CHAPTER 11

Think & Discuss *(p. 659)*

1. 6

2. The base angles of an equilateral triangle measure 60° each. $2(60°) = 120°$, $120° \times 6 = 720°$

Skill Review (p. 660)

1. $A = \frac{1}{2}bh$

$= \frac{1}{2}(12 \text{ in.})(8 \text{ in.})$

$= 48 \text{ in.}^2$

2. $m\angle A + m\angle B + m\angle C = 180°$

$57° + m\angle B + 79° = 180°$

$m\angle B + 136° = 180°$

$m\angle B = 44°$

Exterior angle to A is $180° - 57° = 123°$. Exterior angle to B is $180° - 44° = 136°$. Exterior angle to C is $180° - 79° = 101°$.

3. a. $\frac{XY}{DE} = \frac{12}{8} = \frac{3}{2}$

b. $\frac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle XYZ} = \frac{8}{12} = \frac{2}{3}$

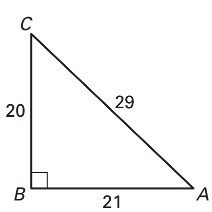
4. *Sample answer:*

$m\angle A = \sin^{-1}\left(\frac{20}{29}\right)$

$\approx 43.6°$

$m\angle C = \sin^{-1}\left(\frac{21}{29}\right)$

$\approx 46.4°$



Developing Concepts Activity (p. 661)

Polygon	*Number of sides*	*Number of triangles*	*Sum of measures of interior angles*
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$
⋮	⋮	⋮	⋮
n-gon	n	$n - 2$	$(n - 2) \cdot 180°$

The sum of the measures of the interior angles of any convex n-gon is $(n - 2)(180°)$.

Lesson 11.1

11.1 Guided Practice (p. 665)

1. Interior angles are $\angle A$, $\angle B$, $\angle D$, $\angle BCD$, and $\angle AED$. Exterior angles are $\angle AEF$, $\angle BCG$, and $\angle DCH$.

2. There are $2n$ exterior angles. No; the Polygon Exterior Angles Theorem specifies only 1 angle at each vertex.

3. The sum of the measures of the interior angles of a pentagon is $(5 - 2)(180°) = 540°$.

$540° - (105° + 115° + 120° + 105°) = x°$

$540° - 445° = x°$

$95° = x°$

4. Sum of measures of interior angles $= (6 - 2)(180°)$

$= 720° = \frac{720°}{6} = 120°$

5. Measure of an exterior angle $= \frac{360°}{n} = \frac{360°}{8} = 45°$

11.1 Practice and Applications (p. 665–668)

6. $(n - 2)(180°)$
$(10 - 2)(180°)$
$1440°$

7. $(n - 2)(180°)$
$(12 - 2)(180°)$
$1800°$

8. $(n - 2)(180°)$
$(15 - 2)(180°)$
$2340°$

9. $(n - 2)(180°)$
$(18 - 2)(180°)$
$2880°$

10. $(n - 2)(180°)$
$(20 - 2)(180°)$
$3240°$

11. $(n - 2)(180°)$
$(30 - 2)(180°)$
$5040°$

12. $(n - 2)(180°)$
$(40 - 2)(180°)$
$6840°$

13. $(n - 2)(180°)$
$(100 - 2)(180°)$
$17{,}640°$

14. $x° + 113° + 80° + 130° + 90° = (5 - 2)(180°)$

$x° + 413° = 540°$

$x° = 127°$

15.
$x° + 125° + 147° + 106° + 98° + 143° = (6 - 2)(180°)$

$x° + 619° = 720°$

$x° = 101°$

16. $x° + 102° + 146° + 120° + 124° + 170° + 158°$

$= (7 - 2)(180°)$

$x° + 820° = 900°$

$x° = 80°$

17. $x° = \frac{(n-2) \cdot 180°}{n}$

$= \frac{(5-2) \cdot 180°}{5}$

$= 108°$

18. $x° = \frac{(n-2) \cdot 180°}{n}$

$= \frac{(7-2)(180°)}{7}$

$\approx 128.57°$

19. $x° = \frac{(n-2) \cdot 180°}{n}$

$= \frac{(8-2) \cdot 180°}{8}$

$= 135°$

20. $x° + 80° + 110° + 80° = (4-2)(180°)$

$x° + 270° = 360°$

$x° = 90°$

21. $x° + 60° + 80° + 120° + 140° = (5-2)(180°)$

$x° + 400° = 540°$

$x° = 140°$

22. $144° = \frac{(n-2)(180°)}{n}$

$144° \cdot n = (n-2)(180°)$

$144° \cdot n = 180° \cdot n - 360°$

$-36° \cdot n = -360°$

$n = 10$

23. $120° = \frac{(n-2)(180°)}{n}$

$120° \cdot n = (n-2)(180°)$

$120° \cdot n = 180° \cdot n - 360°$

$-60° \cdot n = -360°$

$n = 6$

24. $140° = \frac{(n-2)(180°)}{n}$

$140° \cdot n = (n-2)(180°)$

$140° \cdot n = 180° \cdot n - 360°$

$-40° \cdot n = -360°$

$n = 9$

25. $157.5° = \frac{(n-2)(180°)}{n}$

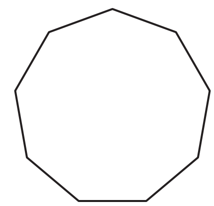
$157.5° \cdot n = (n-2)(180°)$

$157.5° \cdot n = 180° \cdot n - 360°$

$-22.5° \cdot n = -360°$

$n = 16$

26.–28.



29. ext. $\angle$ meas. $= \frac{360°}{n}$

$= \frac{360°}{12}$

$= 30°$

30. ext. $\angle$ meas. $= \frac{360°}{n}$

$= \frac{360°}{11}$

$\approx 32.73°$

31. ext. $\angle$ meas. $= \frac{360°}{n}$

$= \frac{360°}{21}$

$\approx 17.14°$

32. ext. $\angle$ meas. $= \frac{360°}{n}$

$= \frac{360°}{15}$

$= 24°$

33. $n = \frac{360°}{60°}$

$= 6$

34. $n = \frac{360°}{20°}$

$= 18$

35. $n = \frac{360°}{72°}$

$= 5$

36. $n = \frac{360°}{10°}$

$= 36$

37. $x° + 48° + 52° + 55° + 62° + 68° = 360°$

$x° + 285° = 360°$

$x° = 75°$

38. ext. $\angle$ meas. $= \frac{360°}{10}$

$= 36°$

39. The measures of the interior angles of the regular hexagons are 120°. Two of the interior angles of the red triangles form a linear pair with the interior angles of the regular polygons. Therefore, the measure of each interior angle of the triangles is 60°. The measures of the interior angles of the yellow hexagons are 120°. Two of the interior angles of the yellow pentagons form linear pairs with angles in the red triangles, so their measures are 120°. The measure of a third angle of the yellow pentagons is $540° - 120° - 120° - 90° - 90° = 120°$.

40. Regular pentagons have interior angles measuring 108°. The interior angles of the red quadrilaterals which form linear pairs with the interior angles of the pentagons measure 72°. The measure of the angle in the red quadrilateral formed by the joining of two pentagons is 144°. The measure of the 4th angle of the red quadrilateral is $360° - 72° - 72° - 144° = 72°$. For the yellow pentagons, two of the angles form linear pairs with a 72° angle, so their measures are 108°. The measure of the fifth angle is $540° - 108° - 108° - 90° - 90° = 144°$. The measure of each angle of the yellow decagon is 144°.

41. $m\angle 9$ and $m\angle 10$ are 70° because $\angle 9$ and $\angle 10$ form linear pairs with $\angle 4$ and $\angle 5$, respectively. $m\angle 3$ is 140° because $\angle 3$ forms a linear pair with $\angle 8$. $m\angle 7$ is 80° because $\angle 7$ forms a linear pair with $\angle 2$. $m\angle 1$ is 80° because the sum of the measures of the interior angles of a pentagon must equal 540°. $m\angle 6$ is 100° because $\angle 6$ forms a linear pair with $\angle 1$.

42. Any n-gon can be divided into $n - 2$ triangles. The sum of the measures of the interior angles will be equal to $(n - 2)(180°)$. The n-gons do not need to be regular or similar because the sum of the measures of angles of any triangle will always be 180°. What matters is that n is the same.

43. Given pentagon $ABCDE$, where $n = 5$ draw two diagonals from one vertex to divide the pentagon into three triangles. The sum of the measures of the angles of a triangle is 180°. Therefore, the sum of the measures of the angles of the three triangles must be $3(180°)$ or $(n - 2)(180°)$.

44. Let A be a regular n-gon and $x°$ the measure of each interior $\angle$. By the Polygon Interior Angles Thm., the sum of the measures of the interior $\angle$s of A is $(n - 2) \cdot 180°$. That is,

$$n \cdot x° = (n - 2) \cdot 180°, \text{ or } x° = \frac{(n-2) \cdot 180°}{n}.$$

45. In a convex n-gon, extend each side to make an exterior angle with the n-gon. The interior angle and its adjacent exterior angle at any vertex form a linear pair, so the sum of their measures is 180°. Since the n-gon has n sides, there will be n linear pairs composed of an interior and exterior angle, so multiply n by 180°. Then subtract the sum of the measures of the interior angles, which is $(n - 2) \cdot 180°$ to get the total measure of the exterior angles. So, the sum of the measures of the exterior angles of a convex n-gon is $180°n - (n - 2) \cdot 180° = 180°n - 180°n - (-360°) = 360°$.

46. The measure of each interior angle of a regular n-gon is

$$\frac{(n-2)(180°)}{n} = 180° - \frac{360°}{n}.$$

The measure of each exterior angle of a regular n-gon is 180° − measure of interior angle. Therefore, the measure of an exterior angle by substitution is

$$180° - \left[\frac{(n-2) \cdot 180°}{n}\right] = 180° - \left(\frac{180°n - 360°}{n}\right) = \frac{360°}{n}.$$

47. *Sample answer:* If a regular hexagon were constructed, each exterior angle would measure 60° and the sum of the measures of exterior angles would be 360°. Answers will vary.

48. If the measure of an interior angle increases, the measure of the corresp. exterior angle decreases. If the measure of an interior angle decreases, the measure of the corresp. exterior angle increases. The sum of the measures of the exterior angles is always 360°.

49. $3x° + 90° + 90° = (5 - 2)(180°)$

$$3x° + 180° = 540°$$
$$3x° = 360°$$
$$x° = 120°$$

So, $m\angle A = m\angle E = 90°$, $m\angle B = m\angle C = m\angle D = 120°$.

50. $x° + x° + 2x° + 160° + 160° + 150° + 150° = (7 - 2)(180°)$

$$4x° + 620° = 900°$$
$$4x° = 280°$$
$$x° = 70°$$

So, $m\angle P = m\angle V = 70°$ and $m\angle S = 2x° = 140°$

51. $\frac{(n-2)(180°)}{n} = 150°$

$$180°n - 360° = 150°n$$
$$30°n = 360°$$
$$n = 12$$

Yes. The polygon would be a dodecagon.

