CHAPTER 4

Think & Discuss *(p. 191)*

1. right angle
2. $\angle BAC$ and $\angle EDF$, $\angle BDC$ and $\angle EGF$ appear to be acute and congruent. $\angle ABD$ and $\angle DEG$ appear to be obtuse and congruent.

Skill Review *(p. 192)*

1. $180 = 90 + x + 60$

$180 - 90 - 60 = x$

$30 = x$

2. $6 = 2x + 2$

$6 - 2 = 2x$

$4 = 2x$

$2 = x$

3. $2x = 4x - 6$

$0 = 4x - 2x - 6$

$6 = 2x$

$3 = x$

4. $180 = 30 + 2x$

$150 = 2x$

$75 = x$

5. $90 = 3x - 90$

$180 = 3x$

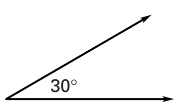
$60 = x$

6. $3x = 27 - 6x$

$9x = 27$

$x = 3$

7.



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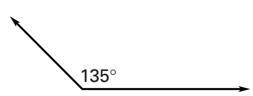

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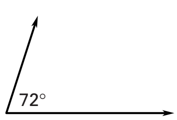


10. Vertical Angles Theorem
11. Alternate Interior Angles Theorem
12. Corresponding Angles Postulate

Developing Concepts (p. 193)
Exploring the Concept

Interior Angles

3. The sum of the measures of the interior angles is 180°.

Exploring the Concept

Exterior Angles

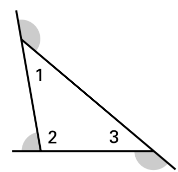
7. The measure of an exterior angle is equal to the sum of the measures of the nonadjacent interior angles.

Drawing Conclusions

1. The sum of the measures of the interior angles of a triangle is 180°.
2. The measure of an exterior angle is equal to the sum of the measures of the interior angles that are not adjacent to it.
3. Subtract the sum of the measures of the two interior angles from 180° to find the measure of the third interior angle.

4.1 Guided Practice (p. 198)

1.



2. hypotenuse **3.** PRQ **4.** base **5.** $\overline{PR}, \overline{RS}$

6. acute isosceles **7.** right scalene

8. equiangular or acute, equilateral or isosceles

9. $180 = 25 + (2x)$

$155 = 2x$

$77.5 = x$; 77.5°, 77.5°

4.1 Practice and Applications (pp. 198–201)

10. B **11.** E **12.** A **13.** D **14.** F **15.** C

16. acute isosceles **17.** right isosceles **18.** obtuse scalene

19. right scalene **20.** obtuse isosceles **21.** acute scalene

22. sometimes **23.** sometimes **24.** always **25.** always

26. never

27. Ex. 17; legs are $\overline{DE}$ and $\overline{DF}$, hypotenuse is $\overline{EF}$

Ex. 19; legs are $\overline{RP}$ and $\overline{RQ}$, hypotenuse is $\overline{PQ}$

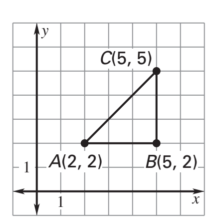
28. Ex. 16; legs are $\overline{CA}$ and $\overline{CB}$, base is $\overline{AB}$

Ex. 17; legs are $\overline{DE}$ and $\overline{DF}$, base is $\overline{EF}$

Ex. 20; legs are $\overline{VT}$ and $\overline{VU}$, base is $\overline{TU}$

Ex. 17 has a base that is also the hypotenuse of a right triangle.

29. The right angle must be at A or B. Since the placement indicates it is not at A, C is directly above B and so the x-coordinate of C is 5. The length of $\overline{AB}$ is 3, so 3 is added to the y-coordinate of B so the y-coordinate of C is 5. The coordinates of C are (5, 5)

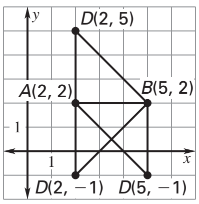


30. There are 3 different possibilities for point D; they are:

a. 3 units above A making $D(2, 5)$

b. 3 units below A making $D(2, -1)$

c. 3 units below B making $D(5, -1)$



31. $m\angle 1 = 180° - 90° - 42° = 48°$

32. $m\angle 1 = 180° - 90° - 40° = 50°$

$\angle 2$ is a vertical angle with the given 40° angle so $m\angle 2 = 40°$.

$m\angle 3 = 180° - 95° - 40° = 45°$

33. $m\angle 1 = 180° - 45° - 56° = 79°$

The sum of 56° and the measures of $\angle 1$ and $\angle 2$ is 180° so $m\angle 2 = 180° - 79° - 56° = 51°$.

$m\angle 3 = 180° - 90° - 51° = 39°$

34. $x° + 2x° + (2x + 15)° = 180°$

$5x + 15 = 180$

$5x = 165$

$x = 33$

$m\angle A = x° = 33°$, $m\angle B = 2x° = 66°$, $m\angle C = (2x + 15)° = 81°$; acute

35. $x° + 7x° + x° = 180$

$9x = 180$

$x = 20$

$m\angle R = m\angle T = x° = 20°$, $m\angle S = 7x° = 140°$; obtuse

36. $(x - 15)° + (2x - 165)° = 90°$

$3x - 180 = 90 : 3x = 270 : x = 90$

$m\angle W = (x - 15)° = 75°$, $m\angle Y = (2x - 165)° = 15°$; right

37. $2x - 8 = x + 31$

$2x = x + 39$

$x = 39$

$(2x - 8)° = 70°$

38. $10x + 9 = 38 + 7x + 1$

$10x = 30 + 7x$

$3x = 30$

$x = 10$

$(10x + 9)° = 109°$

39. Exterior angle $= (180 - x)°$

$(2x - 21)° + x° = 90°$

$3x - 21 = 90$

$3x = 111$

$x = 37$

$(180 - x)° = 143°$

40. *Sample answer:* To demonstrate the △ sum Thm., draw a △, measure its three interior angles, and verify that their sum is 180°. To demonstrate the Ext. ∠ Thm., extend one side of the triangle to form an exterior angle, measure the exterior angle and its two remote interior angles, and verify that the measure of the exterior angle equals the sum of the measures of the two remote interior angles.

41. $m\angle P + m\angle Q + m\angle R = 180°$

$36° + 5x° + x° = 180°$

$6x = 144$

$x = 24$

$m\angle R = x° = 24°$, $m\angle Q = 5x° = 120°$

42. $x° + x° = 120°$

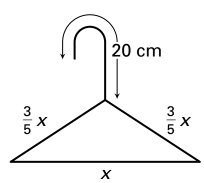
$2x = 120$

$x = 60$

The angles will have measure 60°, 60° and 60°.

43. Yes; the total length needed is 3×33.5, or 100.5 cm.

44.



$\frac{3}{5}x + \frac{3}{5}x + x = 88 - 20$

$\frac{11}{5}x = 68$

$x \approx 30.9 \qquad \frac{3}{5}x \approx 18.5$

about 18.5 cm × 31 cm × 18.5 cm

45. $\overline{MN}$ and $\overline{LN}$ are the legs; $\overline{ML}$ is the hypotenuse.

46. Angle N is a right angle so the measures of the other two angles will add up to 90°. The measure of the down-stream angle should range from 60° to 45°.

47.

Statements	Reasons
4. $m\angle A + m\angle B + m\angle ACB = 180°$	**3.** Linear Pair Postulate
	5. Substitution property of equality
	6. Subtraction property of equality

48.

Statements	Reasons
1. $m\angle A + m\angle B + m\angle C = 180°$	**1.** Triangle Sum Theorem
2. $\angle C$ is a right angle.	**2.** Given
3. $m\angle C = 90°$	**3.** Definition of right angle
4. $m\angle A + m\angle B + 90° = 180°$	**4.** Substitution
5. $m\angle A + m\angle B = 90°$	**5.** Subtraction property of equality
6. $\angle A$ and $\angle B$ are complementary.	**6.** Definition of complementary angles

49. $2x - 5 = x + 7$

$x = 12$

The length of each leg is $12 + 7 = 19$.

The length of the base is $50 - 19 - 19 = 12$.

C

50. B

51.

Statements	Reasons
1. Construct $\overleftrightarrow{PS}$ parallel to $\overleftrightarrow{QR}$.	**1.** Given a line and a point P, not on the line, there is exactly one line through the point that is parallel to the line.
2. $m\angle 1 + m\angle 5 + m\angle 4 = 180°$	**2.** Angle Addition Postulate and definition of straight angle
3. $\angle 5 \cong \angle 2$	**3.** Alternate interior angles theorem
4. $\angle 4 \cong \angle 3$	**4.** Corresponding angles postulate
5. $m\angle 5 = m\angle 2$, $m\angle 4 = m\angle 3$	**5.** Definition of congruent angles
6. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**6.** Substitution property of equality

4.1 Mixed Review (p. 201)

52. true **53.** true **54.** false **55.** false **56.** true

57. yes; Alternate Interior Angles Converse

58. yes; Alternate Exterior Angles Converse

59. yes; Corresponding Angles Converse

60. $y + 2 = 0(x - 0)$

$y + 2 = 0$

$y = -2$

61. $y - 7 = x - 4$

$y = x + 3$

62. $y + 5 = -(x + 3)$

$y + 5 = -x - 3$

$y = -x - 8$

63. $y + 1 = \frac{2}{3}(x - 9)$

$y + 1 = \frac{2}{3}x - 6$

$y = \frac{2}{3}x - 7$

64. $y + 1 = \frac{3}{4}(x + 1)$

$y + 1 = \frac{3}{4}x + \frac{3}{4}$

$y = \frac{3}{4}x - \frac{1}{4}$

65. $y + 3 = -\frac{7}{2}(x + 2)$

$y + 3 = -\frac{7}{2}x - 7$

$y = -\frac{7}{2}x - 10$

66. $y - 2 = 0(x - 5)$

$y - 2 = 0$

$y = 2$

67. $y - 3 = -\frac{3}{2}(x - 8)$

$y - 3 = -\frac{3}{2}x + 12$

$y = -\frac{3}{2}x + 15$

68. $y + 4 = -\frac{1}{3}(x + 6)$

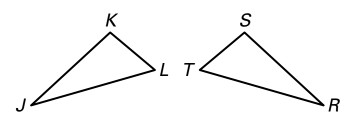
$y + 4 = -\frac{1}{3}x - 2$

$y = -\frac{1}{3}x - 6$

Lesson 4.2

4.2 Guided Practice (p. 205)

1.



Corresponding angles: $\angle J$ and $\angle R$, $\angle K$ and $\angle S$, $\angle L$ and $\angle T$

Corresponding sides: $\overline{KL}$ and $\overline{ST}$, $\overline{JK}$ and $\overline{RS}$, $\overline{JL}$ and $\overline{RT}$

2. Third Angle Theorems

3. No; corresponding sides are not congruent.

4. 105° **5.** 45° **6.** 30° **7.** 30° **8.** $\overline{MN}$ **9.** $\overline{PR}$

4.2 Practice and Applications (pp. 206–209)

10. $\angle T$ **11.** $\overline{CA}$ **12.** $\triangle CAB$ **13.** UV **14.** T; 66°

15. B, C, D

16. $\triangle ABD \cong \triangle CDB$; $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence so the triangles are congruent by the definition of congruence.

