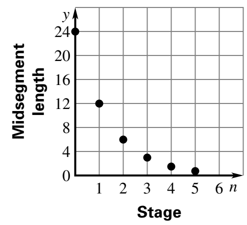
$y = 9 - 2 = 7$

So, $C(3, 7)$.

37.

Stage n	0	1	2	3	4	5
Midsegment length	24	12	6	3	1.5	0.75

38.



$y = 24 \cdot \left(\frac{1}{2}\right)^n$ is the function that gives the length of the midsection at Stage n. From one stage to the next, the length is multiplied by $\frac{1}{2}$.

5.4 Mixed Review (p. 293)

39. $x - 3 = 11$

$x = 14$ Addition property of equality

40. $3x + 13 = 46$

$3x = 33$ Subtraction property of equality

$x = 11$ Division property of equality

41. $8x - 1 = 2x + 17$

$8x = 2x + 18$ Addition property of equality

$6x = 18$ Subtraction property of equality

$x = 3$ Division property of equality

42. $5x + 12 = 9x - 4$

$5x = 9x - 16$ Subtraction property of equality

$-4x = -16$ Subtraction property of equality

$x = 4$ Division property of equality

43. $2(4x - 1) = 14$

$4x - 1 = 7$ Division property of equality

$4x = 8$ Addition property of equality

$x = 2$ Division property of equality

44. $9(3x + 10) = 27$

$3x + 10 = 3$ Division property of equality

$3x = -7$ Subtraction property of equality

$x = -\frac{7}{3}$ Division property of equality

45. $-2(x + 1) + 3 = 23$

$-2(x + 1) = 20$ Subtraction property of equality

$x + 1 = -10$ Division property of equality

$x = -11$ Subtraction property of equality

46. $3x + 2(x + 5) = 40$

$3x + 2x + 10 = 40$ Distributive property

$5x + 10 = 40$ Simplify

$5x = 30$ Subtraction property of equality

$x = 6$ Division property of equality

47. $(x + 2)° + 132° + x° = 180°$

$2x + 134 = 180$

$2x = 46$

$x = 23$

48. $(10x + 22)° + (7x + 1)° + 38° = 180°$

$17x + 61 = 180$

$17x = 119$

$x = 7$

49. $4x° + 61° = (7x + 7)°$

$4x = 7x - 54$

$-3x = -54$

$x = 18$

50. $\angle CAD \cong \angle BAD$ and $\angle BCD \cong \angle ACD$ because $\overrightarrow{AD}$, $\overrightarrow{BD}$, and $\overrightarrow{CD}$ are angle bisectors and angle bisectors divide an angle into two congruent angles.

51. Point D is the *incenter* of $\triangle ABC$ because it is the intersection of the angle bisectors of $\triangle ABC$.

52. $\overline{DE} \cong \overline{DG} \cong \overline{DF}$ because D is the point of intersection of the angle bisectors of $\triangle ABC$ and D is equidistant from the sides of the triangle.

53. $(DE)^2 + (EC)^2 = (CD)^2$

$(DE)^2 + 8^2 = 10^2$

$(DE)^2 + 64 = 100$

$(DE)^2 = 36$

$DE = 6$

But $DE = DF$ because D is equidistant from the sides of $\triangle ABC$.

So, $DF = DE = 6$.

Lesson 5.5

Technology Activity 5.5 (p. 294)

Investigate

1. The longest side is *opposite* the largest angle.
2. The shortest side is *opposite* the smallest angle.
3. The answers are the same.

   The longest side is *opposite* the largest angle.

   The shortest side is *opposite* the smallest angle.
4. The longest side will always be opposite the largest angle. The shortest side will always be opposite the smallest angle. The side with the middle length will be opposite the angle with the middle measure.

Extension

Sample answer:

$$\frac{\text{measure of smallest angle}}{\text{measure of largest angle}} = \frac{41°}{82°} = 0.5$$

$$\frac{\text{length of shortest side}}{\text{length of longest side}} = \frac{63 \text{ mm}}{96 \text{ mm}} = 0.656$$

The statement is *false* because the above example is a counterexample.

5.5 Guided Practice (p. 298)

1. The 1 inch side is opposite the smallest angle of 28°, the $1\frac{7}{8}$ inch side is opposite the middle angle of 62°, and the $2\frac{1}{8}$ inch side is opposite the largest angle of 90°.
2. No, it is not possible to draw a triangle with side lengths of 5 inches, 2 inches, and 8 inches because the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. But $5 + 2$ is not greater than 8.
3. $m\angle D + m\angle E + m\angle F = 180°$

   $32° + m\angle E + 103° = 180°$

   $m\angle E + 135° = 180°$

   $m\angle E = 45°$

   The smallest angle is $\angle D$ and the largest angle is $\angle F$.
4. The shortest side is $\overline{EF}$ and the longest side is $\overline{DE}$.
5. The distance between Guiuan and Masbate has to be greater than $165 - 99 = 66$ miles and less than $165 + 99 = 264$ miles.

5.5 Practice and Applications (pp. 298–301)

6. $m\angle A + m\angle B + m\angle C = 180°$

   $m\angle A + 42° + 71° = 180°$

   $m\angle A + 113° = 180°$

   $m\angle A = 67°$

   $\overline{AC}$ is the shortest side because it is opposite the smallest angle.

   $\overline{AB}$ is the longest side because it is opposite the largest angle.
7. $m\angle R + m\angle S + m\angle T = 180°$

   $m\angle R + 50° + 65° = 180°$

   $m\angle R + 115° = 180°$

   $m\angle R = 65°$

   $\overline{RT}$ is the shortest side. $\overline{RS}$ and $\overline{ST}$ are the longest sides ($\overline{RS} \cong \overline{ST}$).
8. $m\angle J + m\angle H = 90°$

   $m\angle J + 35° = 90°$

   $m\angle J = 55°$

   $\overline{JK}$ is the shortest side. $\overline{HJ}$ is the longest side.

9. $\angle C$ is the smallest angle. $\angle B$ is the largest angle.

10. $\angle R$ is the smallest angle. $\angle Q$ is the largest angle.

11. $\angle H$ is the smallest angle. $\angle F$ is the largest angle.