52. $\frac{(n-2)(180°)}{n} = 90°$

$$180°n - 360° = 90°n$$
$$90°n = 360°$$
$$n = 4$$

Yes. The polygon would be a square.

53. $\frac{(n-2)(180°)}{n} = 72°$

$$180°n - 360° = 72°n$$
$$108°n = 360°$$
$$n = 3\tfrac{1}{3}$$

No. Because n must be an integer, a regular polygon can't have angles of 72°.

54. $\frac{(n-2)(180°)}{n} = 18°$

$$180°n - 360° = 18°n$$
$$162°n = 360°$$
$$n = 2.22$$

No. Because n must be an integer, a regular polygon can't have angles of 18°.

55. The function is showing the relationship between the number of sides, n, and the measure of each interior angle for a regular n-gon. As n gets larger and larger, $f(n)$ approaches 180°.

56. $f(n)$ represents the measure of the exterior angle of a regular polygon. As n gets larger and larger, $f(n)$ decreases.

57. The exterior angles measure 55° and 17°. Each pair of consecutive exterior angles measures $55° + 17° = 72°$. Because $360° \div 72° = 5$, there are 5 pairs of exterior $\angle$s, so $n = 10$.

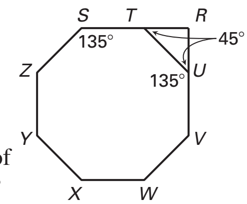
58. B. Column A = 1440°. Column B = 2340°.

59. C. Both columns equal = 360°.

60. A. Column A = 74°. Column B = 73°.

61. D. If these were regular polygons, the quantities could be determined.

62. Each interior angle has a measure of $(8 - 2)(180)/8 = 135°$. Each exterior angle has a measure of $45° = 180° - 135°$. Since the sum of the measures of the angles of a $\triangle$ is $180°$, $m\angle R$ must be $180° - 2(45°) = 90°$.



11.1 Mixed Review (p. 668)

63. $A = \frac{1}{2}bh$

$= \frac{1}{2}(11)(5)$

$= 27.5 \text{ in.}^2$

64. $A = \frac{1}{2}bh$

$= \frac{1}{2}(43)(11)$

$= 236.5 \text{ m}^2$

65. base = 5

height = 15

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(15)$

$= 37.5$ square units

66. base = 6

height = 8

$A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(8)$

$= 24$ square units

67. $9^2 + 13^2 \stackrel{?}{=} 16^2$

$81 + 169 \stackrel{?}{=} 256$

$250 \neq 256$

Not a right $\triangle$.

68. $21^2 + 72^2 \stackrel{?}{=} 75^2$

$441 + 5184 \stackrel{?}{=} 5625$

$5625 = 5625$

Is right $\triangle$.

69. $7^2 + 5^2 \stackrel{?}{=} \left(2\sqrt{17}\right)^2$

$49 + 25 \stackrel{?}{=} 68$

$74 \neq 68$

Not a right $\triangle$.

70. $m\widehat{DH} = 80°$

71. $m\widehat{ED} = m\widehat{FH} - m\widehat{FE} - m\widehat{DH}$

$= 180° - 35° - 80°$

$= 65°$

72. $m\widehat{EH} = m\widehat{FH} - m\widehat{FE}$

$= 180° - 35°$

$= 145°$

73. $m\widehat{EHG} = m\widehat{GD} + m\widehat{DE}$

$= m\widehat{GD} + \left(m\widehat{FH} - m\widehat{FE} - m\widehat{DH}\right)$

$= 180° + 180° - 35° - 80°$

$= 245°$

Lesson 11.2

Activity in a Lesson (p. 670)

1. Six equilateral triangles

2. To find the area of one triangle, one must draw an altitude from one side of the hexagon to the center forming two 30°-60°-90° right triangles. If the side of the hexagon is s, the height of the triangle would be $\frac{\sqrt{3}}{2}s$.

$A = \frac{1}{2}bh = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{1}{4}\sqrt{3}\,s^2$

Area hexagon = 6 · (Area one triangle)

$= 6\left(\frac{1}{4}\sqrt{3}\,s^2\right)$

$= \frac{3}{2}\sqrt{3}\,s^2$

11.2 Guided Practice (p. 672)

1. J **2.** 5

3. *Sample answers:* $\angle BJA$, $\angle AJE$, $\angle BJC$, $\angle EJD$, or $\angle DJC$

4. $\overline{KJ}$ **5.** Divide 360° by the number of sides.

6. $A = \frac{1}{4}\sqrt{3}\,s^2$

$= \frac{1}{4}\sqrt{3}\,(3)^2$

$= \frac{9\sqrt{3}}{4}$

$\approx 3.9 \text{ in.}^2$

7. $\frac{360°}{8} = 45°$

8. Each side length would be 80 inches ÷ 8 or 10 inches. The apothem would be half the height of the sign or 12 inches. The apothem creates a 45°-45°-90° triangle, so the radius of the octagon would be equal to the hypotenuse which is 13 inches.

$A = \frac{1}{2}aP = \frac{1}{2}(12)(80) = 480 \text{ in.}^2$

11.2 Practice and Applications (pp. 672–675)

9. $A = \frac{1}{4}\sqrt{3}\,s^2$

$= \frac{1}{4}\sqrt{3}\,(5)^2$

$= \frac{25\sqrt{3}}{4}$

≈ 10.8 sq. units

10. $A = \frac{1}{4}\sqrt{3}\,s^2$

$= \frac{1}{4}\sqrt{3}\,(11)^2$

$= \frac{121\sqrt{3}}{4}$

≈ 52.4 sq. units

11. $A = \frac{1}{4}\sqrt{3}\,s^2$

$= \frac{1}{4}\sqrt{3}\left(7\sqrt{5}\right)^2$

$= \frac{254\sqrt{3}}{4}$

≈ 106.1 sq. units

12. $\frac{360°}{9} = 40°$

13. $\frac{360°}{12} = 30°$ **14.** $\frac{360°}{15} = 24°$ **15.** $\frac{360°}{180} = 2°$

16. $s = 2\sqrt{8^2 - \left(4\sqrt{2}\right)^2}$

$= 2\sqrt{64 - 32}$

$= 8\sqrt{2}$

$A = \frac{1}{2} a \cdot ns$

$= \frac{1}{2}\left(4\sqrt{2}\right)(4)\left(8\sqrt{2}\right)$

$= 128$ sq. units

17. $s = 2\sqrt{12^2 - 6^2}$

$= 2\sqrt{144 - 36}$

$= 12\sqrt{3}$

$A = \frac{1}{2} a \cdot ns$

$= \frac{1}{2}(6)(3)\left(12\sqrt{3}\right)$

$= 108\sqrt{3}$

≈ 187.1 sq. units

18. $s = 2\sqrt{20^2 - \left(10\sqrt{3}\right)^2}$

$= 2\sqrt{400 - 300}$

$= 20$

$A = \frac{1}{2} a \cdot ns$

$= \frac{1}{2}\left(10\sqrt{3}\right) \cdot (6)(20)$

$= 600\sqrt{3}$

≈ 1039.2 sq. units

19. One side $= 10\sqrt{3}$

$P = ns$

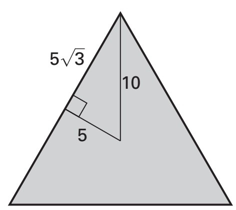
$= 3\left(10\sqrt{3}\right)$

$= 30\sqrt{3} \approx 52.0$ units

$A = \frac{1}{2} aP$

$= \frac{1}{2}(5)\left(30\sqrt{3}\right)$

$= 75\sqrt{3} \approx 129.9$ sq. units



20. $P = ns$

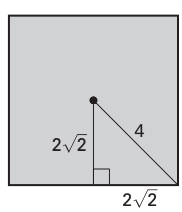
$= 4\left(4\sqrt{2}\right)$

$= 16\sqrt{2}$ units

$A = \frac{1}{2} aP$

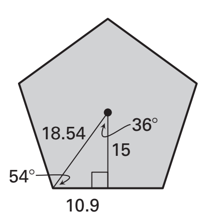
$= \frac{1}{2}\left(2\sqrt{2}\right)\left(16\sqrt{2}\right)$

$= 32$ sq. units



21. $\frac{360°}{5} = 72°$

The segment whose length is the apothem bisects the central angle. If a radius is drawn, a 36°-54°-90° triangle will be formed. Use trigonometry ratios to find the length of the radius ($r = 15/(\cos 36°)$) and a side. The length of a side of the pentagon would be: $2(15 \tan 36°) = 30 \tan 36°$



$P = ns$

$= (5)(30 \tan 36°)$

$= 150 \tan 36°$

≈ 109.0 units

$A = \frac{1}{2} aP$

$= \frac{1}{2}(15)(150 \tan 36°)$

$= 1125 \tan 36°$

≈ 817.36 sq. units

22. $\frac{360°}{6} = 60°$

A segment whose length is the apothem bisects the central angle. So, a 30°-60°-90° triangle is formed by this segment and a radius of the hexagon.

$\frac{a}{7} = \cos 30°$

$a = 7 \cos 30°$

$a = \frac{7\sqrt{3}}{2}$ units

$\frac{x}{7} = \sin 30°$

$x = 7 \sin 30°$

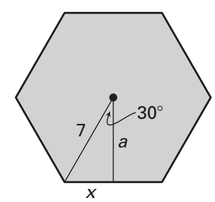
$x \approx 3.5$ units

$s = 2x = 2(3.5) = 7$ units

$P = ns$

$= (6)(7)$

$= 42$ units



$A = \frac{1}{2} aP$

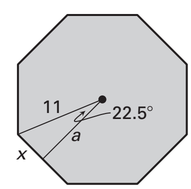
$= \frac{1}{2}\left(\frac{7\sqrt{3}}{2}\right)(42)$

$= \frac{147\sqrt{3}}{2}$

≈ 127.31 sq. units

23. $\frac{360°}{8} = 45°$

A segment whose length is the apothem bisects the central angle. So, $\frac{x}{11} = \sin 22.5°$.



$x = 11(\sin 22.5°)$

One side of the octagon $= 2x = 22 \sin 22.5°$.

$P = ns$

$P = 8(22 \sin 22.5°)$

$= 176 \sin 22.5°$

≈ 67.35 units

$\frac{a}{11} = \cos 22.5°$

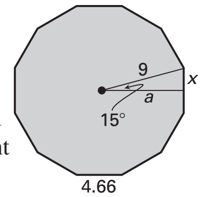
$a = 11(\cos 22.5°)$

$A = \frac{1}{2} aP$

$= \frac{1}{2}(11 \cos 22.5°)(176 \sin 22.5°)$

≈ 342.24 sq. units

24. $\dfrac{360°}{12} = 30°$



The central angle of a dodecagon is 30°. The central angle is bisected by a segment whose length is the apothem forming a 15°-75°-90° triangle.