17. $\triangle FGH \cong \triangle JKH$; $\angle FHG \cong \angle JHK$ by the Vertical Angles Theorem, so the triangles are congruent by the definition of congruence.

18. $ABCD \cong PSRQ$; definition of congruence.

19. $VWXYZ \cong MNJKL$; definition of congruence.

20. $EFJK \cong HJFG$; in a plane, if two lines are perpendicular to the same line, then the lines are parallel to each other, so $\overline{EG} \parallel \overline{KH}$, so $\angle EFJ \cong \angle HJF$ and $\angle KJF \cong \angle GFJ$ by the Alternate Interior Angles Theorem. $\overline{FJ} \cong \overline{FJ}$ by the Reflexive Property of Congruence, so the trapezoids are congruent by the definition of congruence.

21. $\triangle LKR \cong \triangle NMQ$; $\overline{LR}$ and $\overline{NQ}$ are congruent by the Addition property of equality, the Segment Addition Postulate, and the definition of congruence and $\angle NQM \cong \angle LRK$ by the Third Angles Theorem, so the triangles are congruent by the definition of congruence. $LKQS \cong NMRS$; $\angle LSQ \cong \angle NSR$ by the vertical angles theorem and $\angle KQS \cong \angle MRS$ by the Congruent Supplements Theorem, so the quadrilaterals are congruent by the definition of congruence.

22. $\angle G \cong \angle K$, $\angle F \cong \angle J$, $\angle JHK \cong \angle FHG$; $\overline{FG} \cong \overline{JK}$, $\overline{GH} \cong \overline{KH}$, $\overline{FH} \cong \overline{JH}$.

23. 3 pairs of angles are congruent because of the Third Angles Theorem. It cannot be determined whether the triangles are congruent because no corresponding congruent sides are shown.

24. $(10x + 65)^\circ = 135^\circ$

$10x = 70$

$x = 7$

$(4y - 4)^\circ = 28^\circ$

$4y = 32$

$y = 8$

25. $(4a - 4)^\circ = 48^\circ$

$4a = 52$

$a = 13$

$(5b - 3)^\circ = 62^\circ$

$5b = 65$

$b = 13$

26. $m\angle N = 180^\circ - 142^\circ - 24^\circ = 14^\circ$

$(2x - 50)^\circ = 14^\circ$

$2x = 64$

$x = 32$

27. $m\angle S = 40^\circ$

$m\angle U = 180^\circ - 80^\circ - 40^\circ = 60^\circ$

$(5m)^\circ = 60^\circ$

$m = 12$

28. $m\angle B = 180^\circ - 90^\circ - 35^\circ = 55^\circ$

$(3s - 20)^\circ = 55^\circ$

$3s = 75$

$s = 25$

29. $m\angle Y = 180^\circ - 50^\circ - 78^\circ = 52^\circ$

$\frac{4}{5}r^\circ = 52^\circ$

$4r = 260$

$r = 65$

30. $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are corresponding sides of congruent triangles and are congruent. $\triangle ABC$ is equilateral by definition.

31. $m\angle ADB = m\angle CDA = m\angle CDB$ since the angles are corresponding parts of congruent triangles. Each measure must equal $\frac{1}{3}$ of 360°, which is 120°.

32. $m\angle BDC + m\angle DBC + m\angle DCB = 180^\circ$ by the Triangle Sum Theorem

$120^\circ + x^\circ + x^\circ = 180^\circ$

$120 + 2x = 180$

$2x = 60$

$x = 30$

$m\angle DBC = m\angle DCB = 30^\circ$

33. The measure of each of the congruent angles in each small triangle is 30°. By the Angle Addition Postulate, the measure of each angle of $\triangle ABC$ is 60°.

34. Transitive Property of Congruent Triangles

35.

Statements	Reasons
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$	**1.** Given **2.** Definition of congruent angles
7. $\angle C \cong \angle F$	**3.** Triangle Sum Theorem
	4. Substitution property of equality or transitive property of equality
	5. Substitution property of equality
	6. Subtraction property of equality

36. Yes; when the paper is folded, $\overline{EB}$ and $\overline{AB}$ coincide, as do $\overline{EF}$ and $\overline{AF}$.

37. $\triangle ABF$ and $\triangle EBF$; $\overline{BF} \cong \overline{BF}$ by the Reflexive Property of Congruence, and $\angle A$ and $\angle BEF$ are congruent by the Third Angles Theorem, so the triangles are congruent by the definition of congruence.

38. *Sample answer:* $\overline{BE} \cong \overline{BE}$ by the Reflexive Property of Congruence; $\angle GEB \cong \angle FEB$ because they are both right angles since $\overline{BD} \perp \overline{FG}$; $\angle FBE \cong \angle GBE$ by the definition of angle bisector; $\angle EFB \cong \angle EGB$ by the Third Angles Theorem; $\overline{EF} \cong \overline{EG}$ by the definition of segment bisector; since it is given that $\overline{BF} \cong \overline{BG}$, $\triangle FEB \cong \triangle GEB$ by the definition of congruence.

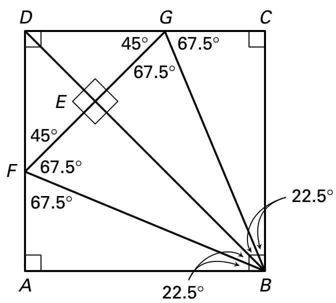
39. **a** and **b.** They are corresponding parts of congruent figures.

c. Linear Pair Postulate and Congruent Supplements Theorem

d. They are both right angles because $\overline{GE} \perp \overline{BD}$. All right angles are congruent.

e. Yes; $\angle BGE \cong \angle DGE$ by the Third Angles Theorem and $\overline{GE} \cong \overline{GE}$ by the Reflexive Property of Congruence. With the given and parts (a)–(d), all corresponding parts are congruent, so $\triangle BEG \cong \triangle DEG$ by the definition of congruent triangles.

40.



4.2 Mixed Review (p. 209)

41. $d = \sqrt{(3-(-1))^2 + (8-(-4))^2}$
$= \sqrt{16 + 144}$
$= \sqrt{160}$
$= 4\sqrt{10}$

42. $d = \sqrt{(3-(-13))^2 + (-8-7)^2}$
$= \sqrt{256 + 225}$
$= \sqrt{481}$

43. $d = \sqrt{(-2-3)^2 + (-6-(-5))^2}$
$= \sqrt{25 + 1}$
$= \sqrt{26}$

44. $d = \sqrt{(0-(-5))^2 + (5-2)^2}$
$= \sqrt{25 + 9}$
$= \sqrt{34}$

45. $d = \sqrt{(0-9)^2 + (-4-2)^2}$
$= \sqrt{81 + 36}$
$= \sqrt{117}$
$= 3\sqrt{13}$

46. $d = \sqrt{(7-0)^2 + (-2-9)^2}$
$= \sqrt{49 + 121}$
$= \sqrt{170}$

47. $M = \left(\frac{-1+(-3)}{2}, \frac{5+(-9)}{2}\right) = (-2, -2)$

48. $M = \left(\frac{5+(-1)}{2}, \frac{7+4}{2}\right) = \left(2, 5\frac{1}{2}\right)$

49. $M = \left(\frac{-6+8}{2}, \frac{-2+2}{2}\right) = (1, 0)$

50. $M = \left(\frac{0+(-6)}{2}, \frac{-7+4}{2}\right) = \left(-3, -1\frac{1}{2}\right)$

51. $M = \left(\frac{12+8}{2}, \frac{0+6}{2}\right) = (10, 3)$

52. $M = \left(\frac{-5+0}{2}, \frac{-7+4}{2}\right) = \left(-2\frac{1}{2}, -1\frac{1}{2}\right)$

53. $m\angle 1 + m\angle 2 = 90°$
$8° + m\angle 2 = 90°$
$m\angle 2 = 82°$

54. $m\angle 1 + m\angle 2 = 90°$
$73° + m\angle 2 = 90°$
$m\angle 2 = 17°$

55. $m\angle 1 + m\angle 2 = 90°$
$62° + m\angle 2 = 90°$
$m\angle 2 = 28°$

56. Slope of Line 1: $\frac{2-(-1)}{1-(-3)} = \frac{3}{4}$

Slope of Line 2: $\frac{1-(-2)}{6-2} = \frac{3}{4}$

The lines are parallel.

57. Slope of Line 1: $\frac{3-(-2)}{-3-(-1)} = -\frac{5}{2}$

Slope of Line 2: $\frac{3-(-1)}{2-4} = \frac{4}{-2} = -2$

The lines are not parallel.

Quiz 1 (p. 210)

1. acute isosceles
2. acute isosceles
3. obtuse scalene
4. $(16x + 20)° = 77° + (7x + 6)°$
   $9x = 63$
   $x = 7$
   $m\angle E = 7(7) + 6 = 55°$; $m\angle F = 77°$;
   $m\angle EDF = 180° - 77° - 55° = 48°$;
   $m\angle CDF = 16(7) + 20 = 132°$
5. $\triangle MNP \cong \triangle QPN$; $\angle M$ and $\angle Q$, $\angle MNP$ and $\angle QPN$, $\angle MPN$ and $\angle QNP$, $\overline{MN}$ and $\overline{QP}$, $\overline{NP}$ and $\overline{PN}$, $\overline{MP}$ and $\overline{QN}$
6. Since $\angle PNQ$ and $\angle NPM$ are congruent, $m\angle NPM = 27°$. By the Triangle Sum Theorem, $m\angle MNP = 180° - 46° - 27° = 107°$.

Math & History

1. $63° + x° + x° = 180°$
   $2x = 180 - 63$
   $2x = 117$
   $x = 58.5$
   58.5°, 58.5°; acute isosceles

Developing Concepts Activity 4.3 (p. 211)

1.–4. Check students' work and drawings. You should notice that step 4 is impossible to do.

5.–8. Check students' work and drawings. You should notice that step 8 is impossible to do.

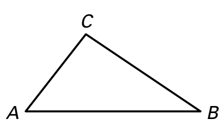
Drawing Conclusions

1. All triangles made with three pencils appear to be congruent. All triangles made with three pencils and a $45°$ angle appear to be congruent.
2. the length of the third side or the measure of the angle between the sides whose lengths are known

Lesson 4.3

4.3 Guided Practice (p. 216)

1. *Sample answer:* This triangle has sides $\overline{AC}$ and $\overline{CB}$ with included angle $\angle C$.