12. $y^\circ + z^\circ = x^\circ$

13. $x^\circ > y^\circ$

$x^\circ > z^\circ$

14. $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$

15. $m\angle D + m\angle E + m\angle F = 180^\circ$

$90^\circ + 30^\circ + m\angle F = 180^\circ$

$120^\circ + m\angle F = 180^\circ$

$m\angle F = 60^\circ$

$\overline{DF}$, $\overline{DE}$, and $\overline{EF}$

16. $m\angle G + m\angle J + m\angle H = 180^\circ$

$m\angle HGJ + 120^\circ + 35^\circ = 180^\circ$

$m\angle G + 155^\circ = 180^\circ$

$m\angle G = 25^\circ$

$\overline{HJ}$, $\overline{JG}$, and $\overline{HG}$

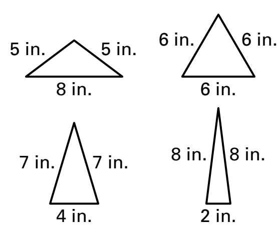
17. $\angle L$, $\angle K$, and $\angle M$

18. $\angle N$, $\angle Q$, and $\angle P$

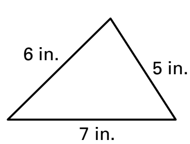
19. $\angle T$, $\angle S$, and $\angle R$

20.–23. Answers may vary; sample answers are given.

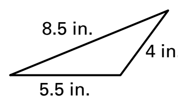
20.



21.



22.



23. The following combinations of lengths will not produce triangles: 4 inches, 4 inches, and 10 inches; 3 inches, 5 inches, and 10 inches; and, 2 inches, 7 inches, and 9 inches.

24. $AB + AC > BC$

$x + 2 + x + 3 > 3x - 2$

$2x - 5 > 3x - 2$

$2x > 3x - 7$

$-x > -7$

$x < 7$

25. $AB + AC > BC$

$x + 2 + x + 4 > 3x - 1$

$2x + 6 > 3x - 1$

$2x > 3x - 7$

$-x > -7$

$x < 7$

26. It is shorter to cut across the empty lot because the sum of the lengths of the two sidewalks is greater than the length of the diagonal across the lot. If the corner of Pleasant Street and Pine Street were labeled point A, the corner of Pine Street and Union Street were labeled point B, and the corner of Union Street and Oak Hill Avenue were labeled point C, $\triangle ABC$ could be formed. By the Triangle Inequality Theorem, $AB + BC > AC$; that is, walking around the sidewalks is longer than walking through the lot.

27. The sides and angles could not be positioned as they are labeled; for example, the longest side is not opposite the largest angle.

28. No, a kitchen triangle cannot have side lengths of 9 feet, 3 feet, and 5 feet because $3 + 5 = 8$ and 8 is not greater than 9.

29. The boom is *raised* when the boom lines are shortened.

30. AB must be less than $100 + 50 = 150$ feet.

31. Yes, when the boom is lowered and length of the boom lines, AB, is greater than 100 feet, then $\angle ABC$ will be larger than $\angle BAC$.

32. The third inequality would be $x + 14 > 10$ and this is not helpful because since x is positive, $x + 14 > 10$ for all values of x.

33. $\overline{MJ} \perp \overline{JN}$, so $\triangle MJN$ is a right triangle. The largest angle in a right triangle is the right angle, so $m\angle MJN > m\angle MNJ$, so $MN > MJ$. (If one angle of a triangle is larger than another $\angle$, then the side opposite the larger angle is longer than the side opposite the smaller angle.)

34.

Statements	Reasons
1. $\triangle ABC$	**1.** Given
2. Extend $\overline{AC}$ to D such that $\overline{AB} \cong \overline{AD}$.	**2.** Ruler Postulate
3. $AD + AC = DC$	**3.** Segment Addition Postulate
4. $\angle 1 \cong \angle 2$	**4.** Base Angles Theorem
5. $m\angle DBC > m\angle 2$	**5.** Protractor Postulate
6. $m\angle DBC > m\angle 1$	**6.** Substitution property of equality
7. $DC > BC$	**7.** If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
8. $AD + AC > BC$	**8.** Substitution property of equality
9. $AB + AC > BC$	**9.** Substitution property of equality

35. $x° > y°$ since the side opposite the angle of $x°$ is longer than the side opposite the angle of $y°$ $(n + 3 > n)$. A

36. $z°$ is the measure of the exterior angle and $x° + y° = z°$. B

37. D

38.

Statements	Reasons
1. $\overline{PC} \perp$ plane M	**1.** Given
2. Let D be a point on plane M distinct from C.	**2.** A plane contains at least three noncollinear points.
3. $\overleftrightarrow{CD} \perp \overline{PC}$	**3.** Definition of a line perpendicular to a plane
4. $\angle PCD$ is a right angle.	**4.** If two lines are perpendicular, then they intersect to form four right angles.
5. $\triangle PCD$ is a right triangle.	**5.** Definition of right triangle
6. $\angle PDC$ is an acute angle.	**6.** The non-right angles in a right triangle are acute angles.
7. $m\angle PDC < 90°$	**7.** Definition of an acute angle
8. $m\angle PCD = 90°$	**8.** Definition of a right angle
9. $m\angle PDC < m\angle PCD$	**9.** Substitution property of equality
10. $PD > PC$	**10.** If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

5.5 Mixed Review (p. 301)

39.–41. Answers may vary. Sample answers are given.

39. The proof for Example 2 on page 230 is a two-column proof.

40. The proof for Example 1 on page 229 is a paragraph proof.

41. The proof for Example 3 on page 158 is a flow proof.

42. $\angle 5$ and $\angle 1$ are corresponding angles. So are $\angle 5$ and $\angle 9$.

43. $\angle 12$ and $\angle 9$ are vertical angles.

44. $\angle 6$ and $\angle 3$ are alternate interior angles. So are $\angle 6$ and $\angle 11$.

45. $\angle 7$ and $\angle 2$ are alternate exterior angles. So are $\angle 7$ and $\angle 10$.

46. slope of $\overline{LM} = m_1 = \frac{3 - 1}{2 - (-2)} = \frac{2}{4} = \frac{1}{2}$

slope of $\overline{MN} = m_2 = \frac{3 - (-1)}{2 - 3} = \frac{4}{-1} = -4$

slope of $\overline{LN} = m_3 = \frac{1 - (-1)}{-2 - 3} = \frac{2}{-5} = -\frac{2}{5}$

The line containing $\overline{AB}$ has slope -4 and passes through $L(-2, 1)$.

$y - 1 = -4(x - (-2))$

$y - 1 = -4(x + 2)$

$y - 1 = -4x - 8$

$y = -4x - 7$

The line containing $\overline{AC}$ has slope $-\frac{2}{5}$ and passes through $M(2, 3)$.

$y - 3 = -\frac{2}{5}(x - 2)$

$y - 3 = -\frac{2}{5}x + \frac{4}{5}$

$y = -\frac{2}{5}x + \frac{19}{5}$

The line containing $\overline{BC}$ has slope $\frac{1}{2}$, and passes through $N(3, -1)$.