$\dfrac{a}{9} = \cos 15°$ $\qquad$ $\dfrac{x}{9} = \sin 15°$

$a = 9 \cos 15°$ $\qquad$ $x = 9 \sin 15°$

$s = 2x = 2(9 \sin 15°) = 18 \sin 15°$

$P = ns$
$= (12)(18 \sin 15°)$
$= 216 \sin 15°$
≈ 55.9 units

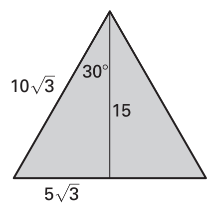
$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(9 \cos 15°)(216 \sin 15°)$
$= 972(\sin 15°)(\cos 15°)$
$= 243$ sq. units

25. $A = \dfrac{1}{2}bh$

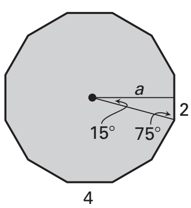
$= \dfrac{1}{2}\left(10\sqrt{3}\right)(15)$

$= 75\sqrt{3}$

$\approx 129.9 \text{ in.}^2$



26. $\dfrac{360°}{12} = 30°$



A segment whose length is the apothem bisects the central angle forming a 15°-75°-90° triangle.

$\dfrac{a}{2} = \tan 75°, \ a = 2 \tan 75°$

$P = ns$
$= (12)(4)$
$= 48$ in.

$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(2 \tan 75°)(48)$
$= 48 \tan 75°$
$\approx 179.14 \text{ in.}^2$

27. True; let θ (read "theta") be the measure of the central angle, and n the number of sides, r the radius, and P the perimeter. As n grows bigger, θ will become smaller, so the apothem, which is given by $r(\cos \dfrac{\theta}{2})$, will get larger. The perimeter of the polygon, which is given by $n(2r \sin \dfrac{\theta}{2})$, will grow larger, too. Although the factor involving the sine will get smaller, the increase in n more than makes up for it. Consequently, the area, which is given by $\dfrac{1}{2}aP$, will increase.

28. True. The apothem and radius are the lengths of a leg and the hypotenuse of a right △ respectively. The hypotenuse is always longer than the legs so the apothem will always be less than the radius.

29. False. It depends on the polygon. They are the same for a regular hexagon.

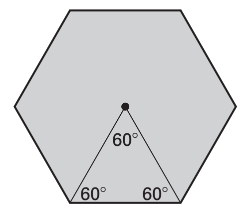
30. $A = \left(16\sqrt{3}\right)(6)$
$= 96\sqrt{3}$
≈ 166.3 sq. units

31. $A = (4 \tan 67.5°)(8)$
$= 32 \tan 67.5°$
≈ 77.3 sq. units

32. $A = (\tan 54°)(5)$
≈ 6.9 sq. units

33. $\dfrac{360°}{6} = 60° =$ measure of central angle

$\dfrac{(6-2)(180°)}{6} = 120° =$ measure of interior angle



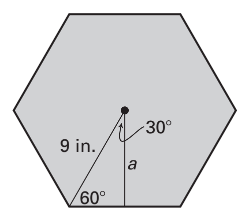
A radius bisects each interior angle, and with a segment whose length is the apothem forms an equilateral △. The area of an equilateral △ is $A = \dfrac{1}{4}\sqrt{3}\,s^2$. Since a regular hexagon can be divided into six equilateral △s, the area of a hexagon is $A = 6\left(\dfrac{1}{4}\sqrt{3}\,s^2\right)$.

34. $\dfrac{a}{9} = \sin 60°$

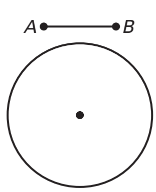
$a = 9 \sin 60°$

$= \dfrac{9\sqrt{3}}{2}$

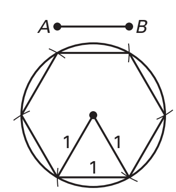
≈ 7.8 in.



35.



36.–37.



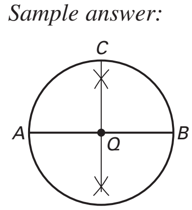
38. $A = 6 \cdot \left(\dfrac{1}{4}\sqrt{3}\,s^2\right)$

$= 6 \cdot \left(\dfrac{1}{4}\sqrt{3}(1)^2\right)$

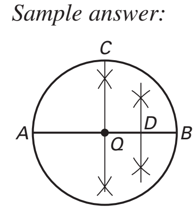
$\approx 2.6 \text{ in.}^2$

39. Do the same construction as in Exs. 35 and 36, but connect every other compass mark.

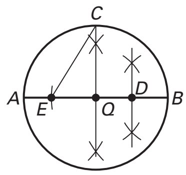
40. *Sample answer:*



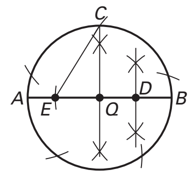
41. *Sample answer:*



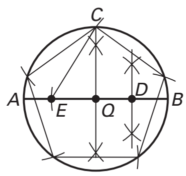
42. *Sample answer:*



43. *Sample answer:*

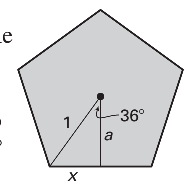


44. *Sample answer:*



Sample answer:

$\frac{360°}{5} = 72° =$ meas. of central angle



A segment whose length is the apothem bisects the central angle so with a radius, it forms a 36°-54°-90° triangle. The apothem can be found by using trigonometry with $r = 1$ in.

$\frac{a}{1} = \cos 36°$ $\qquad$ $\frac{x}{1} = \sin 36°$

$a \approx 0.81$ in. $\qquad$ $x \approx 0.59$ in.

$s = 2x = 1.18$ in.

$P = ns$

$= 5(1.18)$

$= 5.9$ in.

$A = \frac{1}{2}aP$

$= \frac{1}{2}(0.81 \text{ in.})(5.9 \text{ in.})$

$\approx 2.4 \text{ in.}^2$

45. The radius = side length = 0.5 m. A segment whose length is the apothem bisects a side. So, the length of the base of the right triangle $= \frac{1}{2}(0.5) = 0.25$ m.
The triangle formed by a segment whose length is the apothem and a radius is a 30°-60°-90° triangle with the 0.25 m side being opposite the 30° angle. The apothem $= \sqrt{3}\,(0.25) \approx 0.43$ m.

46. $P = ns$ $\qquad$ $A = \frac{1}{2}aP$

$= 6(0.5)$ $\qquad$ $\approx \frac{1}{2}(0.43)(3)$

$= 3$ m $\qquad$ $= 0.65 \text{ m}^2$

Note: See problem 45 for explanation.

47. 3 colors

48. The apothem will be $3\sqrt{3}$ in. based on a 30°-60°-90° triangle. $P = ns = 6(6) = 36$ in.

$A = \frac{1}{2}aP = \frac{1}{2}(3\sqrt{3})(36) = 54\sqrt{3} \approx 93.5 \text{ in.}^2$

49. 6 ft = 72 in., 8 ft = 96 in.

$A = (72)(96) = 6912 \text{ in.}^2$

number tiles $= \frac{6912}{93.6} \approx 74$ tiles for the floor total

$\frac{74}{3} \approx 25$ tiles of each color

50. B. Column $A = 45°$. Column $B \approx 51.43$.

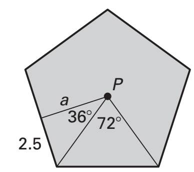
51. A. Column $A \approx 0.92$. Column $B \approx 0.90$.

52. A. Column $A = 6.12$. Column $B = 6.07$.

53. $A = \frac{1}{2}aP$

$= \frac{1}{2}\left(\frac{2.5}{\tan 36°}\right)(5)(5)$

≈ 43 sq. units



Area triangle $= \frac{1}{2}(5 \sin 54°)(5 \cos 54°)(2)$

≈ 11.89 sq. units

Area trapezoid $= \frac{1}{2}(b_1 + b_2)(h)$

$= \frac{1}{2}(5 + 2 \cdot 5 \sin 54°)(5 \cos 18°)$

$\approx \frac{1}{2}(13.09)(4.76) \approx 31.12$ sq. units

Area total $\approx 11.89 + 31.12 \approx 43$ sq. units

Answers should be the same. Differences may arise due to rounding.

11.2 Mixed Review (p. 675)

54. $\frac{x}{6} = \frac{11}{12}$

$12x = 66$

$x = \frac{11}{2}$

55. $\frac{20}{4} = \frac{15}{x}$

$20x = 60$

$x = 3$

56. $\frac{12}{x + 7} = \frac{13}{x}$

$12x = 13(x + 7)$

$12x = 13x + 91$

$-x = 91$

$x = -91$

57. $\frac{x + 6}{9} = \frac{x}{11}$

$11(x + 6) = 9x$

$11x + 66 = 9x$

$66 = -2x$

$-33 = x$

58. True. The lengths of corresponding sides of similar triangles are proportional.

59. True. The ratio of the perimeters of similar triangles equals the ratio of the lengths of any pair of corresponding sides.

60. True. The corresponding angles of similar triangles are congruent.

61. False. The lengths of corresponding sides of similar triangles are proportional, not necessarily equal. Here, the scale factor is 4:3 not 1:1 so $\overline{BC} \ncong \overline{EF}$.

62. $14 \cdot x = 7 \cdot 12$

$14x = 84$

$x = \frac{84}{14}$

$x = 6$

63. $9(9 + x) = 8(8 + 10)$

$81 + 9x = 144$

$9x = 63$

$x = 7$

64. $8^2 = 4(4 + x)$

$64 = 16 + 4x$

$48 = 4x$

$12 = x$

Developing Concepts Activity 11.3 (p. 676)

Exploring the Concept

For example on p. 676-*Sample Answer:*

Original Polygon	*Area*	*Similar Polygon*	*Area*
Rectangle 1	5 sq units	Rectangle 1	20 sq units
Rectangle 2	8 sq units	Rectangle 2	32 sq units
Rectangle 3	6 sq units	Rectangle 3	24 sq units
⋮	⋮	⋮	⋮
Total	11 sq units	Total	44 sq units

Note: The areas for similar polygons should be 4 times the area of the original polygons.

Drawing Conclusions

1. Areas will vary; the ratio of the area of the similar polygon to the area of the original polygon is the square of the scale factor and should be 4:1.

2. The ratio of the areas of two similar polygons is the square of the scale factor.

Lesson 11.3

11.3 Guided Practice (p. 679)

1. $a: b,\ a^2: b^2$

2. True; all corresponding angles are congruent and all side lengths are proportional.

3. False. The scale factor is 1:2, so the ratio of the areas is 1:4 and the area is quadrupled.

4. $P_{red} = 3(6) = 18$ $P_{blue} = 9(6) = 54$ $\frac{P_{red}}{P_{blue}} = \frac{18}{54} = \frac{1}{3}$

The ratios of the areas is equal to the square of the ratio of the perimeters so $\frac{1^2}{3^2} = \frac{1}{9}$.