2. The congruent angles are not the angles included between the congruent sides.
3. yes; SAS Congruence Postulate
4. not enough information
5. yes; SSS Congruence Postulate

4.3 Practice and Applications (pp. 216–219)

6. $\angle JKL$ **7.** $\angle LKP$ **8.** $\angle KLP$ **9.** $\angle KJL$ **10.** $\angle JLK$

11. $\angle KPL$ **12.** not enough information

13. yes; SAS Congruence Postulate

14. yes; SSS Congruence Postulate

15. yes; SAS Congruence Postulate

16. not enough information

17. yes; SSS Congruence Postulate

18. $\overline{AB} \cong \overline{CD}$

19. $\angle ACB \cong \angle CED$

20. (1) Given; (2) Given; (3) Reflexive Property of Congruence; (4) SSS Congruence Postulate

21.

Statements	Reasons
1. $\overline{NP} \cong \overline{QN} \cong \overline{RS} \cong \overline{TR}$, $\overline{PQ} \cong \overline{ST}$	1. Given
2. $\triangle NPQ \cong \triangle RST$	2. SSS Congruence Postulate

22.

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Alternate Interior Angles Theorem
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property of Congruence
4. $\triangle ABC \cong \triangle CDA$	4. SAS Congruence Postulate

23. It is given that $\overline{SP} \cong \overline{TP}$ and that $\overrightarrow{PQ}$ bisects $\angle SPT$. Then by the definition of angle bisector, $\angle SPQ \cong \angle TPQ$. $\overline{PQ} \cong \overline{PQ}$ by the Reflexive Property of Congruence, so $\triangle SPQ \cong \triangle TPQ$ by the SAS Congruence Postulate.

24. It is given that $\overline{PT} \cong \overline{RT}$ and $\overline{QT} \cong \overline{ST}$. $\angle PTQ \cong \angle RTS$ by Vertical Angles Theorem. Then $\triangle PQT \cong \triangle RST$ by SAS Congruence Postulate.

25.

Statements	Reasons
1. $\overline{AC} \cong \overline{BC}$; M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $\overline{CM} \cong \overline{CM}$	3. Reflexive Property of Congruence
4. $\triangle ACM \cong \triangle BCM$	4. SSS Congruence Postulate

26.

Statements	Reasons
1. $\overline{BC} \cong \overline{AE}$, $\overline{BD} \cong \overline{AD}$, $\overline{DE} \cong \overline{DC}$	1. Given
2. $BD = AD$, $DE = DC$	2. Definition of congruent segments
3. $BD + DE = AD + DC$	3. Addition property of equality
4. $BD + DE = BE$, $AD + DC = AC$	4. Segment Addition Postulate
5. $BE = AC$	5. Substitution property
6. $\overline{BE} \cong \overline{AC}$	6. Definition of congruent segments
7. $\overline{AB} \cong \overline{AB}$	7. Reflexive Property of Congruence
8. $\triangle ABC \cong \triangle BAE$	8. SSS Congruence Postulate

27. Since it is given that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ and $\overline{AB} \cong \overline{BC}$, $\triangle PAB \cong \triangle PBC$ by the SSS Congruence Postulate.

28. It is given that $\overline{CR} \cong \overline{CS}$ and that $\overline{QC}$ is perpendicular to both $\overline{CR}$ and $\overline{CS}$. If two lines are perpendicular, they intersect to form four right angles, so $\angle QCR$ and $\angle QCS$ are right angles. By the Right Angle Congruence Theorem, $\angle QCR \cong \angle QCS$. By the Reflexive Property of Congruence, $\overline{QC} \cong \overline{QC}$. Then $\triangle QCR \cong \triangle QCS$ by the SAS Congruence Postulate.

29. The new triangle and the original triangle are congruent.

30. *Sample answer:* The cross pieces form triangles which are rigid, ensuring that the supports keep their shape.

31. *Sample answer:* The struts that go from the body of the plane to the wing form triangles, making the wing structure rigid.

32. The activity on p. 213 demonstrates how to copy a triangle. Use this technique to duplicate your isosceles triangle. The duplicate will be congruent to the original. Note, however, that the same compass setting will be used to construct the legs of the triangle.

33. $AB = ED = 2$, $CB = EF = 4$,

$AC = \sqrt{(2-0)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$DF = \sqrt{(5-3)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$AC = DF = 2\sqrt{5}$, so all three pairs of sides are congruent and $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate.

34. $AC = DF = 2$, $CB = EF = 3$

$AB = \sqrt{(-3-0)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$DE = \sqrt{(0-2)^2 + (6-3)^2} = \sqrt{4+9} = \sqrt{13}$

$AB = DE = \sqrt{13}$, so all three pairs of sides are congruent and $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate.

35. $AB = DE = 3$,

$AC = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$

$DF = \sqrt{(5-4)^2 + (6-3)^2} = \sqrt{1+9} = \sqrt{10}$

$BC = \sqrt{(2-4)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13}$

$EF = \sqrt{(2-4)^2 + (6-3)^2} = \sqrt{4+9} = \sqrt{13}$

$AC = DF = \sqrt{10}$ and $BC = EF = \sqrt{13}$, so all three pairs of sides are congruent and $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Postulate.

36. C **37.** B

38. To show $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate, the Distance Formula could be used to find the lengths of $\overline{OM}$, $\overline{MN}$, $\overline{OP}$, and $\overline{PN}$ and show that $\overline{OP} \cong \overline{NP}$ and $\overline{OM} \cong \overline{NM}$. The triangles share side $\overline{PM}$, and $\overline{PM} \cong \overline{PM}$. The pairs of congruent sides can be used with the SSS Congruence Postulate to show triangle congruence. However, to use SAS, you must show $\angle OMP$ and $\angle NMP$ are right angles. To do so, you should find the slope of $\overline{MP}$ and $\overline{ON}$ and show that they are negative reciprocals of each other so $\overline{PM} \perp \overline{ON}$. Then show right angles $\angle OMP$ and $\angle NMP$ are congruent, $\overline{OM} \cong \overline{NM}$, and $\overline{PM} \cong \overline{PM}$, so $\triangle PMO \cong \triangle PMN$ by the SAS Congruence Postulate. Preferences may vary, but the SAS method involves more computations and mathematical knowledge.

4.3 Mixed Review (p. 219)

39. *Sample answer:* The measure of each of the angles formed by two adjacent "spokes" is about 60°.

40. *Sample answer:* The measure of each of the angles formed by two adjacent sides of a cell in a honeycomb is about 120°.

41. $m\angle 2 = 57°$(Vertical Angles Theorem)

$m\angle 1 = 180° - m\angle 2 = 123°$
(Consecutive Interior Angles Theorem)

42. $m\angle 1 = 180° - 129° = 51°$ (Linear Pair Postulate)

$m\angle 2 = m\angle 1 = 51°$
(Alternate Exterior Angles Theorem)

43. $m\angle 1 = 90°$(Corresponding Angles Postulate)

$m\angle 2 = 90°$(Alternate Interior Angles Theorem or Vertical Angles Theorem).

44. Slope of $\overleftrightarrow{AC} = \dfrac{3-(-2)}{-1-(-2)} = -\dfrac{5}{3}$,

slope of $\overleftrightarrow{BD} = \dfrac{1-(-2)}{3-(-2)} = \dfrac{3}{5}$,

so $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$.

45. Slope of $\overleftrightarrow{EF} = \dfrac{2-(-2)}{-2-0} = -2$,

slope of $\overleftrightarrow{GH} = \dfrac{1-(-1)}{2-3} = -2$,

so $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$.

46. Slope of $\overleftrightarrow{QR} = \dfrac{4-2}{1-4} = -\dfrac{2}{3}$,

slope of $\overleftrightarrow{PQ} = \dfrac{4-1}{1-(-1)} = \dfrac{3}{2}$,

so $\overleftrightarrow{QR} \perp \overleftrightarrow{PQ}$.

Lesson 4.4

4.4 Guided Practice (p. 223)

1. SSS Congruence Postulate, SAS Congruence Postulate, ASA Congruence Postulate, and AAS Congruence Theorem. The AAS Congruence Theorem is a theorem because it is proved, rather than accepted without proof like a postulate is.

2. Yes; ASA Congruence Postulate; two pairs of corresponding angles and the corresponding included sides are congruent.

3. Yes; AAS Congruence Theorem; two pairs of corresponding angles and corresponding nonincluded sides are congruent.

4. No; two pairs of angles are congruent, which is insufficient to prove two triangles are congruent.

5. $\overline{AB} \cong \overline{DE}$ **6.** $\angle A \cong \angle D$

7. By the Right Angle Congruence Theorem, $\angle B \cong \angle D$. Since $\overline{AD} \parallel \overline{BC}$, $\angle CAD \cong \angle ACB$ by the Alternate Interior Angles Theorem. By the Reflexive Property of Congruence, $\overline{AC} \cong \overline{AC}$, so $\triangle ACD \cong \triangle CAB$ by the AAS Congruence Theorem. Then all three pairs of corresponding sides are congruent; that is they have the same length. So, $AB + BC + CA = CD + DA + AC$ and the two courses are the same length.

4.4 Practice and Applications (pp. 223–226)

8. Yes; ASA Congruence Postulate; two pairs of corresponding angles and the corresponding included sides are congruent.

9. Yes; SAS Congruence Postulate; two pairs of corresponding sides and the corresponding included angles are congruent.

10. Yes; AAS Congruence Theorem; two pairs of corresponding angles and the corresponding nonincluded sides are congruent.

11. No; two pairs of corresponding sides are congruent and corresponding nonincluded angles $\angle EGF$ and $\angle JGH$ are congruent by the Vertical Angles Theorem; that is insufficient to prove triangle congruence.

12. No; two pairs of corresponding sides are congruent and corresponding nonincluded angles $\angle K$ and $\angle Q$ are congruent; that is insufficient to prove triangle congruence.

13. Yes; SSS Congruence Postulate; $\overline{XY} \cong \overline{XY}$ by the Reflexive Property of Congruence, so all three pairs of corresponding sides are congruent.

14. $\angle R \cong \angle U$ **15.** $\angle P \cong \angle S$ **16.** $\overline{PR} \cong \overline{SU}$

17. $\overline{QR} \cong \overline{TU}$

18. (1) Given; (2) Given; (3) $\angle W \cong \angle W$; (4) AAS Congruence Theorem

19.

Statements	Reasons
1. $\overline{GF} \cong \overline{GL}$, $\overline{FH} \parallel \overline{LK}$	**1.** Given
2. $\angle F \cong \angle L$, $\angle H \cong \angle K$	**2.** Alternate Interior Angles Theorem
3. $\triangle FGH \cong \triangle LGK$	**3.** AAS Congruence Theorem

20.