$y - (-1) = \frac{1}{2}(x - 3)$

$y + 1 = \frac{1}{2}x - \frac{3}{2}$

$y = \frac{1}{2}x - \frac{5}{2}$

—CONTINUED—

46. **—CONTINUED—**

To find A, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

$$y = -4x - 7$$
$$y = -\frac{2}{5}x + \frac{19}{5}$$
$$-4x - 7 = -\frac{2}{5}x + \frac{19}{5}$$
$$-4x = -\frac{2}{5}x + \frac{54}{5}$$
$$-\frac{18}{5}x = \frac{54}{5}$$
$$x = -3$$
$$y = -4x - 7$$
$$y = -4(-3) - 7$$
$$y = 12 - 7$$
$$y = 5$$

The point A has coordinates $(-3, 5)$.

To find B, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

$$y = -4x - 7$$
$$y = \frac{1}{2}x - \frac{5}{2}$$
$$-4x - 7 = \frac{1}{2}x - \frac{5}{2}$$
$$-4x = \frac{1}{2}x + \frac{9}{2}$$
$$-\frac{9}{2}x = \frac{9}{2}$$
$$x = -1$$
$$y = -4x - 7$$
$$y = -4(-1) - 7$$
$$y = 4 - 7$$
$$y = -3$$

The coordinates of B are $(-1, -3)$.

To find C, find the point of intersection of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

$$y = -\frac{2}{5}x + \frac{19}{5}$$
$$y = \frac{1}{2}x - \frac{5}{2}$$
$$\frac{1}{2}x - \frac{5}{2} = \frac{-2}{5}x + \frac{19}{5}$$
$$5x - 25 = -4x + 38$$
$$5x = -4x + 63$$
$$9x = 63$$
$$x = 7$$
$$y = \frac{1}{2}x - \frac{5}{2}$$
$$y = \frac{1}{2} \cdot 7 - \frac{5}{2}$$
$$y = \frac{7}{2} - \frac{5}{2}$$
$$y = 1$$

The coordinates of C are $(7, 1)$.

47. slope of $\overline{LM} = m_1 = \frac{5 - 2}{-3 - (-2)} = \frac{3}{-1} = -3$

slope of $\overline{MN} = m_2 = \frac{2 - 0}{-2 - (-6)} = \frac{2}{4} = \frac{1}{2}$

slope of $\overline{LN} = m_3 = \frac{5 - 0}{-3 - (-6))} = \frac{5}{3}$

The line containing $\overline{AB}$ has slope $\frac{1}{2}$ and passes through $L(-3, 5)$.

$$y - 5 = \frac{1}{2}(x - (-3))$$
$$y - 5 = \frac{1}{2}(x + 3)$$
$$y - 5 = \frac{1}{2}x + \frac{3}{2}$$
$$y = \frac{1}{2}x + \frac{13}{2}$$

The line containing $\overline{BC}$ has slope $\frac{5}{3}$ and passes through $M(-2, 2)$.

$$y - 2 = \frac{5}{3}(x - (-2))$$
$$y - 2 = \frac{5}{3}(x + 2)$$
$$y - 2 = \frac{5}{3}x + \frac{10}{3}$$
$$y = \frac{5}{3}x + \frac{16}{3}$$

—CONTINUED—

47. **—CONTINUED—**

The line containing $\overline{AC}$ has slope -3 and passes through $N(-6, 0)$.

$$y - 0 = -3(x - (-6))$$
$$y = -3(x + 6)$$
$$y = -3x - 18$$

To find A, find the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

$$y = \frac{1}{2}x + \frac{13}{2}$$
$$y = -3x - 18$$
$$\frac{1}{2}x + \frac{13}{2} = -3x - 18$$
$$\frac{1}{2}x = -3x - \frac{49}{2}$$
$$\frac{7}{2}x = \frac{-49}{2}$$
$$x = -7$$
$$y = -3x - 18$$
$$y = -3(-7) - 18$$
$$y = 21 - 18$$
$$y = 3$$

The coordinates of A are $(-7, 3)$.

To find B, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

$$y = \frac{1}{2}x + \frac{13}{2}$$
$$y = \frac{5}{3}x + \frac{16}{3}$$
$$\frac{1}{2}x + \frac{13}{2} = \frac{5}{3}x + \frac{16}{3}$$
$$3x + 39 = 10x + 32$$
$$3x = 10x - 7$$
$$-7x = -7$$
$$x = 1$$
$$y = \frac{1}{2}x + \frac{13}{2}$$
$$y = \frac{1}{2} \cdot 1 + \frac{13}{2}$$
$$y = \frac{1}{2} + \frac{13}{2}$$
$$y = \frac{14}{2}$$
$$y = 7$$

The coordinates of B are $(1, 7)$.

To find C, find the intersection of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

$$y = -3x - 18$$
$$y = \frac{5}{3}x + \frac{16}{3}$$
$$-3x - 18 = \frac{5}{3}x + \frac{16}{3}$$
$$-3x = \frac{5}{3}x + \frac{70}{3}$$
$$\frac{-14}{3}x = \frac{70}{3}$$
$$x = -5$$
$$y = -3x - 18$$
$$y = -3(-5) - 18$$
$$y = 15 - 18$$
$$y = -3$$

The coordinates of C are $(-5, -3)$.

48. Slope of $\overline{LM} = m_1 = \frac{6 - 5}{3 - 9} = \frac{1}{-6} = -\frac{1}{6}$

Slope of $\overline{MN} = m_2 = \frac{5 - 1}{9 - 8} = \frac{4}{1} = 4$

Slope of $\overline{LN} = m_3 = \frac{6 - 1}{3 - 8} = \frac{5}{-5} = -1$

The line containing $\overline{AB}$ has slope 4 and passes through $L(3, 6)$.

$$y - 6 = 4(x - 3)$$
$$y - 6 = 4x - 12$$
$$y = 4x - 6$$

The line containing $\overline{BC}$ has slope -1 and passes through $M(9, 5)$.

$$y - 5 = -1(x - 9)$$
$$y - 5 = -x + 9$$
$$y = -x + 14$$

The line containing $\overline{AC}$ has slope $-\frac{1}{6}$ and passes through $N(8, 1)$.

$$y - 1 = -\frac{1}{6}(x - 8)$$
$$y - 1 = -\frac{1}{6}x + \frac{8}{6}$$
$$y = -\frac{1}{6}x + \frac{7}{3}$$

—CONTINUED—

48. —CONTINUED—

To find A, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

$$y = 4x - 6$$
$$y = -\frac{1}{6}x + \frac{7}{3}$$
$$4x - 6 = -\frac{1}{6}x + \frac{7}{3}$$
$$4x = -\frac{1}{6}x + \frac{25}{3}$$
$$\frac{25}{6}x = \frac{25}{3}$$
$$x = 2$$
$$y = 4x - 6$$
$$y = 4 \cdot 2 - 6$$
$$y = 8 - 6$$
$$y = 2$$

The coordinates of A are (2, 2).