5. $A_{red} = 6 \cdot 5 = 30$ $A_{blue} = 4 \cdot 3\frac{1}{3} = 13\frac{1}{3}$

$\frac{A_{red}}{A_{blue}} = \frac{30}{\frac{40}{3}} = \frac{90}{40} = \frac{9}{4}$

$\frac{P_{red}}{P_{blue}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

6. The ratio of the lengths of the pieces of paper is 2:1, so the ratio of the areas of the piece of paper is 4:1. The cost of the smaller piece should be $\frac{1}{4}$ as much as the larger sheet or about $.11.

11.3 Practice and Applications (p. 679–681)

7. The ratio of the perimeters is 2:1.
The ratio of the areas is 4:1.

8. The ratio of the perimeter is 5:7.
The ratio of the areas is 25:49.

9. The ratio of the perimeter is 5:6.
The ratio of the areas is 25:36.

10. The ratio of the perimeter is 5:3.
The ratio of the areas is 25:9.

11. sometimes **12.** sometimes **13.** always **14.** 4:25

15. $\sqrt{\frac{49}{100}} = \frac{7}{10}$

16. The scale factor of the hypotenuses is 8:20 or 2:5. The ratio of the areas is the square of the scale factor, or 4:25.

$\frac{4}{25} = \frac{13.9}{x}$

$4x = 347.5$

$x = 86.875 \text{ in.}^2$

17. *Sample answer:* $\overline{AB} \parallel \overline{DC}$, so $\angle AEB \cong \angle DEC$ because vertical angles are $\cong$. $\angle BAE \cong \angle ECD$ because alternate interior angles are $\cong$. $\triangle CDE \sim \triangle ABE$ by the AA Similarity Postulate.

Area of $\triangle ABE = \frac{1}{2}(12)(3) = 18$ sq units

$\left(\frac{7}{3}\right)^2 = \frac{x}{18}$

$\frac{49}{9} = \frac{x}{18}$

$9x = 49 \cdot 18$

$x = 98$

Area of $\triangle CDE = 98$ sq units

18. $\overline{DC} \parallel \overline{AB}$ and $\overline{LK} \parallel \overline{AB}$, so $\overline{DC} \parallel \overline{LK}$ because 2 lines $\parallel$ to the same line are parallel. Then $\angle K \cong \angle C$ and $\angle A \cong \angle J$, and all corresponding $\angle s$ are $\cong$. Ratio of the lengths of any two corresp. sides is 3:1, so the ratio of the areas is $3^2 : 1^2$. The area of $\square ABCD$ is 9 times the area of $\square JBKL = 9(15.3) = 137.7$ sq. in.

19. Perimeter of $ABCDE = 30\sqrt{5}$. Perimeter of $QRSTU = 40$. Ratio of perimeters $= 30\sqrt{5} : 40$ or $3\sqrt{5} : 4$.

20. Perimeter of small square $= 16$.

Ratio of perimeters $= 36{:}16 = 9{:}4$.

Ratio of areas $= 9^2{:}4^2 = 81{:}16$.

21. The ratio of areas is 90:25. The ratio of perimeters is $\sqrt{90} : \sqrt{25} = 3\sqrt{10} : 5$.

22. *Sample answer:* Let $ABCD$ and $EFGH$ be similar rectangles with the lengths of corresp. sides in the ratio $a{:}b$. Let $a\ell$ be the length of $ABCD$ and aw the width, and let $b\ell$ be the length of $EFGH$ and bw the width.

Then $\dfrac{\text{area of } ABCD}{\text{area of } EFGH} = \dfrac{a\ell(aw)}{b\ell(bw)} = \dfrac{a^2}{b^2}$.

23. Area small rug $= 29 \text{ in.} \times 47 \text{ in.} = 1363$ sq in.

Area large rug $= 58 \text{ in.} \times 94 \text{ in.} = 5452$ sq in.

$\dfrac{\text{Area small}}{\text{Area large}} = \dfrac{1363}{5452}$ or 1:4

24. Small rug: $\dfrac{\$79}{1363 \text{ sq in.}} = \$.06$ per sq in.

Large rug: $\dfrac{\$299}{5452 \text{ sq in.}} = \$.05$ per sq in.

The large rug is a good buy because it costs slightly less per square inch than a small rug. Also, the area of the large rug is 4 times the area of the small rug, but the large rug costs less than 4 times the cost of the small rug ($\$79 \times 4 = \$316$).

25. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}(40)(41)$

$= 820 \text{ ft}^2$

26. $AB \approx 1.3 \times 40$ ft

≈ 52 ft

27. Area $\triangle ABC \approx (1.3)^2(820 \text{ ft}^2)$

$\approx 1385.8 \text{ ft}^2$

Area walkway $=$ Area of $\triangle ABC -$ Area of $\triangle DEF$

$\approx 1385.8 - 820$

$\approx 565.8 \text{ ft}^2$

28. $\dfrac{\text{Area outer}}{\text{Area inner}} = \dfrac{466{,}170}{446{,}400} \approx \dfrac{1.04}{1}$

Ratio of perimeters $\approx \dfrac{\sqrt{1.04}}{\sqrt{1}} \approx \dfrac{1.02}{1}$

$\dfrac{1.02}{1} \approx \dfrac{477}{x}$

$1.02x \approx 477$

$x \approx \dfrac{4.77}{1.02}$

≈ 467 ft

29. a. $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \dfrac{15^2}{2^2} = \dfrac{225}{4}$

b. $\dfrac{25x}{x-5} = \dfrac{225}{4}$

$100x = 225(x-5)$

$100x = 225x - 1125$

$-125x = -1125$

$x = 9$

c. 15:2

d. $\dfrac{8+y}{3y-19} = \dfrac{15}{2}$

$2(8+y) = 15(3y-19)$

$16 + 2y = 45y - 285$

$301 = 43y$

$7 = y$

e. Because $\overline{AB}$ and $\overline{DE}$ are corresponding sides of ~ $\triangle$s the ratio of their lengths is the scale factor. You can solve the proportion $\dfrac{22.5}{13z - 10} = \dfrac{15}{z}$ to find z; $z = 1$.

30. *Sample answer:*

$\triangle PVQ \sim \triangle RVT$

$\triangle RVQ \sim \triangle PVU$

$\triangle TQR \sim \triangle TUS$

All pairs are $\sim$ by the AA Similarity Post.

31. $\dfrac{PV}{RV} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$

$\dfrac{3}{5} = \dfrac{PV}{10}$

$PV = 6$

32. $\dfrac{3}{5} = \dfrac{VQ}{VT}$

$\dfrac{3}{5} = \dfrac{VQ}{15}$

$45 = 5VQ$

$9 = VQ$

33. *Sample answer:*

$\dfrac{\text{Area } \triangle PVQ}{\text{Area } \triangle RVT} = \dfrac{9}{25}$

$\dfrac{\text{Area } \triangle RVQ}{\text{Area } \triangle PVU} = \dfrac{45}{16.2} = \dfrac{25}{9}$

$\dfrac{\text{Area } \triangle TQR}{\text{Area } \triangle TUS} = \dfrac{120}{19.2} = \dfrac{25}{4}$

—CONTINUED—

32. —CONTINUED—

$$\frac{VU}{VQ} = \frac{3}{5}$$

$$\frac{VU}{9} = \frac{3}{5}$$

$$5VU = 27$$

$$VU = \frac{27}{5}$$

$$= 5\tfrac{2}{5}$$

$$UT = 15 - 5\tfrac{2}{5} = 9\tfrac{3}{5}$$

11.3 Mixed Review (p. 681)

34. 80° **35.** 145° **36.** 145° **37.** 215°

38. $10x + 8x = 180$ $\quad 17y + 19y = 180$

$18x = 180 \quad 36y = 180$

$x = 10 \quad y = 5$

$m\angle R = 100°, m\angle S = 85°, m\angle T = 80°, m\angle U = 95°$

39. $m\angle 1 = \frac{1}{2}(160°)$

$= 80°$

40. $m\angle 1 = \frac{1}{2}(110° + 50°)$

$= 80°$

41. $m\angle 1 = \frac{1}{2}(126° - 40°)$

$= \frac{1}{2}(86°)$

$= 43°$

Quiz 1 (p. 682)

1. $(20 - 2)(180°) = 3240°$ **2.** $\frac{360°}{25} = 14.4°$

3. $A = \frac{1}{4}\sqrt{3}\, s^2$

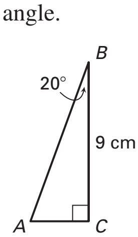
$= \frac{1}{4}\sqrt{3}\,(17)^2$

$\approx 125.1 \text{ in.}^2$

4. The central angle of a nonagon is 40°. A segment whose length is the apothem bisects the central angle.

$\frac{9 \text{ cm}}{AB} = \cos 20° \quad \frac{AC}{9} = \tan 20°$

$AB \approx 9.58 \text{ cm} \quad AC \approx 3.28$



$A = \frac{1}{2}aP$

$= \frac{1}{2}(9)(9 \tan 20°)(2)(9)$

$\approx 265.33 \text{ cm}^2$

5. $\frac{P_{red}}{P_{blue}} = \frac{8}{6} = \frac{4}{3}$

$\frac{A_{red}}{A_{blue}} = \frac{16}{9}$

6. $\frac{P_{red}}{P_{blue}} = \frac{3.25}{5} = \frac{13}{20}$

$\frac{A_{red}}{A_{blue}} = \frac{169}{400}$

7. The scale factor is 3:7 $\left(\frac{9 \text{ feet}}{21 \text{ feet}} = \frac{3}{7}\right)$; therefore the ratio of the areas is $\frac{9}{49}$.

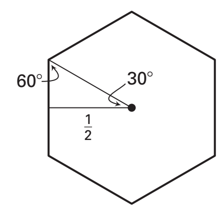
$\frac{9}{49} = \frac{480}{x}$

$9x = 23{,}520$

$x \approx \$2613$

11.3 Math & History (p. 682)

1. The length of each side of the inscribed hexagon will be equal to $\frac{1}{2}$(diameter). So, the perimeter of the inscribed hexagon is equal to 3 · diameter or 3 units. If the apothem is $\frac{1}{2}$ unit, then the length of one side of the hexagon is $2 \cdot \frac{1}{2} \tan 30° = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{3}$ or $\frac{\sqrt{3}}{3}$. The perimeter of the circumscribed hexagon is $6 \cdot \left(\sqrt{3}/3\right) = 2\sqrt{3} \approx 3.46$, so $3 < \pi < 3.46$.



Lesson 11.4

11.4 Guided Practice (p. 686)

1. Arc measure is the number of degrees of the central angle whose endpoints are the ends of the arc. Arc length is the measure of a portion of the circumference.

2. Arc length $\widehat{AB} = \frac{m\angle 1}{360°} \cdot 2\pi r = \frac{m\angle 2}{360°} \cdot 2\pi r =$ arc length $\widehat{CD}$.