Statements	Reasons
1. $\overline{AB} \perp \overline{AD}$, $\overline{DE} \perp \overline{AD}$	**1.** Given
2. $\angle A$ and $\angle D$ are right angles	**2.** If two lines are perpendicular, then they form four right angles.
3. $\angle A \cong \angle D$	**3.** All right angles are congruent.
4. $\angle ACB \cong \angle DCE$	**4.** Vertical Angles Theorem
5. $\overline{BC} \cong \overline{EC}$	**5.** Given
6. $\triangle ABC \cong \triangle DEC$	**6.** AAS Congruence Theorem

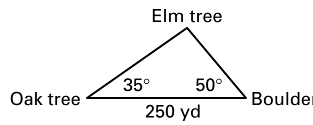
21. It is given that $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, and that $\overline{XW} \parallel \overline{YZ}$. Then, $\angle VXW \cong \angle Y$ by the Corresponding Angles Postulate and $\triangle VXW \cong \triangle XYZ$ by the SAS Congruence Postulate.

22. It is given that $\angle TQS \cong \angle RSQ$ and $\angle R \cong \angle T$. By the Reflexive Property of Congruence, $\overline{QS} \cong \overline{QS}$. Then $\triangle TQS \cong \triangle RSQ$ by the AAS Congruence Theorem.

23. Yes; two sides of the triangle are north-south and east-west lines, which are perpendicular, so the measures of two angles and the length of a nonincluded side are known and only one such triangle is possible.

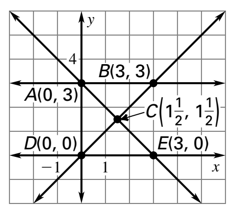
24. Assuming that all three streets can be represented by straight segments, a surveyor needs to measure the bearing of Ellis Avenue with respect to Green Street (which would give a unique triangle by the AAS Congruence Theorem) or Plain Street (which would give a unique triangle by the SAS Congruence Postulate or the ASA Congruence Postulate). The length of the portion of Green Street between Plain Street and Ellis Avenue could also be measured, giving a unique triangle by the SAS Congruence Postulate or the SSS Congruence Postulate.

25.



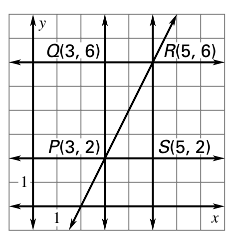
Yes; the measures of two angles and the length of the included side are known and only one such triangle is possible.

26.



Sample answer: $\triangle ABC \cong \triangle EDC$ because, since C is the midpoint of $\overline{AE}$ and $\overline{BD}$ (from the midpoint formula), $AC = CE$, $BC = CD$, and $\angle ACB \cong \angle DCE$ (Vertical Angles Theorem). The congruence follows from the SAS Congruence Postulate.

27.



$\angle PQR \cong \angle RSP$ since they are both right angles, and since $\overline{QR} \parallel \overline{PS}$, $\angle PRQ \cong \angle RPS$ by the Alternate Interior Angles Theorem. $QR = SP = 2$, so $\overline{QR} \cong \overline{SP}$. Then two pairs of corresponding angles and a pair of included sides are congruent, so $\triangle PQR \cong \triangle RSP$ by the ASA Congruence Postulate.

28. SAS Congruence Postulate

29. **a.** $\overline{AB} \cong \overline{AB}$, $\angle ABC \cong \angle ABD$, $\angle BAC \cong \angle BAD$

b. ASA Congruence Postulate

c. By parts (a) and (b), $\triangle ABC \cong \triangle ABD$, so $\overline{BC} \cong \overline{BD}$ since they are corresponding sides of congruent triangles. That is, the distance across the stream is the same as BD.

30. Since only the pair of corresponding sides $\overline{MQ}$ and $\overline{MQ}$ are known to be congruent, Alicia would have to use the ASA Congruence Postulate or the AAS Congruence Theorem. In the first case, she would need to know that $\angle NMQ \cong \angle PQM$ and $\angle NQM \cong \angle PMQ$. In the second case, she would need to know that either $\angle N \cong \angle P$ and $\angle NQM \cong \angle PMQ$ or that $\angle N \cong \angle P$ and $\angle NMQ \cong \angle PQM$. None of the necessary congruences can be deduced from the postulates and theorems concerning parallel lines and transversals.

31. It is given that $\angle XMQ \cong \angle XQM$ and $\angle N \cong \angle P$. $\overline{MQ} \cong \overline{MQ}$ by the Reflexive Property of Congruence, so $\triangle MNQ \cong \triangle QPM$ by the AAS Congruence Theorem.

32. $\frac{5+x}{2} = -1$ $\qquad$ $\frac{7+y}{2} = 0$

$5 + x = -2$ $\qquad$ $7 + y = 0$

$x = -7$ $\qquad$ $y = -7$

$(-7, -7)$

33. $\frac{0+x}{2} = 6$ $\qquad$ $\frac{9+y}{2} = -2$

$x = 12$ $\qquad$ $9 + y = -4$

$y = -13$

$(12, -13)$

34. $\frac{8+x}{2} = -1$ $\qquad$ $\frac{-5+y}{2} = -3$

$8 + x = -2$ $\qquad$ $-5 + y = -6$

$x = -10$ $\qquad$ $y = -1$

$(-10, -1)$

35. $m\angle DBC = 42°$ and $m\angle ABC = 84°$

36. $m\angle ABD = m\angle DBC = 27.5°$

37. $m\angle ABD = 75°$ and $m\angle ABC = 150°$

38. Corresponding Angles Converse

Quiz 2 (p. 227)

1. Yes; SAS Congruence Postulate; $\overline{BD} \cong \overline{BD}$, by the Reflexive Property of Congruence, so two pairs of corresponding sides and the corresponding included angles are congruent.

2. Yes; SSS Congruence Postulate; $\overline{SQ} \cong \overline{SQ}$ by the Reflexive Property of Congruence, so three pairs of corresponding sides are congruent.

3. No; two pairs of corresponding sides and one pair of corresponding nonincluded angles are congruent; that is insufficient to prove triangle congruence.

4. Yes; ASA Congruence Postulate; $\overline{MK} \cong \overline{MK}$ by the Reflexive Property of Congruence, so two pairs of corresponding angles and the corresponding included sides are congruent.

5. No; $\overline{ZB} \cong \overline{ZB}$ by the Reflexive Property of Congruence, so two pairs of corresponding sides are congruent; that is insufficient to prove triangle congruence.

6. Yes; AAS Congruence Theorem; $\angle STR \cong \angle VTU$ by the Vertical Angles Theorem, so two pairs of corresponding angles and corresponding nonincluded sides are congruent.

7.

Statements	Reasons
1. M is the midpoint of $\overline{NL}$, $\overline{NL} \perp \overline{NQ}$, $\overline{NL} \perp \overline{MP}$, $\overline{QM} \parallel \overline{PL}$.	**1.** Given
2. $\angle N$ and $\angle PML$ are right angles	**2.** If two lines are perpendicular, they form four right angles.
3. $\angle N \cong \angle PML$	**3.** Right Angle Congruence Theorem
4. $\overline{NM} \cong \overline{ML}$	**4.** Definition of midpoint
5. $\angle QMN \cong \angle PLM$	**5.** Corresponding Angles Postulate
6. $\triangle NQM \cong \triangle MPL$	**6.** ASA Congruence Postulate

Activity 4.4 (p. 228)

1. Answers will vary, but $BH = BG$.

2. Answers will vary.

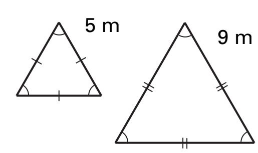
3. $\overline{AB}$ and $\overline{AB}$, $\overline{BG}$ and $\overline{BH}$

4. $\angle BAG$ and $\angle BAH$

5. $\triangle ABG$ and $\triangle ABH$ provide a counterexample.

Extension

It is possible to have two noncongruent triangles in which three angles of one are congruent to three angles of the other. Consider the following drawings which demonstrate that there is no AAA Congruence Postulate or Theorem:



Lesson 4.5

4.5 Guided Practice (p. 232)

1. $\triangle PSQ$ and $\triangle RSQ$ or $\triangle PTQ$ and $\triangle RTQ$
2. Yes; prove that $\triangle PQS \cong \triangle RSQ$ or $\triangle PQR \cong \triangle RSP$ by ASA to get $\overline{PQ} \cong \overline{RS}$. Therefore $\triangle PQT \cong \triangle RST$ by the ASA Congruence Postulate.
3. *Sample answer:* A, G, C, F, E, B, D

Statements	Reasons
1. $\overline{QS} \perp \overline{RP}$	1. Given
2. $\angle PTS$ and $\angle RTS$ are right angles	2. If two lines are perpendicular, then they form four right angles.
3. $\angle PTS \cong \angle RTS$	3. Right Angle Congruence Theorem
4. $\overline{TS} \cong \overline{TS}$	4. Reflexive Property of Congruence
5. $\overline{PT} \cong \overline{RT}$	5. Given
6. $\triangle PTS \cong \triangle RTS$	6. SAS Congruence Postulate
7. $\overline{PS} \cong \overline{RS}$	7. Corresp. parts of $\cong$ ◬ are $\cong$.

4.5 Practice and Applications (pp. 232–235)

4. $\triangle NUP$ and $\triangle PUQ$ are both isosceles, so $\overline{UN} \cong \overline{UP} \cong \overline{UQ}$. Since $\overline{NP}$ and $\overline{PQ}$ are also congruent, $\triangle NUP \cong \triangle PUQ$ by the SSS Congruence Postulate.
5. You can use the method in the answer to Exercise 4 to show that $\triangle QUR \cong \triangle PUQ$, so by the Transitive Property of Congruent Triangles, $\triangle NUP \cong \triangle QUR$. (You could instead use the Transitive Property of Congruence to show that $\overline{UN} \cong \overline{UP} \cong \overline{UQ} \cong \overline{UR}$.)
6. Yes; the procedure from Exercise 5 can be used to prove that any of the triangles is congruent to $\triangle NUP$. The Transitive Property of Congruent Triangles can then be used to show all the triangles are congruent.
7. $\triangle UNP \cong \triangle UPQ$ by Exercise 4. Since corresponding parts of congruent triangles are congruent, $\angle UNP \cong \angle UPQ$.
8. AAS Congruence Theorem; if $\triangle MNL \cong \triangle QPL$, then $\overline{ML} \cong \overline{QL}$ because corresponding parts of congruent triangles are congruent.
9. SSS Congruence Postulate, if $\triangle STV \cong \triangle UVT$, then $\angle STV \cong \angle UVT$ because correponding parts of congruent triangles are congruent.
10. *Sample answer:* Since $\overline{JK} \parallel \overline{MN}$, we know $\angle J \cong \angle M$. The triangles are therefore congruent by the SAS Congruence Postulate. If $\triangle JKL \cong \triangle MNL$, then $\overline{KL} \cong \overline{NL}$ because corresponding parts of congruent triangles are congruent.
11. Since $\triangle AGD \cong \triangle FHC$, their corresponding parts are congruent, and $\overline{GD} \cong \overline{HC}$.
12. Since $\triangle BFC \cong \triangle ECF$, their corresponding parts are congruent, and $\angle CBH \cong \angle FEH$.
13. Since $\triangle EDA \cong \triangle BCF$, their corresponding parts are congruent, and $\overline{AE} \cong \overline{FB}$.

14.