To find B, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

$$y = 4x - 6$$
$$y = -x + 14$$
$$4x - 6 = -x + 14$$
$$4x = -x + 20$$
$$5x = 20$$
$$x = 4$$
$$y = 4x - 6$$
$$y = 4 \cdot 4 - 6$$
$$y = 16 - 6$$
$$y = 10$$

The coordinates of B are (4, 10).

To find C, find the point of intersection of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

$$y = -\frac{1}{6}x + \frac{7}{3}$$
$$y = -x + 14$$
$$-\frac{1}{6}x + \frac{7}{3} = -x + 14$$
$$-\frac{1}{6}x = -x + \frac{35}{3}$$
$$\frac{5}{6}x = \frac{35}{3}$$
$$x = 14$$
$$y = -x + 14$$
$$y = -14 + 14$$
$$y = 0$$

The coordinates of C are (14, 0).

49. slope of $\overline{LM} = m_1 = \frac{-2 - (-4)}{3 - 0} = \frac{2}{3}$

slope of $\overline{MN} = m_2 = \frac{-4 - (-6)}{0 - 3} = \frac{2}{-3} = -\frac{2}{3}$

slope of $\overline{LN} = m_3 = \frac{-2 - (-6)}{3 - 3} = \frac{4}{0} =$ undefined

$\overline{LN}$ is vertical.

The line containing $\overline{AB}$ has slope $-\frac{2}{3}$ and passes through $L(3, -2)$.

$$y - (-2) = -\frac{2}{3}(x - 3)$$
$$y + 2 = -\frac{2}{3}x + 2$$
$$y = -\frac{2}{3}x$$

The line containing $\overline{BC}$ is a vertical line passing through $M(0, -4)$.

$x = 0$

The line containing $\overline{AC}$ has slope $\frac{2}{3}$ and passes through $N(3, -6)$.

$$y - (-6) = \frac{2}{3}(x - 3)$$
$$y + 6 = \frac{2}{3}x - 2$$
$$y = \frac{2}{3}x - 8$$

To find A, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

$$y = -\frac{2}{3}x$$
$$y = \frac{2}{3}x - 8$$
$$-\frac{2}{3}x = \frac{2}{3}x - 8$$
$$-\frac{4}{3}x = -8$$
$$x = 6$$
$$y = -\frac{2}{3}x$$
$$y = -\frac{2}{3} \cdot 6$$
$$y = -4$$

The coordinates of A are (6, −4).

—CONTINUED—

49. **—CONTINUED—**

To find B, find the point of intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

$y = -\frac{2}{3}x$

$x = 0$

$y = -\frac{2}{3}x$

$y = -\frac{2}{3} \cdot 0$

$y = 0$

The coordinates of B are $(0, 0)$.

To find C, find the point of intersection of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

$y = \frac{2}{3}x - 8$

$x = 0$

$y = \frac{2}{3}x - 8$

$y = \frac{2}{3} \cdot 0 - 8$

$y = 0 - 8$

$y = -8$

The coordinates of C are $(0, -8)$.

Lesson 5.6

5.6 Guided Practice (p. 305)

1. An indirect proof might also be called a *proof by contradiction* because in an indirect proof, you prove that a statement is true by first assuming that its opposite is true. If this assumption leads to a contradiction, then you have proved that the original statement is true.

2. To use an indirect proof to show that two lines m and n are parallel, you would first make the assumption that lines m and n are not parallel.

3. $m\angle 1 > m\angle 2$ **4.** $KL < NQ$ **5.** $DC < FE$

6. In $\triangle ABC$, if you wanted to prove that $BC > AC$, you would use the two cases $BC < AC$ and $BC = AC$ in an indirect proof.

5.6 Practice and Applications (pp. 305–307)

7. $RS < TU$ **8.** $m\angle 1 = m\angle 2$ **9.** $m\angle 1 > m\angle 2$

10. $XY > ZY$ **11.** $m\angle 1 = m\angle 2$ **12.** $m\angle 1 < m\angle 2$

13. $AB > CB$ **14.** $UT > SV$ **15.** $m\angle 1 > m\angle 2$

16. The correct answer is C because $BD = CD$, $AD = AD$, and $AC > AB$ so by the Converse of the Hinge Theorem $m\angle 4 < m\angle 5$.

17. The correct answer is B because $AB = DC$, $AD = AD$, and $m\angle 3 < m\angle 5$ so by the Hinge Theorem $AC > BD$.

18. $x > 9$ because $70° > 60°$.

19. $3x + 1 > x + 3$ because $115° > 45°$.

$3x > x + 2$

$2x > 2$

$x > 1$

20. $(4x - 5)° < 65°$ because $2 < 4$.

$4x < 70$

$x < 17.5$

21. Given that $RS + ST \neq 12$ in. and $ST = 5$ in., assume that $RS = 7$ in.

22. Given $\triangle MNP$ with Q the midpoint of $\overline{NP}$, assume $\overline{MQ}$ is not a median.

23. Given $\triangle ABC$ with $m\angle A + m\angle B = 90°$, assume $m\angle C \neq 90°$.

24. C Assume that there are two points, P and Q, where m and n intersect.

B Then there are two lines (m and n) through points P and Q.

A But this contradicts Postulate 5, which states that there is exactly one line through any two points.

D It is false that m and n can intersect in two points, so they must intersect in exactly one point.

25. **Case l:** Assume that $EF < DF$. If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side, so $m\angle D < m\angle E$. But this contradicts the given information that $m\angle D > m\angle E$.

Case 2: Assume that $EF = DF$. By the Converse of the Base Angles Theorem, $m\angle E = m\angle D$. But this contradicts the given information that $m\angle D > m\angle E$.

Since both cases produce a contradiction, the assumption that EF is not greater than DF must be incorrect and $EF > DF$.

26. Assume $m \nparallel n$. Then m and n intersect in a point and the triangle shown in the diagram is formed. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ by the Triangle Sum Theorem. Then $m\angle 1 + m\angle 2 = 180° - m\angle 3$ by the Subtraction property of equality. But $m\angle 1 + m\angle 2 = 180°$ because $\angle 1$ and $\angle 2$ are supplementary. So $180° = 180° - m\angle 3$ by the Substitution property of equality. Then $m\angle 3 = 0$ by simplifying both sides. But this is not possible; angle measures in a triangle cannot be zero.