3. F **4.** D **5.** C **6.** B **7.** A **8.** E

9. False. Arc length depends on the radius of the circle. So if the radii are different, the arc lengths will be different.

10. False. The circumference is proportional to the radius. If the radius doubles, the circumference doubles.

11. False. Arc length is determined by the measure of the arc and the radius of the circle. If you solve for the measure of the arc, then it is dependent upon both the arc length and the radius of the circle.

12. Length $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$= \frac{140°}{360°} \cdot 2\pi \cdot 29.5$

$\approx 22.9\pi$ cm or 72.1 cm

13. $\text{Length } \widehat{CD} = \dfrac{m\widehat{CD}}{360^\circ} \cdot 2\pi r$

$= \dfrac{160^\circ}{360^\circ} \cdot 2 \cdot \pi \cdot 29$

$\approx 25.8\pi \text{ cm or } 81.0 \text{ cm}$

14. $\text{Length } \widehat{EF} = \dfrac{m\widehat{EF}}{360^\circ} \cdot 2\pi r$

$67.6 \text{ cm} = \dfrac{m\widehat{EF}}{360^\circ} \cdot 2\pi(25)$

$\dfrac{(360^\circ)(67.6 \text{ cm})}{50\pi} = m\widehat{EF}$

$155^\circ \approx m\widehat{EF}$

11.4 Practice and Applications (pp. 686-689)

15. $C = 2\pi r$

$= 2\pi(5)$

$= 10\pi$

$\approx 31.4 \text{ in.}$

16. $C = 2\pi r$

$44 = 2\pi r$

$\dfrac{44}{2\pi} = r$

$7.0 \text{ ft} \approx r$

17. $C = \pi d$

$= \pi(8)$

$\approx 25.1 \text{ m}$

18. $C = 2\pi r$

$= 2\pi(15)$

$= 30\pi \text{ in.}$

$\approx 94.2 \text{ in.}$

19. $C = 2\pi r$

$32 = 2\pi r$

$\dfrac{32}{2\pi} \approx r$

$5.1 \text{ yd} \approx r$

20. $\text{Length } \widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

$= \dfrac{45^\circ}{360^\circ} \cdot 2\pi(3)$

$\approx 2.36 \text{ cm}$

21. $\text{Length } \widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

$= \dfrac{60^\circ}{360^\circ} \cdot 2\pi(7)$

$\approx 7.33 \text{ in.}$

22. $\text{Length } \widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

$= \dfrac{120^\circ}{360^\circ} \cdot 2\pi(10)$

$\approx 20.9 \text{ ft}$

23.

Radius	12	3	0.6	3.5	5.1	$3\sqrt{3}$
$m\widehat{AB}$	45°	30°	120°	192°	90°	107°
Length of $\widehat{AB}$	3π	0.5π	0.4π	3.73π	2.55π	3.09π

24. $\text{Length of } \widehat{XY} = \dfrac{m\widehat{XY}}{360^\circ} \cdot 2\pi r$

$= \dfrac{30^\circ}{360^\circ} \cdot 2\pi(8)$

$\approx 4.2 \text{ units}$

25. $\text{Length of } \widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

$5.5 = \dfrac{55^\circ}{360^\circ} \cdot 2\pi r$

$5.5\left(\dfrac{360^\circ}{55^\circ}\right) = 2\pi r$

$36 \text{ units} = 2\pi r$

26. $\text{Length of } \widehat{CD} = \dfrac{m\widehat{CD}}{360^\circ} \cdot 2\pi r$

$20 = \dfrac{160^\circ}{360^\circ} \cdot 2\pi r$

$7.16 \text{ units} \approx r$

27. $\text{Length of } \widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$

$= \dfrac{118^\circ}{360^\circ} \cdot 2\pi(10.14)$

$\approx 20.88 \text{ units}$

28. $\text{Length of } \widehat{ST} = \dfrac{m\widehat{ST}}{360^\circ} \cdot 2\pi r$

$12.4 = \dfrac{84^\circ}{360^\circ} \cdot 2\pi r$

$53.14 \text{ units} \approx 2\pi r$

29. $\text{Length of } \widehat{LM} = \dfrac{m\widehat{LM}}{360^\circ} \cdot 2\pi r$

$42.56 = \dfrac{240^\circ}{360^\circ} \cdot 2\pi r$

$10.16 \text{ units} \approx r$

30. $P = 2(l) + 2\pi r$
$= 2(12) + 2\pi(3.5)$
≈ 45.99 units

31. $P = 5 + 5 + 5 + 2\pi(2)$
$\approx 15 + 15.71$
$= 30.71$ units

32. $P = 4(2) + 2\pi(2)$
$\approx 8 + 12.57$
≈ 20.57 units

33. $2x + 15 = 135$
$2x = 120$
$x = 60$

$$\text{Length of arc} = \frac{m\text{ arc}}{360^\circ} \cdot 2\pi r$$
$$(y - 3)\pi = \frac{135^\circ}{360^\circ} \cdot 2\pi(8)$$
$$y - 3 = 6$$
$$y = 9$$

34. $15y - 30 = 270$
$15y = 300$
$y = 20$

$$\text{Length of arc} = \frac{m\text{ arc}}{360^\circ} \cdot 2\pi r$$
$$(13x + 2)\pi = \frac{270^\circ}{360^\circ} \cdot 2\pi(10)$$
$$13x + 2 = 15$$
$$13x = 13$$
$$x = 1$$

35. $18x = 45$
$x = 2.5$

$$\text{Length of arc} = \frac{m\text{ arc}}{360^\circ} \cdot 2\pi r$$
$$(14y - 3)\pi = \frac{45^\circ}{360^\circ} \cdot 2\pi(7)$$
$$14y - 3 = 1.75$$
$$14y = 4.75$$
$$y \approx 0.34$$

36. $r = 3$
$C = 2\pi r$
$C = 6\pi$

37. $r = 2\sqrt{7}$
$C = 2\pi r$
$C = 4\pi\sqrt{7}$

38. $r = 2$
$C = 2\pi r$
$C = 4\pi$

39. Tire A = 15 in. + 4.6 in. + 4.6 in.
= 24.2 in.
Tire B = 16 in. + 4.43 in. + 4.43 in.
= 24.86 in.
Tire A = 17 in. + 4.33 in. + 4.33 in.
= 25.66 in.

40. 500 ft = 6000 in.

Tire A: $\dfrac{6000\text{ in.}}{\pi(24.2\text{ in.})} \approx 79$ rev.

Tire B: $\dfrac{6000\text{ in.}}{\pi(24.86\text{ in.})} \approx 77$ rev.

Tire C: $\dfrac{6000\text{ in.}}{\pi(25.66\text{ in.})} \approx 74$ rev.

41. The sidewall width must be added twice to the rim diameter to get the tire diameter.

42. L = straightaways + curves

$$L = 30 + 8 + 12 + 12 + 30 + 20 + 6\left(\frac{1}{4}\right)(2\pi)(3) + \frac{1}{2}(2\pi)(3) + 2\left(\frac{1}{2}\right)(2\pi)(2.25)$$
$$= 100 + 2\pi(8.25)$$
$$\approx 163.84 \text{ m}$$

43. $\dfrac{1609\text{ m}}{164\text{ m/lap}} \approx 9.8$ laps

44. measure of arc = 4.2°, radius of Earth ≈ 4000 mi

$$\text{Length of arc} = \frac{4.2^\circ}{360^\circ} \cdot 2\pi(4000)$$
$$\approx 293.22 \text{ miles}$$

45. Length of arc (rear sprocket) $= \dfrac{164^\circ}{360^\circ} \cdot 8$
≈ 3.64 in.

Length of arc (front sprocket) $= \dfrac{196^\circ}{360^\circ} \cdot 22$
≈ 11.98 in.

Length of chain ≈ 3.64 + 16 + 11.98 + 16
≈ 47.62 in.

46. number of teeth ≈ 47.62 in. ÷ (0.5 in./tooth)
≈ 95 teeth

47. $P = \dfrac{3}{4}(2\pi r)$
$= \dfrac{3}{4}(2\pi)(8)$
≈ 37.7 ft

48. B. Length of $\widehat{XZ} = \frac{m\widehat{XZ}}{360°} \cdot 2\pi r$

$= \frac{60°}{360°} \cdot 2\pi(4)$

$= \frac{4}{3}\pi$ units

49. B. $x = \frac{2(360°)}{2\pi r}$ $\quad y = \frac{360°}{2\pi r}$

$\frac{x}{y} = \frac{\frac{2(360°)}{2\pi r}}{\frac{360°}{2\pi r}}$

$= \frac{2}{1}$

50. The four semicircles form 2 complete circles so that the length would be $2(2\pi r) = 4\pi r$.

51. 8 segments = 4 complete circles = $4\left[2\pi\left(\frac{1}{2}\right)r\right] = 4\pi r$.

16 segments = 8 complete circles = $8\left[2\pi\left(\frac{1}{4}\right)r\right] = 4\pi r$.

n segments = $\frac{n}{2}$ complete circles = $\frac{n}{2}\left[2\pi\left(\frac{1}{\frac{n}{4}}\right)r\right] = 4\pi r$.

No. The number of segments doesn't matter.

11.4 Mixed Review (p. 689)

52. $A = \pi r^2$

$= \pi(9)^2$

$\approx 254.47 \text{ ft}^2$

53. $A = \pi r^2$

$= \pi(3.3)^2$

$\approx 34.21 \text{ in.}^2$

54. $A = \pi r^2$

$= \pi\left(\frac{27}{5}\right)^2$

$\approx 91.61 \text{ cm}^2$

55. $A = \pi\left(4\sqrt{11}\right)^2$

$\approx 552.92 \text{ m}^2$

56. $y = mx + b$

$-2 = \frac{2}{3}(9) + b$

$-2 = 6 + b$

$-8 = b$

$y = \frac{2}{3}x - 8$

57. $\frac{x}{3.5} = \frac{5}{6}$

$6x = 17.5$

$x \approx 2.92$

58. $\frac{30}{15} = \frac{y}{18}$

$540 = 15y$

$36 = y$

59. 96° **60.** 176° **61.** 258° **62.** 31°

Technology Activity 11.4 (p. 690)

1. The perimeter increases as n increases, getting closer and closer to 2π.

2. $n = 6;\ P = 2(6)\sin\left(\frac{180°}{6}\right) = 12 \sin 30° = 6$

3. 12-gon: $P \approx 6.21$ units

15-gon: $P \approx 6.24$ units

18-gon: $P \approx 6.25$ units

24-gon: $P \approx 6.27$ units

Lesson 11.5

11.5 Guided Practice (p. 695)

1. A sector of a circle is bounded by two radii of the circle and their intercepted arc.

2. The radius is one half of the diameter or $\frac{1}{2} \cdot 4$. The area is $\pi r^2 = \pi\left(\frac{1}{2} \cdot 4\right)^2$.