Statements	Reasons
6. $\triangle BAC \cong \triangle DBE$	2. Definition of midpoint
	5. Corresponding Angles Postulate
	7. Corresp. parts of $\cong$ ◬ are $\cong$.

15.

Statements	Reasons
3. $\overline{CF} \cong \overline{CF}$	1. Given
6. $\angle AFB \cong \angle EFD$	2. Given
	4. AAS Congruence Theorem
	5. Corresp. parts of $\cong$ ◬ are $\cong$.
	7. ASA Congruence Postulate

16.

Statements	Reasons
1. L is the midpoint of $\overline{JN}$, $\overline{PJ} \cong \overline{QN}$, $\overline{PL} \cong \overline{QL}$, $\angle PKJ$ and $\angle QMN$ are right angles.	1. Given
2. $\angle PKJ \cong \angle QMN$	2. Right Angle Congruence Theorem
3. $\overline{LJ} \cong \overline{LN}$	3. Definition of midpoint
4. $\triangle PLJ \cong \triangle QLN$	4. SSS Congruence Postulate
5. $\angle J \cong \angle N$	5. Corresponding parts of congruent triangles are congruent.
6. $\triangle PKJ \cong \triangle QMN$	6. AAS Congruence Theorem

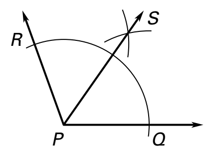
17.

Statements	Reasons
1. $\overline{UR} \parallel \overline{ST}$, $\angle R$ and $\angle T$ are right angles	1. Given
2. $\angle R \cong \angle T$	2. Right Angle Congruence Thm.
3. $\angle RUS \cong \angle TSU$	3. Alternate Interior Angles Theorem
4. $\overline{US} \cong \overline{US}$	4. Reflexive Property of Congruence
5. $\triangle RSU \cong \triangle TUS$	5. AAS Congruence Theorem
6. $\angle RSU \cong \angle TUS$	6. Corresp. parts of $\cong$ ◬ are $\cong$.

18. It is given that $\overline{BD}$ bisects $\overline{AC}$, so $\overline{AD} \cong \overline{DC}$ by the definition of a segment bisector. We also know by the Reflexive Property of Congruence that $\overline{BD} \cong \overline{BD}$. Since all right angles are congruent, $\angle BDA \cong \angle BDC$. Therefore, $\triangle BDA \cong \triangle BDC$ by the SAS Congruence Postulate. In a right triangle, the two acute angles are complementary, so $m\angle ABD + m\angle BAD = 90°$. Since corresponding parts of congruent triangles are congruent, we know $\angle BAD \cong \angle BCD$, or $m\angle BAD = m\angle BCD$, so by substitution, $m\angle ABD + m\angle BCD = 90°$. That is, $\angle ABD$ and $\angle BCD$ are complementary.

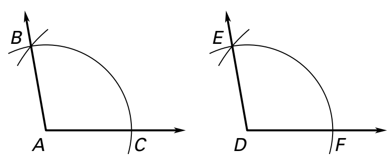
19. It is given that $\overline{AB} \cong \overline{AC}$ and $\overline{BD} \cong \overline{CD}$. By the Reflexive Property of Congruence, $\overline{AD} \cong \overline{AD}$. So, $\triangle ACD \cong \triangle ABD$ by the SSS Congruence Postulate. Then, since corresponding parts of congruent triangles are congruent, $\angle CAD \cong \angle BAD$. Then by definition, $\overrightarrow{AD}$ bisects $\angle C$.

20.



$\overline{PR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{QS}$ (Given); $\overline{PS} \cong \overline{PS}$ (Reflexive Prop. of Cong.) → $\triangle PRS \cong \triangle PQS$ (SSS Congruence Post.) → $\angle RPS \cong \angle QPS$ (Corresp. parts of ≅ ⚠ are ≅.) → $\overrightarrow{PS}$ bisects $\angle RPQ$ (Def. of angle bisector)

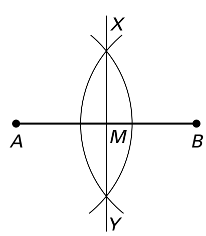
21.



$\overline{AB} \cong \overline{AC} \cong \overline{DE} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$ (Given) → $\triangle ABC \cong \triangle DEF$ (SSS Congruence Post.) → $\angle A \cong \angle D$ (Corresp. parts of ≅ ⚠ are ≅.)

22. A **23.** D

24.



Sample answer: It is given that $\overline{AX}$, $\overline{AY}$, $\overline{BX}$, and $\overline{BY}$ are congruent. Also, $\overline{AB} \cong \overline{AB}$ by the Reflexive Property of Congruence, so $\triangle AXB \cong \triangle BYA$ by the SSS Congruence Postulate. Then $\angle XAB \cong \angle YBA$ and $\overline{AX} \cong \overline{BY}$ because they are corresponding parts of congruent triangles. Also $\angle XMA \cong \angle YMB$ by the Vertical Angles Theorem, so $\triangle AXM \cong \triangle BYM$ by the AAS Congruence Theorem and corresponding parts $\overline{AM}$ and $\overline{BM}$ are congruent. Then M is the midpoint of $\overline{AB}$ by definition.

4.5 Mixed Review (p. 235)

25. $P = 2(55) + 2(30) = 170$ m

$A = (55)(30) = 1650$ m^2.

26. $P = 43.5 + 30.8 + 53.3 = 127.6$ m

$A = \frac{1}{2}(30.8)(43.5) = 669.9$ m^2

27. $P = 2\pi(12) = 75.36$ cm

$A = \pi(12)^2 = 452.16$ cm^2

28. $x - 2 = 10$

$x = 12$ (Addition property of equality)

29. $x + 11 = 21$

$x = 10$ (Subtraction property of equality)

30. $9x + 2 = 29$

$9x = 27$ (Subtraction property of equality)

$x = 3$ (Division property of equality)

31. $8x + 13 = 3x + 38$

$5x = 25$ (Subtraction property of equality)

$x = 5$(Division property of equality)

32. $3(x - 1) = 16$

$3x - 3 = 16$ (Distributive property)

$3x = 19$ (Addition property of equality)

$x = \frac{19}{3}$ (Division property of equality)

33. $6(2x - 1) + 15 = 69$

$6(2x - 1) = 54$(Subtraction property of equality)

$2x - 1 = 9$(Division property of equality)

$2x = 10$(Addition property of equality)

$x = 5$(Division property of equality)

34. acute isosceles; legs: $\overline{AC}$ and $\overline{BC}$, base: $\overline{AB}$

35. right scalene; legs: $\overline{MN}$ and $\overline{MP}$, hypotenuse: $\overline{NP}$

36. acute isosceles; legs: $\overline{XZ}$ and $\overline{YZ}$, base: $\overline{XY}$

Lesson 4.6

Developing Concepts Activity 4.6 (p. 236)

1., 2. Constructions will vary

3. The base angles of an isosceles triangle are congruent.

4.6 Guided Practice (p. 239)

1. Equilateral means all sides congruent, and equiangular means all angles congruent.

2. $m\angle C = 50°$ (Base Angles Theorem), $m\angle B = 80°$ (Triangle Sum Theorem)

3. 5 cm (Converse of the Base Angles Theorem)

4. 60° (Corollary to the Base Angles Theorem)

5. Yes; the hypotenuese and one leg of one right triangle are congruent to the hypotenuse and one leg of the other.

6. No; both triangles are equiangular, but it cannot be determined whether the sides of one triangle are congruent to the sides of the other.

7. No; it cannot be shown that $\triangle ABC$ is equilateral.

4.6 Practice and Applications (pp. 239–242)

8. $x = 46, y = 88$ **9.** $x = 70, y = 70$

10. $x = 54, y = 63$

11. Yes; the triangles can be proved congruent using the SSS Congruence Postulate.

12. No; there are two pairs of corresponding congruent sides, but no angles can be shown to be congruent.

13. Yes; the triangles can be proved congruent using the ASA Congruence Postulate, the SSS Congruence Postulate, the SAS Congruence Postulate or the AAS Congruence Theorem.

14. Yes; the triangles can be proved congruent using the SAS Congruence Postulate.

15. Yes; the triangles can be proved congruent using the HL Congruence Theorem.

16. No; it cannot be shown that any of the sides of $\triangle ABC$ are congruent to any of the sides of $\triangle DEF$.

17. $x + 13 = 24$

$x = 11$

18. $2x = 12$

$x = 6$

19. $8x = 56$

$x = 7$

20. All triangles pictured are equilateral, so $x° = 60°$ and $y° = 60°$

21. $2x° + 75° = 180°$

$2x = 105$

$x = 52.5$

$y = 180 - 2(52.5) = 75$

22. $x° + 40° + 40° = 180°$

$x = 100$

$y = 140$ (Alternate Interior Angles Theorem.)

23. $x = 30$

$y = 180 - 30 - 30 = 120$

24. $2x° + 140° = 180°$

$x = 20$

$20° + y° = 90°$

$y = 70$

25. $x = 60$

$y = 90 - 60 = 30$

26. GIVEN: $\angle B \cong \angle C$; PROVE: $\overline{AB} \cong \overline{AC}$

Draw $\overrightarrow{AD}$, the bisector of $\angle BAC$. By construction, $\angle BAD \cong \angle CAD$. It is given that $\angle B \cong \angle C$, and $\overline{AD} \cong \overline{AD}$ by the Reflexive Property of Congruence. Then $\triangle ABD \cong \triangle ACD$ by AAS Congruence Theorem and $\overline{AB} \cong \overline{AC}$ because corresponding parts of congruent triangles are congruent.

27. GIVEN: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$; PROVE: $\angle A \cong \angle B \cong \angle C$

Since $\overline{AB} \cong \overline{AC}$, $\angle B \cong \angle C$ by the Base Angles Theorem. Since $\overline{AB} \cong \overline{BC}$, $\angle A \cong \angle C$ by the Base Angles Theorem. Then by the Transitive Property of Congruence, $\angle A \cong \angle B \cong \angle C$ and $\triangle ABC$ is equiangular.

28. GIVEN: $\angle A \cong \angle B \cong \angle C$; PROVE: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$

Since $\angle B \cong \angle C$, $\overline{AB} \cong \overline{AC}$ by the Converse of the Base Angles Theorem. Since $\angle A \cong \angle C$, then $\overline{BC} \cong \overline{AB}$ by the Converse of the Base Angles Theorem. Then by the Transitive Property of Congruence, $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ and $\triangle ABC$ is equilateral.

29. $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles, so $\overline{AB} \cong \overline{CB}$ and $\triangle ABC$ is isosceles by definition.

30. $\angle BAE \cong \angle BCE$ by the Base Angles Theorem.