So the assumption that $m \nparallel n$ must be false. Therefore, $m \parallel n$.

27. **Case 1:** Assume that $RS > RT$. Then $m\angle T > m\angle S$ by the Converse of the Hinge Theorem. But $\triangle RUS \cong \triangle RUT$ by the ASA Congruence Postulate, so $\angle S \cong \angle T$ or $m\angle S = m\angle T$. This is a contradiction. So $RS \le RT$.

Case 2: Assume $RS < RT$. Then $m\angle T < m\angle S$ by the Converse of the Hinge Theorem. But $\triangle RUS \cong \triangle RUT$ by the ASA Congruence Postulate, so $\angle S \cong \angle T$ or $m\angle S = m\angle T$. This is a contradiction. So $RS \ge RT$.

Therefore $RS = RT$ and $\triangle RST$ is an isosceles triangle.

28. The paths are described by two triangles in which two sides of one triangle are congruent to two sides of another triangle, but the included angle in your friend's triangle is larger than the included angle of your triangle, so the side representing the distance from the airport is longer in your friend's triangle.

29. The paths are described by two triangles in which two sides of one triangle are congruent to two sides of another triangle, but the included angle in your friend's triangle is larger than the included angle in your triangle, so the side representing the distance to the airport is longer in your friend's triangle.

30. **a.** As ED increases, $m\angle EBD$ increases because $\angle EBD$ is the angle opposite $\overline{ED}$.

As ED increases, $m\angle DBA$ decreases because $m\angle EBD$ increases and $\angle DBA$ and $\angle EBD$ are supplementary.

b. As ED increases, AD decreases because as ED increases $m\angle EBD$ increases and $m\angle ABD$ decreases making AD decrease.

c. The cleaning arm illustrates the Hinge Theorem because the lengths of $\overline{BE}$ and $\overline{BD}$ remain constant while $m\angle EBD$ and ED change. In $\triangle EBD$ and $\triangle ABD$, $\overline{BE} \cong \overline{BD} \cong \overline{BA}$. The included angle in $\triangle EBD$, $\angle EBD$, is larger than the included angle in $\triangle ABD$, $\angle ABD$. So $\overline{ED}$ is longer than $\overline{AD}$.

31.

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$	**1.** Given
2. Construct a ray from B to construct an angle in the interior of $\angle ABC$ that is congruent to $\angle DEF$.	**2.** Protractor Postulate
3. Locate P on the constructed ray such that $\overline{BP} \cong \overline{ED}$.	**3.** Ruler Postulate
4. $\triangle PBC \cong \triangle DEF$	**4.** SAS Congruence Postulate
5. $\overline{PC} \cong \overline{DF}$	**5.** Corresponding parts of congruent triangles are congruent
6. Locate H on $\overline{AC}$ so that $\overrightarrow{BH}$ bisects $\angle PBA$.	**6.** Protractor Postulate
7. $\angle PBH \cong \angle ABH$	**7.** Definition of angle bisector
8. $\overline{PB} \cong \overline{AB}$	**8.** Transitive property of congruence (Steps 1, 3)
9. $\overline{BH} \cong \overline{BH}$	**9.** Reflexive property of congruence
10. $\triangle ABH \cong \triangle PBH$	**10.** SAS Congruence Postulate
11. $\overline{AH} \cong \overline{PH}$	**11.** Corresponding parts of congruent triangles are congruent
12. $AH = PH$	**12.** Definition of congruent segments
13. $AC = AH + HC$	**13.** Segment Addition Postulate
14. $AC = PH + HC$	**14.** Substitution property of equality
15. $PH + HC > PC$	**15.** Triangle Inequality
16. $AC > PC$	**16.** Substitution property of equality
17. $PC = DF$	**17.** Definition of congruent segments (Step 5)
18. $AC > DF$	**18.** Substitution property of equality

5.6 Mixed Review (p. 308)

32. isosceles **33.** equilateral, equiangular, and isosceles

34. scalene **35.** isosceles

36. equiangular, equilateral, and isosceles

37. isosceles

38. $(x + 13)° + (x + 19)° = 3x°$

$$2x + 32 = 3x$$

$$32 = x$$

39. $m\angle B = (x + 19)°$

$m\angle B = 32° + 19°$

$m\angle B = 51°$

40. $m\angle C = (x + 13)°$

$m\angle C = 32° + 13°$

$m\angle C = 45°$

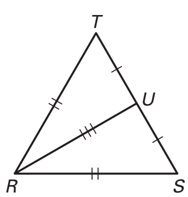
41. $m\angle BAC + m\angle B + m\angle C = 180°$

$$m\angle BAC + 51° + 45° = 180°$$

$$m\angle BAC + 96° = 180°$$

$$m\angle BAC = 84°$$

42.



$\overline{RU}$ is a median, an altitude, an angle bisector, and a perpendicular bisector. $\overline{RU}$ divides $\triangle RST$ into two congruent triangles. It is shorter than each side of $\triangle RST$.

5.6 Quiz 2 (p. 308)

1. $\overline{FG} \parallel \overline{CE}$ **2.** If $FG = 8$, then $CE = 16$.

3. If the perimeter of $\Delta CDE = 42$, then the perimeter of $\Delta GHF = 21$

4. $m\angle L + m\angle M + m\angle Q = 180°$

$$75° + m\angle M + 74° = 180°$$

$$m\angle M + 149° = 180°$$

$$m\angle M = 31°$$

$\overline{LQ}, \overline{LM}, \overline{MQ}$

5. $m\angle M + m\angle P + m\angle Q = 180°$

$$m\angle M + 49° + 50° = 180°$$

$$m\angle M + 99° = 180°$$

$$m\angle M = 81°$$

$\overline{MQ}, \overline{MP}, \overline{PQ}$

6. $m\angle M + m\angle N + m\angle P = 180°$

$m\angle M + 48° + 75° = 180°$

$m\angle M + 123° = 180°$

$m\angle M = 57°$

$\overline{MP}, \overline{NP}, \overline{MN}$

7. $\overline{DE}$ is longer than $\overline{AB}$ because two sides of $\triangle ABC$ are congruent to two sides of $\triangle DEF$ and the included angle $\angle DFE$ is larger than included angle $\angle BCA$ so $DE > AB$.

8. The 2nd Group is farther from the camp because the groups' paths form two triangles with 2 pairs of congruent sides and the included angle for the 2nd group is larger than the included angle for the 1st group.