3. $A = \pi r^2$

$= \pi(9)^2$

$\approx 254.47 \text{ in.}^2$

4. $A = \pi r^2$

$= \pi(3.8)^2$

$\approx 45.36 \text{ cm}^2$

5. $A = \pi r^2$

$= \pi\left(\frac{1}{2} \cdot 12\right)^2$

$\approx 113.10 \text{ ft}^2$

6. $A = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \frac{110°}{360°} \cdot \pi(6)^2$

$\approx 34.56 \text{ ft}^2$

7. $A = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \frac{70°}{360°} \cdot \pi(10)^2$

$\approx 61.09 \text{ m}^2$

8. $A =$ Area of circle $-$ Area of sector

$= \pi(3)^2 - \left(\frac{60°}{360°}\right) \cdot \pi(3)^2$

$\approx 28.27 - 4.71$

$\approx 23.56 \text{ in.}^2$

9. $A = \frac{45°}{360°}\pi(8)^2$

$\approx 25.13 \text{ in.}^2$

11.5 Practice and Applications (p. 695–698)

10. $A = \pi(31)^2$

$\approx 3019.07 \text{ ft}^2$

11. $A = \pi(0.4)^2$

$\approx 0.50 \text{ cm}^2$

12. $A = \pi(4)^2$

$\approx 50.27 \text{ m}^2$

13. $A = \pi(10)^2$

$\approx 314.16 \text{ in.}^2$

14. $A = \frac{60°}{360°} \cdot \pi(11)^2$

$\approx 63.36 \text{ ft}^2$

15. $A = \frac{80°}{360°} \cdot \pi\left(\frac{7}{2}\right)^2$

$\approx 8.55 \text{ in.}^2$

16. $A = \frac{293°}{360°} \cdot \pi(10)^2$

$\approx 255.69 \text{ cm}^2$

17. $A = \frac{120°}{360°} \cdot \pi(4.6)^2$

$\approx 22.16 \text{ m}^2$

18. $A = \frac{250°}{360°} \cdot \pi(8)^2$

$\approx 139.63 \text{ in.}^2$

19. $A = \pi(10)^2$

$\approx 314.16 \text{ ft}^2$

20. $50 = \pi(r^2)$

$15.92 \approx r^2$

$3.99 \text{ m} \approx r$

21. $A = \frac{m\,\widehat{AB}}{360°} \cdot \pi r^2$

$59 = \frac{40°}{360°} \cdot \pi r^2$

$169.02 \approx r^2$

$13.00 \text{ in.} \approx r$

22. $A = \frac{m \text{ arc}}{360°} \cdot \pi\left(\frac{1}{2}d\right)^2$

$277 = \frac{288°}{360°} \cdot \pi\left(\frac{1}{4}\right)d^2$

$440.86 \approx d^2$

$21.00 \text{ m} \approx d$

23. A = Area large circle − Area small circle

$= \pi(24)^2 - \pi(6)^2$

$\approx 1696.46 \text{ m}^2$

24. A = Area large semicircle − Area small semicircle

$= \frac{1}{2}\pi(38)^2 - \frac{1}{2}\pi(19)^2$

$\approx 1701.17 \text{ cm}^2$

25. A = Area of circle − Area of pentagon

$= \pi(4)^2 - \frac{1}{2}(4 \sin 54°)(2)(5)(4 \cos 54°)$

$\approx 12.22 \text{ ft}^2$

26. Area $= \pi(3)^2 - \pi(2)^2$

$= \pi(9 - 4)$

$\approx 15.7 \text{ ft}^2$

27. A = Area square − Area circles

$= (18)^2 - 4\pi(4.5)^2$

$\approx 69.53 \text{ in.}^2$

28. A = Area semicircle − Area triangle

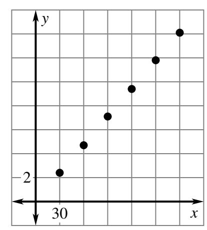
$= \frac{1}{2}\pi(2)^2 - \frac{1}{4}\sqrt{3}(2)^2$

$\approx 4.55 \text{ cm}^2$

29.

Measure of arc, x	30°	60°	90°	120°	150°	180°
Area of corresponding sector, y	2.4	4.7	7.1	9.4	11.8	14.1

30.



31. Yes it appears to be linear. The equation would be

$y = \frac{\pi}{40}x.$

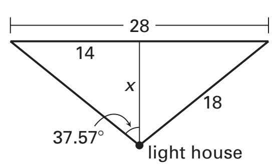
32. With a 5-inch radius, the area of the sectors would be larger. However the relationship would still be linear.

$y = \frac{5\pi}{72}x.$

33. $A = \frac{245°}{360°} \cdot \pi(18)^2$

$\approx 692.72 \text{ mi}^2$

34.



$14^2 + x^2 = 18^2$

$196 + x^2 = 324$

$x^2 = 128$

$x \approx 11.31$ miles

35. A = Area of sector − Area of triangle

$= \frac{60}{360} \cdot \pi(6)^2 - \frac{1}{2}(2)(3)(3\sqrt{3})$

$\approx 3.26 \text{ cm}^2$

36. A = Area of sector − Area of triangle

$= \frac{90°}{360°} \cdot \pi(14)^2 - \frac{1}{2}(2)(7\sqrt{2})(7\sqrt{2})$

$\approx 55.94 \text{ m}^2$

37. A = Area of sector − Area of triangle

$= \frac{120°}{360°} \cdot \pi(48)^2 - \frac{1}{2}(24)(2)(24\sqrt{3})$

$\approx 1415.08 \text{ cm}^2$

38. A = Total area − Area of smaller sector

$= \frac{72°}{360°} \cdot \pi(3.5)^2 - \frac{72°}{360°} \cdot \pi(3)^2$

$\approx 2.04 \text{ ft}^2$

39. To find the area of the sector, you would need to either know the arc length or the measure of the central angle, and the radius. To find the area of the kite you would have to know the lengths of the diagonals or be given sufficient information to determine the length of the diagonals, such as a side with an angle or two sides.

40. Area $ABCD$ = Area of sector formed by $\angle APB$ − Area of sector formed by $\angle DPC$

$= \frac{45°}{360°} \cdot \pi(4)^2 - \frac{45°}{360°} \cdot \pi(2)^2$

$\approx 4.71 \text{ ft}^2$

41. If you double the radius, the area is quadrupled because it is proportional to the square of the radius. If you double the radius, the circumference doubles also because the circumference is directly proportional to the radius.

42. As n gets larger, the area of the polygon approaches π, the area of the circle.

Sample spreadsheet:

n-gon Data

	A	B
1	# of sides	Area
2	n	.5*cos(180/n)*2*n*sin(180/n)
3	3	1.299038106
4	4	2
5	5	2.377641291
6	6	2.598076211
7	7	2.736410189
8	8	2.828427125
9	9	2.892544244
10	10	2.938926261
11	11	2.973524496
12	12	3
13	13	3.020700618
14	14	3.037186174
15	15	3.050524823
16	16	3.061467459

Continuing the spreadsheet, you can find that
$A \approx 3.14151$ when $n = 500$,
$A \approx 3.14157$ when $n = 1000$,
$A \approx 3.14159$ when $n = 2500$, etc.

43. C

$A = \pi(6)^2 - \pi(3)^2$

$= 36\pi - 9\pi$

$= 27\pi$

44. D

$A = \frac{108°}{360°} \cdot \pi(6)^2$

$= 10.8\pi$

45. A = Area of triangle − 3 · Area of sector

$= \frac{1}{4}\sqrt{3}(12)^2 - 3 \cdot \frac{60°}{360°} \cdot \pi(6)^2$

$\approx 62.35 - 56.55$

$\approx 5.81 \text{ cm}^2$

11.5 Mixed Review (p. 698)

46. $\frac{2}{5}$ **47.** $\frac{3}{16}$ **48.** $\frac{4}{21}$ **49.** $\frac{4}{11}$

50. $15\sqrt{2} \approx 21.21$ units

51. $DB = \frac{18}{\sin 68°}$

≈ 19.4 cm

52. $DC = 18$ cm

53. $m\angle DBC = 68°$

54. $BC = \frac{18}{\tan 68°}$

≈ 7.3 cm

55. $(x + 2)^2 + (y + 7)^2 = 36$

56. $x^2 + (y + 9)^2 = 100$

57. $(x + 4)^2 + (y - 5)^2 = 10.24$

58. $(x - 8)^2 + (y - 2)^2 = 11$

59. $C = 2\pi(12.5)$

≈ 78.54 in.

60. Length of $\widehat{AB} = \frac{53°}{360°} \cdot 2\pi(13)$

≈ 12.03 ft

61. $31.6 = \frac{129°}{360°} \cdot 2\pi(r)$

$31.6 \approx 2.25r$

$14.04 \approx r$

Lesson 11.6

11.6 Guided Practice (p. 701)

1. A geometric probability involves geometric measures such as length and area instead of counting events or outcomes.

2. I would use the area method because a triangular region is two-dimensional.

3. Length method should be used because time can be related to a line.

4. $P = \frac{AB}{AF} = \frac{2}{18} = \frac{1}{9} \approx 11\%$

5. $P = \frac{BD}{AF} = \frac{9}{18} = \frac{1}{2} = 50\%$

6. $P = \frac{BF}{AF} = \frac{16}{18} = \frac{8}{9} \approx 89\%$

7. $\overline{AF} = \overline{AB} + \overline{BF}$. Therefore any point not on $\overline{AB}$ must be on $\overline{BF}$. So the sum of the probabilities is

$\frac{1}{9} + \frac{8}{9} = 1.$

8. $P = \frac{\text{Area shaded}}{\text{Area total}}$

$= \frac{\frac{1}{2}(5)(4) + \frac{1}{2}(4)(4)}{\frac{1}{2}(7 + 16)(4)}$

$= \frac{18}{46} \approx 39\%$

11.6 Practice and Applications (pp. 702–704)

9. $P = \frac{GH}{GN} = \frac{2}{14} = \frac{1}{7} \approx 14\%$

10. $P = \frac{JL}{GN} = \frac{4}{14} = \frac{2}{7} \approx 29\%$

11. $P = \frac{JN}{GN} = \frac{8}{14} = \frac{4}{7} \approx 57\%$

12. $P = \frac{GJ}{GN} = \frac{6}{14} = \frac{3}{7} \approx 43\%$

13. $P = \frac{PQ}{PU} = \frac{12}{48} = \frac{1}{4} = 25\%$

14. $P = \frac{PS}{PU} = \frac{28}{48} = \frac{7}{12} \approx 58\%$

15. $P = \frac{SU}{PU} = \frac{20}{48} = \frac{5}{12} \approx 42\%$

16. $P = \frac{PU}{PU} = 1 = 100\%$

17. $P = \frac{\text{Area square} - \text{Area circle}}{\text{Area square}}$

$\approx \frac{144 - 113.1}{144} \approx 21.5\%$

18. $P = \frac{\text{Area circle}}{\text{Area square} + \text{Area circle}}$

$\approx \frac{50.27}{64 + 50.27}$

$\approx 44\%$

19. $P = \frac{\text{Area triangle}}{\text{Area rectangle}}$

$= \frac{\frac{1}{2}(16 \cdot 5)}{160}$

$= \frac{40}{160}$

$= 25\%$

20. $P = \frac{\text{Area of semicircle}}{\text{Area of triangle}}$

$= \frac{\frac{1}{2}\pi(5)^2}{\frac{1}{2}(20)(10)}$

$\approx \frac{39.27}{100}$

$\approx 39.27\%$

21. $P = \frac{\text{Area of circle}}{\text{Area of hexagon}}$

$= \frac{\pi(1.5)^2}{6\left(\frac{1}{4}\right)\left(\sqrt{3}\right)(14)^2}$

$\approx \frac{7.07}{509.22}$

$\approx 1.39\%$

22. Since the probability is between 1 and 2%, 1 to 2 darts would probably hit the bull's-eye.

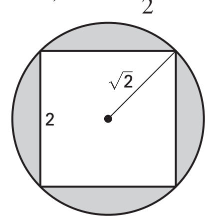
23. $P \approx \frac{28.28}{509.22}$

$\approx 5.55\%$

24.