31. Since $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles, $\overline{AB} \cong \overline{BC}$ and $\angle ABD \cong \angle CBD$. By the Base Angles Theorem, $\angle BAE \cong \angle BCE$. Then, $\triangle ABE \cong \triangle CBE$ by the AAS Congruence Theorem. Moreover, by the Linear Pair Postulate, $m\angle AEB + m\angle CEB = 180°$. But $\angle AEB$ and $\angle CEB$ are corresponding parts of congruent triangles, so they are congruent, that is $m\angle AEB = m\angle CEB$. Then, by the Substitution Property, $2m\angle AEB = 180°$ and $m\angle AEB = 90°$. So, $\angle AEB$ and $\angle CEB$ are both right angles, and $\triangle AEB$ and $\triangle CEB$ are congruent right triangles.

32. $m\angle BAE = 30°$

33.

Statements	**Reasons**
1. D is the midpoint of $\overline{CE}$, $\angle BCD$ and $\angle FED$ are right angles.	**1.** Given
2. $\angle BCD \cong \angle FED$	**2.** Right Angle Congruence Theorem
3. $\overline{CD} \cong \overline{ED}$	**3.** Definition of midpoint
4. $\overline{BD} \cong \overline{FD}$	**4.** Given
5. $\triangle BCD \cong \triangle FED$	**5.** HL Congruence Theorem

34. It is given that $\overline{VW} \parallel \overline{ZY}$ and that $\overline{VZ}$ and $\overline{WY}$ are both perpendicular to $\overline{VW}$. By the Perpendicular Transversal Theorem, it follows that $\overline{VZ}$ and $\overline{WY}$ are both perpendicular to $\overline{ZY}$. $\angle UZV$ and $\angle XYW$ are both right angles and $\triangle UZV$ and $\triangle XYW$ are both right triangles. Since $\overline{UV} \cong \overline{XW}$ and $\overline{UZ} \cong \overline{XY}$, $\triangle UZV \cong \triangle XYW$ by the HL Congruence Theorem. Therefore, $\angle U \cong \angle X$ because corresponding parts of congruent triangles are congruent.

35. Each of the triangles is isosceles and every pair of adjacent triangles have a common side, so the legs of all the triangles are congruent by the Transitive Property of Congruence. The common vertex angles are congruent, so any two of the triangles are congruent by the SAS Congruence Postulate.

36. Let $x°$ be the measure of a base angle.

$2x° + 30° = 180°$, so $x = 75$

The measures of the base angles are 75°.

37. equilateral

38. red, yellow, blue; red-orange, yellow-green, blue-purple; purple, orange, green; red-purple, yellow-orange, blue-green.

39. It is given that $\angle CDB \cong \angle ADB$ and that $\overline{DB} \perp \overline{AC}$. Since perpendicular lines form right angles, $\angle ABD$ and $\angle CBD$ are right angles. By the Right Angle Congruence Theorem, $\angle ABD \cong \angle CBD$. By the Reflexive Property of Congruence, $\overline{DB} \cong \overline{DB}$, so $\triangle ABD \cong \triangle CBD$ by the ASA Congruence Postulate.

40. Since $\triangle ABD \cong \triangle CBD$, their corresponding parts are congruent. Therefore, $\overline{AD} \cong \overline{CD}$, so $\triangle ACD$ is isosceles.

41. No; the measure of $\angle ADB$ will decrease, as will the measure of $\angle CDB$ and the amount of reflection will remain the same.

42. C **43.** C

44. Each of the six triangles in the first figure is equilateral. By connecting every other vertex of the hexagon, three isosceles triangles are formed. For each triangle, the measure of the angle with vertex at the center is $2 \cdot 60° = 120°$, so each of the congruent base angles has measure 30°. Then the measure of each angle in the third figure is $2 \cdot 30° = 60°$ and the triangle is equiangular and, therefore, equilateral.

4.6 Mixed Review (p. 242)

45. $AB = \sqrt{(0-5)^2 + (-4-8)^2} = \sqrt{25 + 144} = 13$

$AC = \sqrt{(0-(-12))^2 + (-4-1)^2} = \sqrt{144 + 25} = 13$

The two segments are congruent.

46.
$$\begin{aligned} AB &= \sqrt{(0-(-6))^2 + (0-(-10))^2} \\ &= \sqrt{36 + 100} \\ &= \sqrt{136} \\ &= 2\sqrt{34} \\ AC &= \sqrt{(0-6)^2 + (0-10)^2} \\ &= \sqrt{36+100} \\ &= \sqrt{136} \\ &= 2\sqrt{34} \end{aligned}$$

The two segments are congruent.

47.
$$\begin{aligned} AB &= \sqrt{(1-(-8))^2 + ((-1)-7)^2} \\ &= \sqrt{81 + 64} \\ &= \sqrt{145} \end{aligned}$$

$AC = \sqrt{(1-8)^2 + ((-1)-7)^2} = \sqrt{49 + 64} = \sqrt{113}$

The two segments are not congruent.

48. $\left(\frac{4+10}{2}, \frac{9+7}{2}\right) = (7, 8)$

49. $\left(\frac{0+8}{2}, \frac{11-3}{2}\right) = (4, 4)$

50. $\left(\frac{1-5}{2}, \frac{7-5}{2}\right) = (-2, 1)$

51. $\left(\frac{-2+5}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

52. $\left(\frac{0+2}{2}, \frac{-13-1}{2}\right) = (1, -7)$

53. $\left(\frac{-3+0}{2}, \frac{-5-20}{2}\right) = \left(-\frac{3}{2}, -\frac{25}{2}\right)$

54.
$$\begin{aligned} y - 1 &= \tfrac{1}{3}(x-1) \\ y - 1 &= \tfrac{1}{3}x - \tfrac{1}{3} \\ y &= \tfrac{1}{3}x + \tfrac{2}{3} \end{aligned}$$

55.
$$\begin{aligned} y &= -(x + 0) \\ y &= -x \end{aligned}$$

56.
$$\begin{aligned} y + 12 &= \tfrac{9}{10}(x-5) \\ y + 12 &= \tfrac{9}{10}x - \tfrac{9}{2} \\ y &= \tfrac{9}{10}x - \tfrac{33}{2} \end{aligned}$$

57.
$$\begin{aligned} y - 4 &= -\tfrac{3}{2}(x+3) \\ y - 4 &= -\tfrac{3}{2}x - \tfrac{9}{2} \\ y &= -\tfrac{3}{2}x - \tfrac{1}{2} \end{aligned}$$

Lesson 4.7

Developing Concepts Activity (p. 243)

1., 2. Drawings will vary.

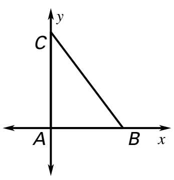
3. *Sample answer:* with one vertex at the origin and the hypotenuse along the positive *x*-axis.

4.7 Guided Practice (p. 246)

1. Coordinate proofs involve placing geometric figures in a coordinate plane. All the methods of proof involve providing a logical argument that shows that the given information leads to that which is to be proved.

2. The first; each vertex has at least one coordinate that is 0.

Sample figure:



3. Using the origin as the vertex with the right angle: (4, 0) and (−4, 0); using (0, 7) as the vertex with the right angle: (4, 7) and (−4, 7).

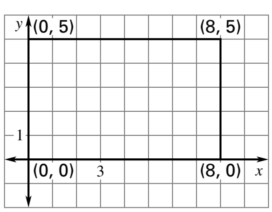
4. Use the Distance Formula to show $\overline{OG} \cong \overline{HG}$. Then show that since $\overline{GJ}$ bisects $\angle OGH$, $\angle OGJ \cong \angle HGJ$ and that $\overline{GJ} \cong \overline{GJ}$ by the Reflexive Property of Congruence. Then $\triangle GJO \cong \triangle GJH$ by the SAS congruence postulate.

5. Use the Distance Formula to show that $\overline{AB} \cong \overline{AC}$.

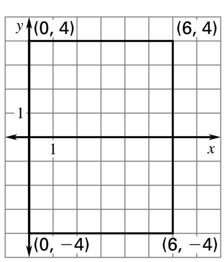
4.7 Practice and Applications (pp. 247–249)

6.–11. Good placements should include vertices for which at least one coordinate is 0.

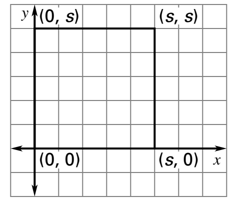
6. *Sample figure:*



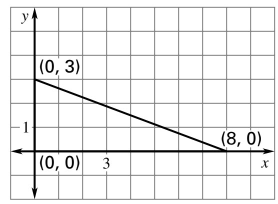
7. *Sample figure:*



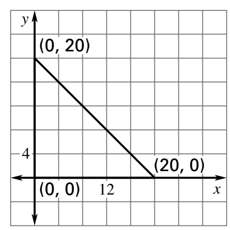
8. *Sample figure:*



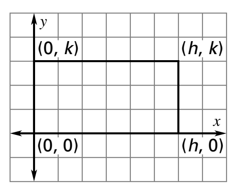
9. *Sample figure:*



10. *Sample figure:*



11. *Sample figure:*



12. Units are ten so $B(0, 50)$ and $C(30, 0)$.

13. $BC = \sqrt{(30 - 0)^2 + (0 - 50)^2}$
$= \sqrt{900 + 2500}$
≈ 58.31

14. $d = \sqrt{(7 - 0)^2 + (9 - 0)^2}$
$= \sqrt{49 + 81}$
$= \sqrt{130}$

15. $d = \sqrt{(5 - 0)^2 + (4 - 0)^2}$
$= \sqrt{25 + 16}$
$= \sqrt{41}$

16. $d = \sqrt{(3 - 0)^2 + (3 - 0)^2}$
$= \sqrt{9 + 9}$
$= \sqrt{18}$
$= 3\sqrt{2}$

17. $d = \sqrt{(3-0)^2 + (3-0)^2}$
$= \sqrt{9+9}$
$= \sqrt{18}$
$= 3\sqrt{2}$

18. $H = \left(\frac{80-0}{2}, \frac{80-0}{2}\right)$
$= (40, 40)$

19. $H = \left(\frac{90-0}{2}, \frac{70-0}{2}\right)$
$= (45, 35)$

20. Use the Distance Formula to show that $\overline{OR} \cong \overline{OT}$. Then show that since $\angle R \cong \angle T$ by the Base Angles Theorem and $\angle OSR \cong \angle OST$ (Right Angle Congruence Theorem), $\triangle OSR \cong \triangle OST$ (AAS Congruence Theorem). Then $\angle TOS \cong \angle ROS$ and $\overrightarrow{OS}$ bisects $\angle TOR$.

21. Show that, since $\overline{HJ}$ and $\overline{OF}$ both have a slope of 0, they are parallel, so that alternate interior angles $\angle H$ and $\angle F$ are congruent. $\overline{HG} \cong \overline{FG}$ by the definition of midpoint. Then use the Distance Formula to show that $\overline{HJ} \cong \overline{OF}$ so that $\triangle GHJ \cong \triangle GFO$ by the SAS Congruence Postulate.