Review (pp. 310–312)

1. If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

2. If $\overline{UR} \cong \overline{UT}$, then U must be on the perpendicular bisector $\overleftrightarrow{SQ}$ of $\overline{RT}$.

3. If Q is equidistant from $\overrightarrow{SR}$ and $\overrightarrow{ST}$, then Q is on the bisector of $\angle RST$.

4. Let X be the midpoint of $\overline{ST}$. Then $XT = \frac{1}{2}ST$ or $XT = \frac{1}{2} \cdot 32$ or $XT = 16$.

$$(KX)^2 + (XT)^2 = (KT)^2$$

$$12^2 + 16^2 = KT^2$$

$$144 + 256 = KT^2$$

$$400 = KT^2$$

$$20 = KT$$

But $KR = KT = 20$ because K is equidistant from R, S, and T.

5. $(WA)^2 + (AY)^2 = (WY)^2$

$$(WA)^2 + 8^2 = 10^2$$

$$(WA)^2 + 64 = 100$$

$$(WA)^2 = 36$$

$$WA = 6$$

Since W is equidistant from the sides of $\triangle XYZ$, $WB = WA = 6$.

6. The special segments are angle bisectors and the point of concurrency is the incenter

7. The special segments are perpendicular bisectors and the point of concurrency is the circumcenter.

8. The special segments are medians and the point of concurrency is the centroid.

9. The special segments are altitudes and the point of concurrency is the orthocenter.

10. midpoint of $\overline{XY} = \left(\frac{-4+0}{2}, \frac{0+0}{2}\right)$

$= \left(\frac{-4}{2}, \frac{0}{2}\right)$

$= (-2, 0)$

midpoint of $\overline{XZ} = \left(\frac{0+0}{2}, \frac{6+0}{2}\right) = \left(\frac{0}{2}, \frac{6}{2}\right) = (0, 3)$

midpoint of $\overline{YZ} = \left(\frac{-4+0}{2}, \frac{0+6}{2}\right)$

$= \left(\frac{-4}{2}, \frac{6}{2}\right)$

$= (-2, 3)$

$m_1 = \frac{3-0}{0-(-4)} = \frac{3}{4}$

An equation of the median to $\overline{XZ}$ is

$y - 0 = \frac{3}{4}(x - (-4))$.

$y = \frac{3}{4}(x + 4)$

$y = \frac{3}{4}x + 3$

$m_2 = \frac{3-0}{-2-0} = \frac{3}{-2}$

An equation of the median to $\overline{YZ}$ is

$y - 0 = -\frac{3}{2}(x - 0)$.

$y = -\frac{3}{2}x$

The centroid is the point of intersection of the two lines.

$y = \frac{3}{4}x + 3$

$y = -\frac{3}{2}x$

$\frac{3}{4}x + 3 = \frac{-3}{2}x$

$3 = \frac{-9}{4}x$

$\frac{-4}{3} = x$

$y = -\frac{3}{2}x$

$y = -\frac{3}{2}\left(-\frac{4}{3}\right)$

$y = 2$

The coordinates of the centroid of $\triangle XYZ$ are $\left(-\frac{4}{3}, 2\right)$.

11. The coordinates of the orthocenter of $\triangle XYZ$ are $(0, 0)$ since $\triangle XYZ$ is a right triangle and the two legs are also altitudes of $\triangle XYZ$.

12. The slope of $\overline{LM} = m_1 = \frac{3-3}{8-4} = \frac{0}{4} = 0$

The slope of $\overline{MN} = m_2 = \frac{3-1}{8-6} = \frac{2}{2} = 1$

The slope of $\overline{LN} = m_3 = \frac{3-1}{4-6} = \frac{2}{-2} = -1$

The line containing $\overline{HJ}$ has slope -1 and passes through $M(8, 3)$.

$y - 3 = -1(x - 8)$

$y - 3 = -x + 8$

$y = -x + 11$.

The line containing $\overline{HK}$ has slope 1 and passes through $L(4, 3)$.

$y - 3 = 1(x - 4)$

$y - 3 = x - 4$

$y = x - 1$.

The line containing $\overline{JK}$ has slope 0 and passes through $N(6, 1)$.

$y - 1 = 0(x - 6)$

$y - 1 = 0$

$y = 1$.

H is the point of intersection of $\overleftrightarrow{HJ}$ and $\overleftrightarrow{HK}$.

$y = -x + 11$

$y = x - 1$

$-x + 11 = x - 1$

$-x = x - 12$

$-2x = -12$

$x = 6$

$y = -x + 11$

$y = -6 + 11$

$y = 5$

The coordinates of H are $(6, 5)$

J is the point of intersection of $\overleftrightarrow{HJ}$ and $\overleftrightarrow{JK}$.

$y = -x + 11$

$y = 1$

$1 = -x + 11$

$-10 = -x$

$10 = x$

The coordinates of J are $(10, 1)$.

—CONTINUED—

12. —CONTINUED—

K is the point of intersection of $\overleftrightarrow{HK}$ and $\overleftrightarrow{JK}$.

$y = x - 1$

$y = 1$

$1 = x - 1$

$2 = x$

The coordinates of K are $(2, 1)$.

13. Let L be the midpoint of $\overline{HJ}$, M be the midpoint of $\overline{JK}$, and N be the midpoint of $\overline{HK}$.

The slope of $\overline{LM} = 0 =$ the slope of $\overline{HK}$, so $\overline{LM} \parallel \overline{HK}$.

The slope of $\overline{MN} = 1 =$ the slope of $\overline{HJ}$, so $\overline{MN} \parallel \overline{HJ}$.

The slope of $\overline{LN} = -1 =$ the slope of $\overline{JK}$, so $\overline{LN} \parallel \overline{JK}$.

14. $BG = GC = 9$

$BG + GC = BC$

$9 + 9 = BC$

$18 = BC$

$GF = \frac{1}{2}BD$

$12 = \frac{1}{2}BD$

$24 = BD$

$P = BC + CD + BD$

$P = 18 + 22 + 24$

$P = 64$

15. $RU = UQ = 9$

$RU + UQ = RQ$

$9 + 9 = RQ$

$18 = RQ$

$ST = \frac{1}{2}RQ$

$ST = \frac{1}{2} \cdot 18$

$ST = 9$

$TU = \frac{1}{2}PQ$

$TU = \frac{1}{2} \cdot 24$

$TU = 12$

$P = ST + TU + SU$

$P = 9 + 12 + 10$

$P = 31$

16. The angle measurements in order from least to greatest are $m\angle C$, $m\angle A$, and $m\angle B$. The side measurements in order from least to greatest are *AB, BC,* and *AC*.