Let L be the midpoint of $\overline{JM}$ and N be the midpoint of $\overline{MK}$. Then $\overline{LN}$ is the part of the line closer to M than to J or K, so $P = \frac{1}{2} = 50\%$.

25.



Area of square = 4 sq. units

Area of circle $= \pi\left(\sqrt{2}\right)^2 \approx 6.28$ sq. units

Area shaded $\approx 6.28 - 4 \approx 2.28$ sq. units

$P = \frac{\text{Area shaded}}{\text{Area of circle}} \approx \frac{2.28}{6.28} \approx 36.31\%$

26. $P = \frac{3.5}{7.5} \approx 46.7\%$

27. $P = \frac{10}{60} = \frac{1}{6} \approx 16.7\%$ **28.** $P = \frac{3}{15} = \frac{1}{5} = 20\%$

29. $A = (2000)(5000) = 10{,}000{,}000 \text{ yd}^2$

30. Area of circle $= \pi\left(\frac{500}{3}\right)^2 \approx 87{,}266.4 \text{ yd}^2$

$P \approx \frac{87{,}266.46}{10{,}000{,}000} \approx 0.87\%$

31. Area Total $= \pi(24)^2 \approx 1809.58$

Area of 10 point region $= \pi(2.4)^2 \approx 18.1$

$P \approx \frac{18.1}{1809.58} \approx 1\%$

32. Area yellow region $= \pi(4.8)^2 \approx 72.38$

$P \approx \frac{72.38}{1809.58} \approx 4\%$

33. Area white region $=$ Total area $-$ Area regions 3–10

$\approx 1809.58 - \pi(19.2)^2$

≈ 651.46

$P \approx \frac{651.46}{1809.58} \approx 36\%$

34. Area regions 5–10 $= \pi(14.4)^2 \approx 651.44$

Area regions 6–10 $= \pi(12)^2 \approx 452.39$

Area regions 1–4 $\approx 1809.58 - 651.44$

≈ 1158.14

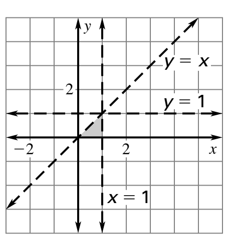
Area region 5 $\approx 1809.58 - 452.39 - 1158.14$

≈ 199.05

$P \approx \frac{199.05}{1809.58} \approx 11\%$

35. *Sample answer:* It would not hold because an expert archer is more likely to hit the bull's eye, so it is no longer true that every point on the target has an equal probability of being hit.

36.



Total Area $= 1$

$P = \frac{1}{2} = 50\%$

37. $\frac{1}{3}(360°) = 120° \div 2 = 60°$

38. $\frac{1}{4}(360°) = 90° \div 2 = 45°$

39. $\frac{1}{6}(360°) = 60° \div 2 = 30°$

40. $P = \frac{(15)^2}{(200)(250)} = \frac{225}{50{,}000} = 0.45\%$

41. $P = \frac{450}{50{,}000} = 0.9\%$

If the area is doubled, the probability is doubled.

42. $P = \frac{900}{50{,}000} = 1.8\%$

The probability is four times as much.

43. a. Area of jar $= \pi(12.5)^2 \approx 490.87 \text{ cm}^2$

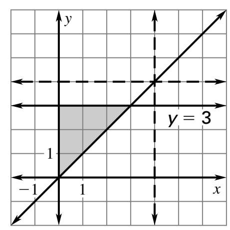
Area of dish $= \pi(2.5)^2 \approx 19.63 \text{ cm}^2$

$P \approx \frac{19.63}{490.87} \approx 4\%$

b. 25 coins **c.** $\frac{(250)(5)}{25} = 50$ prizes

d. Yes, the probability will change because now pennies just have to touch the circle, not be inside it so the target area is larger and the probability increases.

44.



$P = \frac{4.5}{16} = \frac{9}{32} \approx 28\%$

11.6 Mixed Review (p.705)

45. $\overleftrightarrow{AB}$ is not tangent to $\odot C$ because ABC is not a right $\triangle$ as follows.

$10^2 + 4^2 \neq 11^2$

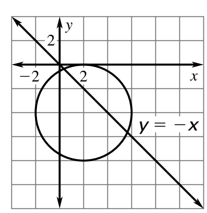
46. $\overleftrightarrow{AB}$ is tangent to $\odot C$. ABC is a right $\triangle$ because

$5^2 + 12^2 = 13^2$, and $\overline{CB} \perp \overleftrightarrow{AB}$.

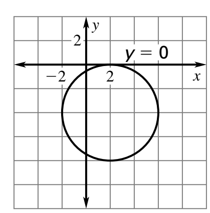
47. $\overleftrightarrow{AB}$ is tangent to $\odot C$. ABC is a right $\triangle$ as follows.

$24^2 + 7^2 = 25^2$

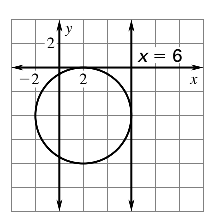
48. secant



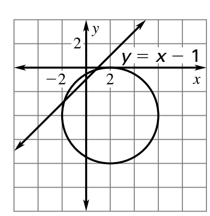
49. tangent



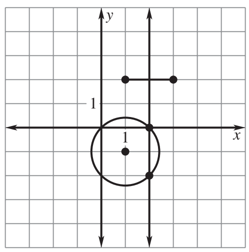
50. tangent



51. secant



52. The locus is all points on the $\perp$ bisector of the segment with endpoints (3, 2) and (1, 2) that are on or inside the circle with center $(-1, 1)$ and radius $\sqrt{2}$, all points $(2, y)$ where $-2 < y < 0$.



11.6 Quiz 2 (p.705)

1. $8.2 = \frac{68°}{360°} \cdot 2\pi r$

$43.41 \text{ m} \approx 2\pi r$

2. Length of $\widehat{AB} = \frac{88°}{360°} \cdot 26\pi$

≈ 19.97 in.

3. $24.6 \text{ ft} = \frac{138°}{360°} \cdot 2\pi r$

$24.6 \text{ ft}\left(\frac{360°}{138°}\right) \cdot \left(\frac{1}{2\pi}\right) = r$

$10.21 \text{ ft} \approx r$

4. $A = \pi r^2$

$= \pi(50)^2$

$\approx 7854.0 \text{ mi}^2$

5. $A = \frac{105°}{360°} \cdot \pi(7)^2$

$\approx 44.9 \text{ cm}^2$

6. $A = 2\left(\frac{145°}{360°}\right) \cdot \pi(10 \text{ ft})^2$

$\approx 253.07 \text{ ft}^2$

7. Area of triangle $= \frac{1}{4}\sqrt{3} \cdot 5^2 \approx 10.83$

Area of square $= (20)^2 = 400$

$P \approx \frac{10.83}{400}$

$\approx 2.7\%$

11.6 Technology Activity (p. 706)

Conjecture

1. This is the equation for the region inside the circle centered at the origin with radius 0.5.

2. Answers may vary. *Sample answer:* Experimental probability is approximately 27.5% with the graphing calculator program and theoretical probability is 19.6%.

3. Answers may vary.

4. The more trials there are, the closer the theoretical and experimental probabilities will be to each other.

Extension

Sample answer: A calculator simulation would be easier because it would not involve the physical actions of tossing and retrieving the darts and recording the results. It would be more accurate because it would not involve such variables as skill level, effort, or physical factors such as air currents.

Chapter 11 Review *(p. 708–710)*

1. measure of interior $\angle = \frac{(9-2)(180°)}{9} = 140°$

measure of exterior $\angle = 180° - 140° = 40°$

2. measure of interior $\angle = \frac{(13-2)(180°)}{13} \approx 152.3°$

measure of exterior $\angle \approx 180° - 152.3° \approx 27.7°$

3. measure of interior $\angle = \frac{(16-2)(180°)}{16} = 157.5°$

measure of exterior $\angle = 180° - 157.5° = 22.5°$

4. measure of interior $\angle = \frac{(24-2)(180°)}{24} = 165°$

measure of exterior $\angle = 180° - 165° = 15°$

5. $\frac{(n-2)(180°)}{n} = 172°$

$180°n - 360° = 172°n$

$8°n = 360°$

$n = 45$

6. $\frac{(n-2)(180°)}{n} = 135°$

$180°n - 360° = 135°n$

$45°n = 360°$

$n = 8$

7. $\frac{(n-2)(180°)}{n} = 150°$

$180°n - 360° = 150°n$

$30°n = 360°$

$n = 12$

8. $\frac{(n-2)(180°)}{n} = 170°$

$180°n - 360° = 170°n$

$10°n = 360°$

$n = 36$

9. $A = \frac{1}{4}\sqrt{3}s^2$

$= \frac{1}{4}\sqrt{3}(12)^2$

$\approx 62.35 \text{ cm}^2$

10. $S = \frac{6}{\tan 60°}$

$= 4\sqrt{3}$

$A = \frac{1}{4}\sqrt{3}s^2$

$= \frac{1}{4}\sqrt{3}(4\sqrt{3})^2$

$\approx 20.78 \text{ in.}^2$

11. $A = 6 \cdot \frac{1}{4}\sqrt{3}\ 5^2$

$= 6 \cdot \frac{1}{4}\sqrt{3}(5)^2$

$\approx 64.95 \text{ m}^2$

12. Measure of interior angle $= \frac{(10-2)180°}{10} = 144°$

$a = 0.75 \tan 72°$

$A = \frac{1}{2}aP$

$= \frac{1}{2}(0.75 \tan 72°)(10)(1.5)$

$\approx 17.31 \text{ ft}^2$

13. sometimes **14.** sometimes **15.** always

16. $\frac{\text{Perimeter } CDE}{\text{Perimeter } BDF} = \frac{3}{9} = \frac{1}{3}$