22. $M(0, k), N(h, k), P(h, 0); MP = \sqrt{h^2 + k^2}$

23. $F(2h, 0), E(2h, h), OE = \sqrt{(2h)^2 + h^2} = \sqrt{5h^2} = h\sqrt{5}$

24. $N(h, k); ON = \sqrt{h^2+k^2}, MN = \sqrt{(2h-h)^2+k^2} = \sqrt{h^2+k^2}$

25. $O(0, 0), R(k, k), S(k, 2k), T(2k, 2k), U(k, 0)$:
$OT = \sqrt{(2k)^2 + (2k)^2}$
$\sqrt{8k^2} = 2k\sqrt{2}$

26. Since $OP = 2h$ and $OM = 2h$, $\overline{OP} \cong \overline{OM}$. According to the Midpoint Formula, N is the midpoint of $\overline{PM}$, so $\overline{PN} \cong \overline{MN}$. By the Reflexive Property of Congruence, $\overline{ON} \cong \overline{ON}$. Then $\triangle NPO \cong \triangle NMO$ by the SSS Congruence Postulate.

27. Since $OC = \sqrt{h^2 + k^2}$ and $EC = \sqrt{h^2 + k^2}$, $\overline{OC} \cong \overline{EC}$ and since $BC = k$ and $DC = k$, $\overline{BC} \cong \overline{DC}$. Then, since vertical angles $\angle OCB$ and $\angle ECD$ are congruent, $\triangle OBC \cong \triangle EDC$ by the SAS Congruence Postulate.

28. $\triangle OBC$ is not isosceles. By the Distance Formula, $OB = \sqrt{12^2 + 48^2} = \sqrt{2448} = 12\sqrt{17} \approx 49.5$ and $BC = \sqrt{(12-18)^2 + 48^2} = 6\sqrt{65} \approx 48.4$. $\overline{OB}$ is longer. The plant stand is leaning to the right, which is causing the instability.

29. isosceles; no; no

30. AC and AB are equal; they remain equal; they remain equal.

31. The triangle in Exercise 5 has vertices which can be used to describe $\triangle ABC$. Point A is on the y-axis and points B and C are on the x-axis, equidistant from the origin. The proof shows that any such triangle is isosceles.

32. A **33.** A

34. The Distance Formula verifies that $\overline{AD} \cong \overline{AE}$ and since H is the midpoint of $\overline{AD}$ and G is the midpoint of $\overline{AE}$, $AH = \frac{1}{2}AD$ and $AG = \frac{1}{2}AE$ implying that $AH = AG$ and $\overline{AH} \cong \overline{AG}$. $\angle A$ is common to both $\triangle AEH$ and $\triangle ADG$ and $\angle EAH \cong \angle DAG$ by the Reflexive Property of Congruence, so $\triangle AEH \cong \triangle ADG$ by the SAS Congruence Postulate. This makes $\overline{DG} \cong \overline{EH}$ because they are corresponding parts of the two congruent triangles.

4.7 Mixed Review (p. 250)

35. $15x° = (4x + 55)°$
$11x = 55$
$x = 5$

36. $m\angle CGF = (4x + 55)° + 15x°$
$= 19x° + 55°$
$= 19(5°) + 55°$
$= 150°$

37. true **38.** false **39.** true **40.** true

41. If two triangles are congruent, then the corresponding angles of the triangles are congruent; true.

42. If the corresponding angles of two triangles are congruent, then the triangles are congruent; false.

43. If two triangles are not congruent, then the corresponding angles of the two triangles are not congruent; false.

Quiz 3 (p. 250)

1.

Statements	Reasons
1. $\overline{DF} \cong \overline{DG}, \overline{ED} \cong \overline{HD}$	**1.** Given
2. $\angle EDF \cong \angle HDG$	**2.** Vertical Angles Theorem
3. $\triangle EDF \cong \triangle HDG$	**3.** SAS Congruence Postulate
4. $\angle EFD \cong \angle HGD$	**4.** Corresp. parts of $\cong$ △s are $\cong$.

2.

Statements	Reasons
1. $\overline{ST} \cong \overline{UT} \cong \overline{VU}$, $\overline{SU} \parallel \overline{TV}$	**1.** Given
2. $\angle S \cong \angle SUT, \angle UTV \cong \angle V$	**2.** Base Angles Theorem
3. $\angle SUT \cong \angle UTV$	**3.** Alternate Interior Angles Theorem
4. $\angle S \cong \angle SUT \cong \angle UTV \cong \angle V$	**4.** Transitive Property of Congruence
5. $\triangle STU \cong \triangle TUV$	**5.** AAS Congruence Theorem

3. Use the Distance Formula to show that OP, PM, NM, and ON are all equal, so that $\overline{OP} \cong \overline{PM} \cong \overline{ON} \cong \overline{NM}$. Since $\overline{OM} \cong \overline{OM}$ by the Reflexive Property of Congruence, $\triangle OPM \cong \triangle ONM$ by the SSS Congruence Postulate and both triangles are isosceles by definition.

Chapter 4 Review *(pp. 252–254)*

1. isosceles right **2.** obtuse scalene

3. obtuse isosceles **4.** equiangular or acute; equilateral or isosceles

5. $90° - 47° = 53°$

6. $m\angle M + m\angle N + m\angle P = 180°$

$m\angle N = 5(m\angle P), m\angle M = 24°$

$24° + 5(m\angle P) + m\angle P = 180°$

$6(m\angle P) = 156°$

$m\angle P = 26°$

$m\angle N = 5(26°) = 130°$

7. $\angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z,$
$\overline{AB} \cong \overline{XY}, \overline{BC} \cong \overline{YZ}, \overline{AC} \cong \overline{XZ}$

8. $m\angle Y = 180° - 48° - 37° = 95°$

9. Yes; ASA Congruence Postulate; two pairs of corresponding angles are congruent and the corresponding included sides are congruent.

10. No; the triangles cannot be proved congruent with the given information.

11. Yes; AAS Congruence Theorem; because $\overline{HF} \parallel \overline{JE}$, $\angle HFG \cong \angle E$ (Corresponding Angles Postulate), so two pairs of corresponding angles are congruent and two nonincluded sides are congruent.

12. Yes; by the ASA Congruence Postulate or the AAS Congruence Theorem

13. $\overline{PQ}$

14. $2x + 3 = 17$

$2x = 14$

$x = 7$

15. $x + x + 72 = 180$

$2x = 108$

$x = 54$

16. $4x - 2 = 3x + 3$

$x = 5$

17. $35 + 35 + x = 180$

$70 + x = 180$

$x = 110$

18. Since both $\overline{AB}$ and $\overline{OC}$ have slope 0, $\overline{AB} \parallel \overline{OC}$. Then $\angle OCA \cong \angle BAC$ by the Alternate Interior Angles Theorem. Since $OC = h$ and $AB = h$, $\overline{OC} \cong \overline{AB}$ by the definition of congruence. Also, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. Therefore $\triangle OAC \cong \triangle BCA$ by the SAS Congruence Postulate.

Chapter 4 Test *(p. 255)*

1. $\triangle QPS, \triangle QSR$ **2.** $\triangle QPS$ **3.** $\triangle QPR$ **4.** $\triangle QPS$

5. $\triangle QSR$ **6.** $\triangle QPR$

7. $m\angle A + m\angle B + m\angle C = 180°$

Let $x° = m\angle C$

$116° + 3x° + x° = 180°$

$4x = 64$

$x = 16$

Therefore $m\angle C = 16°$ and $m\angle B = 3(16°) = 48°$

8. Yes; AAS Congruence Theorem; two pairs of corresponding angles are congruent and two corresponding nonincluded sides are congruent.

9. Yes; SAS Congruence Postulate; $\overline{HJ} \cong \overline{KG}$ (given), and $\overline{GJ} \cong \overline{GJ}$ (Reflexive Property of congruence). Since $\overline{HJ} \parallel \overline{GK}$, $\angle HJG \cong \angle JGK$ (Alternate Interior Angles Theorem), so two pairs of corresponding sides are congruent and two corresponding included angles are congruent.

10. Yes; ASA Congruence Postulate; since $\angle LMP \cong \angle NPM$ and $\angle NMP \cong \angle LPM$ (given), and $\overline{MP} \cong \overline{MP}$ (Reflexive Property of Congruence), two pairs of corresponding angles are congruent and two corresponding included angles are congruent.

11. No; the triangles cannot be proved congruent from the given information.

12. Yes; HL Congruence Theorem; since $\overline{WX} \cong \overline{XY}$ (given) and $\overline{XZ} \cong \overline{XZ}$ (Reflexive Property of Congruence), two corresponding hypotenuses and two corresponding legs of two right triangles are congruent.

13. Yes; HL Congruence Theorem; since it is given that $\triangle GMH$ and $\triangle LMK$ are right triangles and $\overline{HG} \cong \overline{KJ}$ and $\overline{KM} \cong \overline{MH}$, two corresponding hypotenuses and two corresponding legs of two right triangles are congruent.

14. $70° + x° + x° = 180°$

$2x = 110$

$x = 55$

15. $3x - 4 = 2x + 1$

$x = 5$

16. $x° + 2(65°) = 180°$

$x + 130 = 180$

$x = 50$

17.

Statements	Reasons
1. $\overline{BD} \cong \overline{EC}, \overline{AC} \cong \overline{AD}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Base Angles Theorem
3. $\triangle ABD \cong \triangle AEC$	3. SAS Congruence Postulate
4. $\overline{AB} \cong \overline{AE}$	4. Corresp. parts of $\cong$ ⚠ are $\cong$.

18.

Statements	Reasons
1. $\overline{XY} \parallel \overline{WZ}, \overline{XZ} \parallel \overline{WY}$	1. Given
2. $\angle XYZ \cong \angle WZY$, $\angle XZY \cong \angle WYZ$	2. Alternate Interior Angles Theorem
3. $\overline{ZY} \cong \overline{ZY}$	3. Reflexive Property of Congruence
4. $\triangle XYZ \cong \triangle WZY$	4. SAS Congruence Postulate
5. $\angle X \cong \angle W$	5. Corresp. parts of $\cong$ ⚠ are $\cong$.

19. Using the Pythagorean Theorem, $\sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$.

20. $M = \left(\frac{s+0}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$

Chapter 4 Standardized Test *(pp. 256–257)*

1. $m\angle J = 180° - 42° - 42° = 96°$

C

2. $m\angle BCD = 90° + 35° = 125°$

D

3. B **4.** B **5.** B **6.** C

7. B

$2x + 5 = 3x + 2$

$-x = -3$

$x = 3$

8. D **9.** A

10. Because $\overline{RU} \perp \overline{QS}$, $\angle RUS$ and $\angle RUQ$ are right angles and $\triangle RUS$ and $\triangle RUQ$ are right triangles. It is given that $\overline{RQ} \cong \overline{RS}$. $\overline{RU} \cong \overline{RU}$ by the Reflexive Property of Congruence. $\triangle RUS \cong \triangle RUQ$ by the HL Congruence Theorem.