17. The angle measurements in order from least to greatest are $m\angle D$, $m\angle E$, and $m\angle F$. The side measurements in order from least to greatest are *EF, DF,* and *DE*.

18. $m\angle J + m\angle H + m\angle G = 180°$

$m\angle J + 50° + 70° = 180°$

$m\angle J + 120° = 180°$

$m\angle J = 60°$

The angle measurements in order from least to greatest are $m\angle H$, $m\angle J$, and $m\angle G$. The side measurements in order from least to greatest are *GJ, GH,* and *HJ.*

19. $m\angle L + m\angle K = 90°$

$m\angle L + 55° = 90°$

$m\angle L = 35°$

The angle measurements in order from least to greatest are $m\angle L$, $m\angle K$, and $m\angle M$. The side measurements in order from least to greatest are *KM, LM,* and *KL.*

20. The length of the third side must be less than the sum of the lengths of the other two sides. So the length of the third side must be less than 300 feet $(100 + 200)$. So the maximum length of fencing needed is 600 feet $(100 + 200 + 300)$ of fencing.

21. $AB < CB$ **22.** $m\angle 1 < m\angle 2$ **23.** $TU = VS$

24. In $\triangle MPQ$, if $\angle M \cong \angle Q$, then $MP \neq QP$.

25. Assume $\triangle ABC$ has two right angles at $\angle A$ and $\angle B$. Then $m\angle A + m\angle B = 180°$ and, since $m\angle C > 0°$, $m\angle A + m\angle B + m\angle C > 180°$. This contradicts the Triangle Sum Theorem. Then the assumption that there is such a $\triangle ABC$ must be incorrect and no triangle has two right angles.

Chapter 5 Test *(p. 313)*

1. If P is the circumcenter of $\triangle RST$, then *PR, PS,* and *PT* are *always* equal.

2. If $\overrightarrow{BD}$ bisects $\angle ABC$, then $\overline{AD}$ and $\overline{CD}$ are *sometimes* congruent.

3. The incenter of a triangle *never* lies outside the triangle.

4. The length of a median of a triangle is *sometimes* equal to the length of a midsegment.

5. If $\overline{AM}$ is the altitude to side $\overline{BC}$ of $\triangle ABC$, then $\overline{AM}$ is *always* shorter than $\overline{AB}$.

6. a. $HC = \frac{2}{3}CG$

$HC = \frac{2}{3}(HC + HG)$

$HC = \frac{2}{3}(HC + 6)$

$HC = \frac{2}{3}HC + 4$

$\frac{1}{3}HC = 4$

$HC = 12$

b. $(HG)^2 + (GB)^2 = (HB)^2$

$6^2 + 8^2 = (HB)^2$

$36 + 64 = (HB)^2$

$100 = (HB)^2$

$10 = HB$

c. $HE = \frac{1}{3}EB$

$HE = \frac{1}{3}(HE + HB)$

$HE = \frac{1}{3}(HE + 10)$

$HE = \frac{1}{3}HE + \frac{10}{3}$

$\frac{2}{3}HE = \frac{10}{3}$

$HE = 5$

d. $BC = CF + FB$

$BC = 9.9 + 9.9$

$BC = 19.8$

7. Point H is the centroid of the triangle.

8. $\overline{CG}$ is a(n) median, perpendicular bisector, altitude, and angle bisector of $\triangle ABC$.

9. $EF = \frac{1}{2}AB$ and $\overline{EF} \parallel \overline{AB}$ by the Midsegment Theorem.

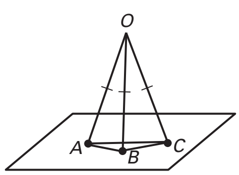
10. $m\angle BAC > m\angle ACB$ because the side opposite $\angle BAC$ is longer than the side opposite $\angle ACB$.

11. To locate the pool so that its center is equidistant from the sidewalks, find the incenter of the triangle by constructing angle bisectors of two angles of the triangle and locating the point of intersection of the bisectors. This point will be equidistant from each sidewalk.

12. The converse of the Hinge Theorem guarantees that the angles between the legs get larger as the legs are spread apart.

13. The maximum distance between the end of two legs is 10 feet because the length of the third side of the triangle must be less than the sum of the lengths of the other two sides.

14.



If $m\angle AOC > m\angle BOC$, then $\overline{AC}$ is longer than $\overline{BC}$ because two sides of one triangle are congruent to two sides in another triangle and the measure of the included angle of one triangle is larger than the measure of the included angle of the other triangle (Hinge Thm.).

15.

Statements	Reasons
1. $AC = BC$	**1.** Given
2. $AC + CE = AE$	**2.** Segment Addition Post.
3. $BC + CE = AE$	**3.** Substitution property of equality
4. $BE < BC + CE$	**4.** Triangle Inequality Theorem
5. $BE < AE$	**5.** Substitution property of equality

16. Assume $m\angle D = m\angle ABC$. Then $\overline{AD} \cong \overline{AB}$ because if two angles of a triangle are congruent, then the sides opposite them are congruent. So $AD = AB$ by the definition of congruent segments. But this contradicts the given statement that $AD \neq AB$. Therefore, the assumption must be false. So $m\angle D \neq m\angle ABC$.

Chapter 5 Standarized Test *(pp. 314–315)*

1. $4x - 9 = \frac{2}{3}x + 21$ **2.** D **3.** B

$4x = \frac{2}{3}x + 30$

$\frac{10}{3}x = 30$

$x = 9$

$\frac{9}{2}y - 4 = 5y - 6$

$\frac{9}{2}y = 5y - 2$

$-\frac{1}{2}y = -2$

$y = 4$

B

4. The midpoint of $\overline{FG}$ is $M = \left(\frac{-12 + (-2)}{2}, \frac{1 + 1}{2}\right)$

$= \left(\frac{-14}{2}, \frac{2}{2}\right)$

$= (-7, 1)$.

The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

$CH = \frac{2}{3}MH$

But $MH = 1 - (-11) = 12$.

So $CH = \frac{2}{3}MH$

$CH = \frac{2}{3} \cdot 12$

$CH = 8$

The coordinates of C are $(-7, -11 + 8)$ or $(-7, -3)$.

E

5. $KP = 16$, so $PL = 16$

$MK = 12$, so $NP = 12$

$(NP)^2 + (PL)^2 = (NL)^2$

$12^2 + 16^2 = (NL)^2$

$144 + 256 = (NL)^2$

$400 = (NL)^2$

$20 = NL$

So $P = NP + PL + NL$

$P = 12 + 16 + 20$

$P = 48$

C

6. A

7. The length of the side has to be greater than $28 - 16$ or 12 inches and less than $28 + 16 = 44$ inches. A

8. A

9. $x + y = 90$ because the sum of the measures of the acute angles of a right triangle is $90°$.

10. $x° > y°$ because the side opposite $\angle G$ is longer than the side opposite $\angle H$.

11. If $x = y$, then $x = 45$. But $x > y$ so $x > 45$. C

12. The location of the point of intersection of the perpendicular bisectors is the midpoint of $\overline{GH}$ because $\triangle GHJ$ is a right triangle.

13. Let M be the midpoint of $\overline{AB}$.

$$M = \left(\frac{0 + 12}{2}, \frac{0 + 6}{2}\right) = \left(\frac{12}{2}, \frac{6}{2}\right) = (6, 3)$$

Let N be the midpoint of $\overline{BC}$.

$$N = \left(\frac{12 + 18}{2}, \frac{6 + 0}{2}\right) = \left(\frac{30}{2}, \frac{6}{2}\right) = (15, 3)$$

Let P be the midpoint of $\overline{AC}$.

$$P = \left(\frac{0 + 18}{2}, \frac{0 + 0}{2}\right) = \left(\frac{18}{2}, \frac{0}{2}\right) = (9, 0)$$

The slope of $\overleftrightarrow{AN} = m_1 = \frac{3 - 0}{15 - 0} = \frac{3}{15} = \frac{1}{5}$

An equation of $\overleftrightarrow{AN}$ is $y - 0 = \frac{1}{5}(x - 0)$

$$y = \frac{1}{5}x$$

The slope of $\overleftrightarrow{BP} = m_2 = \frac{6 - 0}{12 - 9} = \frac{6}{3} = 2$

An equation of $\overleftrightarrow{BP}$ is $y - 0 = 2(x - 9)$

$$y = 2x - 18$$

The slope of $\overleftrightarrow{CM} = m_3 = \frac{3 - 0}{6 - 18} = \frac{3}{-12} = -\frac{1}{4}$

An equation of $\overleftrightarrow{CM}$ is $y - 0 = -\frac{1}{4}(x - 18)$

$$y = -\frac{1}{4}x + \frac{9}{2}$$

The centroid is the point of intersection of $\overleftrightarrow{AN}$, $\overleftrightarrow{PB}$, and $\overleftrightarrow{CM}$.

$y = \frac{1}{5}x$

$y = 2x - 18$

$\frac{1}{5}x = 2x - 18$

$-\frac{9}{5}x = -18$

$x = 10$

$y = \frac{1}{5}x$

$y = \frac{1}{5} \cdot 10$

$y = 2$

The coordinates of the centroid are (10, 2).

14. The slope of $\overleftrightarrow{AC} = m_1 = \frac{0 - 0}{18 - 0} = \frac{0}{18} = 0$

The slope of the line perpendicular to $\overleftrightarrow{AC}$ is undefined. So the line perpendicular to $\overleftrightarrow{AC}$ that passes through B is the line $x = 12$.

The slope of $\overleftrightarrow{AB} = m_2 = \frac{6 - 0}{12 - 0} = \frac{6}{12} = \frac{1}{2}$

The slope of the line perpendicular to $\overleftrightarrow{AB}$ is -2 because $\frac{1}{2} \cdot (-2) = -1$.

An equation of the line perpendicular to $\overleftrightarrow{AB}$ and passing through $C(18, 0)$ is $y - 0 = -2(x - 18)$

$$y = -2x + 36.$$

The slope of $\overleftrightarrow{BC} = m_3 = \frac{6 - 0}{12 - 18} = \frac{6}{-6} = -1.$

The slope of the line perpendicular to $\overleftrightarrow{BC}$ is 1 because $1 \cdot (-1) = -1$.

An equation of the line parallel to $\overleftrightarrow{BC}$ and passing through $A(0, 0)$ is $y - 0 = 1(x - 0)$

$$y = x.$$

The orthocenter is the point of intersection of $\overleftrightarrow{BM}$, $\overleftrightarrow{AN}$, and $\overleftrightarrow{CP}$.

$x = 12$

$y = -2x + 36$

$y = -2 \cdot 12 + 36$

$y = -24 + 36$

$y = 12$

The coordinates of the orthocenter are (12, 12).

15. a. The coordinates of the centroid are (10, 2). The coordinates of the orthocenter are (12, 12). Find the equation of the line passing through the centroid (10, 2) and the orthocenter (12, 12), then show that the circumcenter $(9, -3)$ is also on the line.

$$\text{slope} = m = \frac{12 - 2}{12 - 10} = \frac{10}{2} = 5$$

An equation of the line passing through the centroid and the orthocenter is

$y - 2 = 5(x - 10)$

$y - 2 = 5x - 50$

$y = 5x - 48$

Substitute the coordinates of the circumcenter into this equation.

—CONTINUED—

15. —CONTINUED—

$-3 = 5 \cdot 9 - 48$

$-3 = 45 - 48$

$-3 = -3$

Since $-3 = -3$ is true, the circumcenter is on the same line as the centroid and the orthocenter. Therefore, they are all collinear.

b. The distance from the circumcenter C to the centroid D is CD.

$$CD = \sqrt{(10 - 9)^2 + (2 - (-3))^2}$$
$$= \sqrt{1^2 + 5^2}$$
$$= \sqrt{1 + 25}$$
$$= \sqrt{26}$$

The distance from the circumcenter C to the orthocenter P is CP.

$$CP = \sqrt{(12 - 9)^2 + (12 - (-3))^2}$$
$$= \sqrt{3^2 + 15^2}$$
$$= \sqrt{9 + 225}$$
$$= \sqrt{234}$$
$$= \sqrt{9}\sqrt{26}$$
$$= 3\sqrt{26}$$

$$CD = \frac{1}{3}CP$$

$$\sqrt{26} = \frac{1}{3}\left(3\sqrt{26}\right)$$

$$\sqrt{26} = \sqrt{26}$$

So the distance from the circumcenter to the centroid is one third the distance from the circumcenter to the orthocenter.

Project Chapters 4–5 *(pp. 316–317)*

Investigation

1. The lines are *medians* because they are the lines that contain the line segments whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

2. The balancing point of the triangle is the *centroid* because it is the point of intersection of the medians.

3. Answers will vary.

4. **Conjecture:** The balancing point of a square, a rectangle, a parallelogram, or a rhombus is the point of intersection of its diagonals.

5. Answers will vary.

Sample answer: I tested the conjecture by making more example shapes of each kind. The results were the same each time. The balancing point was the point of intersection of the diagonals.

Present Your Results

Projects may vary.

Extension

The conjecture does not work for *all* four–sided shapes. The following is an example for which it was not true.