11. It is given that $\overline{PT} \cong \overline{TS}$. Because $\overline{QT} \perp \overline{PS}$, $\angle QTP$ and $\angle QTS$ are right angles. $\angle PTQ \cong \angle STQ$ because all right angles are congruent. $\overline{QT} \cong \overline{QT}$ by the Reflexive Property of Congruence, so $\triangle QTP \cong \triangle QTS$ by the SAS Congruence Postulate.

12. In Ex.10, it was proved that $\triangle RUQ \cong \triangle RUS$. You can use the HL Congruence Theorem to prove that $\triangle QTP \cong \triangle RUS$. Then, by the Transitive Property of Congruence, $\triangle QTP \cong \triangle RUQ$. In Ex.11, it was proved that $\triangle QTP \cong \triangle QTS$. By the Transitive Property of Congruence, $\triangle QTS \cong \triangle RUQ$ and $\triangle QTS \cong \triangle RUS$. Because corresponding parts of congruent triangles are congruent, $\angle QPT \cong \angle RQU$ and $\angle QST \cong \angle RSU$. It is given that $\overline{PQ} \cong \overline{QR}$. So, by the AAS Congruence Theorem, $\triangle PQS \cong \triangle QRS$.

13. Both triangles are equilateral equiangular triangles. In Ex. 11, it was proved that $\triangle QTP \cong \triangle QTS$. Because corresponding parts of congruent triangles are congruent, $\overline{QP} \cong \overline{QS}$. It is given that $\overline{QP} \cong \overline{RQ} \cong \overline{RS}$, so by the Transitive Property of Congruence, $\overline{QS} \cong \overline{RQ}$ and $\overline{QS} \cong \overline{RS}$. Thus, all three sides of $\triangle QRS$ are congruent, so $\triangle RQS$ is equilateral. It is also equiangular. In Ex. 12, it was proved that $\triangle PQS \cong \triangle QRS$. Therefore, $\triangle PQS$ is also equilateral and equiangular.

14. Check drawing.

15. Check drawing.

16. Yes; *Sample answer:* By the Distance Formula, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. Since $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence, the triangles are congruent by the SSS Congruence Postulate.

17. isosceles

18. isosceles

19. Yes; the triangles will always be isosceles because either diagonal will give you a pair of triangles each with two congruent sides.

Algebra Review (pp. 258–259)

1. $d = \sqrt{(3-0)^2 + (6-(-2))^2}$
$= \sqrt{9 + 64}$
$= \sqrt{73}$

2. $d = \sqrt{(5-(-6))^2 + (-2-5)^2}$
$= \sqrt{121 + 49}$
$= \sqrt{170}$

3. $d = \sqrt{(3-1)^2 + (4-4)^2}$
$= \sqrt{16}$
$= 4$

4. $d = \sqrt{(-6-(-3))^2 + (-6-(-2))^2}$
$= \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$

5. $d = \sqrt{(8-(-3))^2 + (-2-(-6))^2}$
$= \sqrt{121 + 16}$
$= \sqrt{137}$

6. $d = \sqrt{(-8-(-1))^2 + (5-1)^2}$
$= \sqrt{49 + 16}$
$= \sqrt{65}$

7. $6x + 11y - 4x + y = 6x - 4x + 11y + y = 2x + 12y$

8. $-5m + 3q + 4m - q = -5m + 4m + 3q - q$
$= -m + 2q$

9. $-3p - 4t - 5t - 2p = -3p - 2p - 4t - 5t$
$= -5p - 9t$

10. $9x - 22y + 18x - 3y = 9x + 18x - 22y - 3y$
$= 27x - 25y$

11. $3x^2y - 5xy^2 - 6x^2y = 3x^2y + 6x^2y - 5xy^2$
$= 9x^2y - 5xy^2$

12. $5x^2 + 2xy - 7x^2 + xy = 5x^2 - 7x^2 + 2xy + xy$
$= -2x^2 + 3xy$

13. $3x + 5 = 2x + 11$
$x + 5 = 11$
$x = 6$

14. $-14 + 3a = 10 - a$
$-14 + 4a = 10$
$4a = 24$
$a = 6$

15. $8m + 1 = 7m - 9$
$m + 1 = -9$
$m = -10$

16. $y - 18 = 6y + 7$
$-18 = 5y + 7$
$-25 = 5y$
$-5 = y$

17. $2s + 1 = 7s + 1$
$1 = 5s + 1$
$0 = 5s$
$0 = s$

18. $3a - 12 = -6a - 12$
$9a - 12 = -12$
$9a = 0$
$a = 0$

19. $-2t + 10 = -t$
$-t + 10 = 0$
$-t = -10$
$t = 10$

20. $11q - 6 = 3q + 8q$
$11q - 6 = 11q$
$-6 \neq 0$
No solution

21. $-7x + 7 = 2x - 11$
$7 = 9x - 11$
$18 = 9x$
$2 = x$

22. $-x + 2 > 7$
$-x > 5$
$x < -5$

23. $x - 18 < 10$
$c < 28$

24. $-5 + m < 21$
$m < 26$

25. $x - 5 < 4$
$x < 9$

26. $z + 6 > -2$
$z > -8$

27. $-3x + 4 \leq -5$
$-3x \leq -9$
$x \geq 3$

28. $5 - 2x < -3x - 6$
$5 + x < -6$
$x < -11$

29. $-m + 3 \geq -4m + 6$
$3m + 3 \geq 6$
$3m \geq 3$
$m \geq 1$

30. $2b + 4 > -3b + 7$
$5b + 4 > 7$
$5b > 3$
$b > \frac{3}{5}$

31. $13 - 6x > 10 + 4x$
$13 - 10x > 10$
$-10x > -3$
$x < \frac{3}{10}$

32. $4z + 8 \leq 12$
$4z \leq 4$
$z \leq 1$

33. $14 - 5t \geq 28$
$-5t \geq 14$
$t \leq -\frac{14}{5}$

34. $6 - 3r < 24$
$-3r < 18$
$r > -6$

35. $16 - 2x \leq 28$
$-12x \leq 12$
$x \geq -1$

36. $-3x + 11 \geq 32$
$-3x \geq 21$
$x \leq -7$

37. $|x + 5| = 12$
$x + 5 = 12$ or $x + 5 = -12$
$x = 7$ or $x = -17$

38. $|x - 2| = 10$
$x - 2 = 10$ or $x - 2 = -10$
$x = 12$ or $x = -8$

39. $|5 - x| = 3$
$5 - x = 3$ or $5 - x = -3$
$-x = -2$ or $-x = -8$
$x = 2$ or $x = 8$

40. $|1 - x| = 6$
$1 - x = 6$ or $1 - x = -6$
$-x = 5$ or $-x = -7$
$x = -5$ or $x = 7$

41. $|x + 3| = 17$
$x + 3 = 17$ or $x + 3 = -17$
$x = 14$ or $x = -20$

42. $|-5x + 2| = 7$
$-5x + 2 = 7$ or $-5x + 2 = -7$
$-5x = 5$ or $-5x = -9$
$x = -1$ or $x = \frac{9}{5}$

43. $|2x - 3| = 11$
$2x - 3 = 11$ or $2x - 3 = -11$
$2x = 14$ or $2x = -8$
$x = 7$ or $x = -4$

44. $|7x + 8| = 20$
$7x + 8 = 20$ or $7x + 8 = -20$
$7x = 12$ or $7x = -28$
$x = \frac{12}{7}$ or $x = -4$

45. $|-4x + 5| = 13$
$-4x + 5 = 13$ or $-4x + 5 = -13$
$-4x = 8$ or $-4x = -18$
$x = -2$ or $x = \frac{9}{2}$

46. $|3x + 8| = 4$
$3x + 8 = 4$ or $3x + 8 = -4$
$3x = -4$ or $3x = -12$
$x = -\frac{4}{3}$ or $x = -4$

47. $|x + 13| \geq 23$

$x + 13 \geq 23$ or $x + 13 \leq -23$

$x \geq 10$ or $x \leq -36$

48. $|x - 6| > 8$

$x - 6 > 8$ or $x - 6 < -8$

$x > 14$ or $x < -2$

49. $|x - 2| \leq 8$

$-8 \leq x - 2 \leq 8$

$-6 \leq x \leq 10$

50. $|15 - x| \geq 7$

$15 - x \geq 7$ or $15 - x \leq -7$

$-x \geq -8$ or $-x \leq -22$

$x \leq 8$ or $x \geq 22$

51. $|16 - x| < 4$

$-4 < 16 - x < 4$

$-20 < -x < -12$

$20 > x > 12$

$12 < x < 20$

52. $|6x - 4| < 8$

$-8 < 6x - 4 < 8$

$-4 < 6x < 12$

$-\frac{2}{3} < x < 2$

53. $|-2x + 4| \leq 10$

$-10 \leq -2x + 4 \leq 10$

$-14 \leq -2x \leq 6$

$7 \geq x \geq -3$

$-3 \leq x \leq 7$

54. $|9x - 6| \leq 21$

$-21 \leq 9x - 6 \leq 21$

$-15 \leq 9x \leq 27$

$-\frac{5}{3} \leq x \leq 3$

55. $|11x - 11| \geq 33$

$11x - 11 \geq 33$ or $11x - 11 \leq -33$

$11x \geq 44$ or $11x \leq -22$

$x \geq 4$ or $x \leq -2$

56. $|2x + 3| > 13$

$2x + 3 > 13$ or $2x + 3 < -13$

$2x > 10$ or $2x < -16$

$x > 5$ or $x < -8$

57. $|10x + 20| < 40$

$-40 < 10x + 20 < 40$

$-60 < 10x < 20$

$-6 < x < 2$

58. $|4x - 6| > 14$

$4x - 6 > 14$ or $4x - 6 < -14$

$4x > 20$ or $4x < -8$

$x > 5$ or $x < -2$

59. $|x + 2| \geq 4$

$x + 2 \geq 4$ or $x + 2 \leq -4$

$x \geq 2$ or $x \leq -6$

60. $|5x - 9| < 14$

$-14 < 5x - 9 < 14$

$-5 < 5x < 23$

$-1 < x < \frac{23}{5}$

61. $|11x + 1| > 21$

$11x + 1 > 21$ or $11x + 1 < -21$

$11x > 20$ or $11x < -22$

$x > \frac{20}{11}$ or $x < -2$

62. $|-7x - 2| \leq -21$

No solution

63. $|3x - 2| > 10$

$3x - 2 > 10$ or $3x - 2 < -10$

$3x > 12$ or $3x < -8$

$x > 4$ or $x < -\frac{8}{3}$

64. $|12x + 16| \leq 20$

$-20 \leq 12x + 16 \leq 20$

$-36 \leq 12x \leq 4$

$-3 \leq x \leq \frac{1}{3}$

65. $|5x + 8| \geq -32$

$5x + 8 \geq -32$ or $5x + 8 \leq 32$

$5x \geq -40$ or $5x \leq 24$

$x \leq -8$ or $x \leq \frac{24}{5}$

All real numbers

66. $7 + |x + 1| \leq 8$

$|x + 1| \leq 1$

$-1 \leq x + 1 \leq 1$

$-2 \leq x \leq 0$