$\frac{\text{Area } CDE}{\text{Area } BDF} = \frac{1^2}{3^2} = \frac{1}{9}$

17. $\frac{\text{Perimeter } ADG}{\text{Perimeter } BDF} = \frac{15}{9} = \frac{5}{3}$

$\frac{\text{Area } ADG}{\text{Area } BDF} = \frac{5^2}{3^2} = \frac{25}{9}$

18. $C = 2\pi r$

$= 2\pi(4)$

$\approx 25.13 \text{ cm}$

$\text{Length of } \widehat{AB} = \frac{35°}{360°} \cdot 2\pi(4)$

$\approx 2.44 \text{ cm}$

19. $C = 2\pi r$

$= 2\pi(7.6)$

$\approx 47.75 \text{ m}$

$\text{Length of } \widehat{AB} = \frac{162°}{360°} \cdot 2\pi(7.6)$

$\approx 21.49 \text{ m}$

20. $C = 2\pi r$

$= 2\pi(12)$

$\approx 75.4 \text{ ft}$

$\text{Length of } \widehat{AB} = \frac{118°}{360°} \cdot 2\pi(12)$

$\approx 24.71 \text{ ft}$

21. $r = \frac{C}{2\pi}$

$= \frac{12}{2\pi}$

$\approx 1.91 \text{ in.}$

22. $d = \frac{C}{\pi}$

$= \frac{15\pi}{\pi}$

$= 15 \text{ m}$

23. $A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(10)^2$

$\approx 157.08 \text{ in.}^2$

24. $A = \frac{135°}{360°} \cdot \pi(5.5)^2$

$\approx 35.64 \text{ ft}^2$

25. $A = \pi(15)^2 - \pi(8)^2$

$\approx 505.8 \text{ cm}^2$

26. $A = \frac{300°}{360°} \cdot \pi(6)^2$

$\approx 94.25 \text{ cm}^2$

27. $A = \pi(14)^2$

$\approx 615.75 \text{ ft}^2$

28. $A = \pi r^2$

$\sqrt{(A/\pi)} = r$

$\sqrt{40/\pi} = r$

$3.57 \text{ in.} \approx r$

29. $P = \frac{\text{Length of } \overline{LM}}{\text{Length of } \overline{JN}} = \frac{12}{40} = \frac{3}{10} = 30\%$

30. $P = \frac{\text{Length of } \overline{JL}}{\text{Length of } \overline{JN}} = \frac{20}{40} = \frac{1}{2} = 50\%$

31. $P = \frac{\text{Length of } \overline{KM}}{\text{Length of } \overline{JN}} = \frac{24}{40} = \frac{3}{5} = 60\%$

32. Radius of a circle $= \frac{1}{2}\sqrt{8^2 + 6^2} = 5$

$P = \frac{\text{Area of rectangle}}{\text{Area of circle}}$

$= \frac{(8)(6)}{\pi(5)^2}$

$\approx \frac{48}{78.54}$

$\approx 0.611 \text{ or } 61.1\%$

33. $P = \frac{\text{Area shaded}}{\text{Area of semicircle}}$

$= \frac{2\pi r_1{}^2}{\frac{1}{2}\pi r_2{}^2}$

$= \frac{2\pi(6)^2}{\frac{1}{2}\pi(24)^2}$

$= \frac{1}{4}$

$= 25\%$

34. $P = \frac{\text{Area shaded}}{\text{Total area}}$

$= \frac{(7\sqrt{2})^2}{(14)^2}$

$= \frac{98}{196}$

$= \frac{1}{2}$

$= 50\%$

Chapter 11 Test *(p. 711)*

1. $x° + 120° + 135° + 90° + 115° + 120° = (6 - 2)(180°)$

$x° + 580° = 720°$

$x° = 140°$

2. $40° + 60° + 45° + 90° + 65° + 60° = 360°$

3. measure of interior $\angle = \frac{(n-2)(180°)}{n}$

$= \frac{(30-2)(180°)}{30}$

$= 168°$

4. meas. of exterior $\angle = \frac{360°}{27}$

$= 13\frac{1}{3}°$

5. $A = \frac{1}{4}\sqrt{3}s^2$

$= \frac{1}{4}\sqrt{3}(10)^2$

$\approx 43.30 \text{ ft}^2$

6. Measure of interior angle $= \frac{(5-2)(180°)}{5} = 108°$

$A = \frac{1}{2}aP$

$= \frac{1}{2}(8)(5)(2)\frac{8}{\tan 54°}$

$\approx 232.49 \text{ in.}^2$

7. $A = 6 \cdot \frac{1}{4}\sqrt{3}(9)^2$

$\approx 210.44 \text{ cm}^2$

8. Measure of interior angle $= \frac{(9-2)(180°)}{9} = 140°$

$A = \frac{1}{2}aP$

$= \frac{1}{2}(\cos 70°)(9)(2)(\sin 70°)$

$\approx 2.89 \text{ m}^2$

9. $\frac{\text{Perimeter } ABCD}{\text{Perimeter } EFGH} = \frac{8}{6} = \frac{4}{3}$

$\frac{\text{Area of } ABCD}{\text{Area of } EFGH} = \frac{4^2}{3^2} = \frac{16}{9}$

10. $\frac{56}{x} = \frac{16}{9}$

$16x = 504$

$x = 31.5 \text{ cm}^2$

11. $C = 2\pi r$

$= 2\pi(5)$

≈ 31.42 cm

$A = \pi r^2$

$= \pi(5)^2$

$\approx 78.54 \text{ cm}^2$

12. Length of $\widehat{AB} = \frac{m\,\widehat{AB}}{360°} \times 2\pi r$

$= \frac{105°}{360°} \cdot 2\pi(5)$

≈ 9.16 cm

13. Area sector $ARB = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \frac{105°}{360°} \cdot \pi(5)^2$

$\approx 22.91 \text{ cm}^2$

14. $A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(15)^2$

$\approx 353.43 \text{ ft}^2$

15. Area of sector $= \frac{245°}{360°} \cdot \pi(16)^2$

$\approx 547.34 \text{ in.}^2$

16. Area of sectors $= \frac{120°}{360°} \cdot \pi(7)^2$

$\approx 51.31 \text{ m}^2$

17. $P = \frac{\text{Area of circle}}{\text{Area of square}}$

$= \frac{\pi(10)^2}{(20)^2}$

≈ 0.785 or 78.5%

18. $P = \frac{\text{Area of triangle}}{\text{Area of square}}$

$= \frac{\frac{1}{2}(10)(10)}{20^2}$

$= \frac{50}{400}$

$= \frac{1}{8}$ or 12.5%

19. boat $= 2\pi(80) \approx 502.65$ ft

skiier $= 2\pi(110) \approx 691.15$ ft

$691.15 - 502.65 = 188.50$ ft

20. $P = \frac{20 \text{ min}}{120 \text{ min}} = \frac{1}{6} \approx 16.67\%$

Chapter 11 Standardized Test *(p. 712–713)*

1. C $\frac{(n-2)(180°)}{n} = 160°$

$180°n - 360° = 160°n$

$20°n = 360°$

$n = 18$

2. A $180° - x° = (7-2)(180°) - (130° + 128° + 133° + 139° + 110° + 136°)$

$180° - x° = 900° - 776°$

$180° - x° = 124°$

$56° = x°$

3. A Polygon a: $a = 15 \cos 36° \approx 12.14$

Polygon b: $a = 14 \cos 30° \approx 12.12$

4. A Polygon A: $P = 2(5)(15 \sin 36°) \approx 88.17$ units

Polygon B: $P = 6(14)2(6)(14 \sin 30°) \approx 84$ units

5. A Polygon A: $A \approx \frac{1}{2}(12.14)(88.17)$

≈ 535.19 sq. units

Polygon B: $A \approx \frac{1}{2}(12.12)(84)$

≈ 509.04 sq. units

6. A $\frac{\text{Area of smaller octagon}}{\text{Area of larger octagon}} = \frac{12^2}{18^2} = \frac{144}{324} = \frac{4}{9}$

7. B Length of $\widehat{AB} = \frac{86°}{360°} \cdot 2\pi(8.3)$

≈ 12.46 ft

8. E Area shaded $= \frac{(360° - 86°)}{360°} \cdot \pi(8.3)^2$

$\approx 164.72 \text{ ft}^2$

9. C Area shaded $= \frac{(90° + 30°)}{360°} \cdot \pi(8)^2$

$\approx 67.02 \text{ cm}^2$

10. C $P = \frac{\text{Area} \odot P - \text{Area} \odot Q}{\text{Area} \odot P}$

$= \frac{\pi(6)^2 - \pi(3)^2}{\pi(6)^2}$

$= 75\%$

11. A $P = \frac{24}{60} = \frac{2}{5} = 0.4$

12. $C = 2\pi(16)$

≈ 100.53 m

$A = \pi(16)^2$

$\approx 804.25 \text{ m}^2$

13. $\frac{(8 - 2)(180°)}{8} = 135°$

14. $180° - 135° = 45°$

15. $P = 8(2)(16 \sin 22.5°)$

≈ 97.97 m

$A = \frac{1}{2}(16 \cos 22.5°)(8)(2)(16 \sin 22.5°)$

$\approx 724.08 \text{ m}^2$

16. Length of $\widehat{AB} = \frac{45°}{360°} \cdot 2\pi(16)$

≈ 12.57 m

17. *Sample answer:* One method is to find the area of the circle and from that subtract the area of the octagon. A second method is to find the area of a sector. From that, subtract the area of a triangle. Then multiply by 8.

From problems 12–16, Area $\odot J \approx 804.25 \text{ m}^2$ and area octagon $\approx 724.08 \text{ m}^2$

Shaded area $\approx 804.25 - 724.08 \approx 80.17 \text{ m}^2$

Area sector $= \frac{45°}{360°} \cdot \pi(16)^2 \approx 100.53 \text{ m}^2$

Area triangle $= \frac{1}{2}(2)(16 \sin 22.5°)(16 \cos 22.5°)$

$\approx 90.51 \text{ m}^2$

Shaded Area $\approx 8(100.53 - 90.51)^2 \approx 80.16 \text{ m}^2$

Each method yields the same result.

18. $r = 3$ in.

$C = \pi(6) \approx 18.85$ in.

$A = \pi(3)^2 \approx 28.27 \text{ in.}^2$

19. $P = \frac{\text{Area of circle}}{\text{Area of dart board}} = \frac{\pi(3)^2}{(36)(24)} \approx 3.27\%$

20. $P = \frac{\text{Area of blue region}}{\text{Area of dart board}} = \frac{(22)(5)}{(36)(24)} = 12.73\%$

21. $P = \frac{\text{area green} + \text{area red} + \text{area blue}}{\text{area dart board}}$

$= \frac{\pi(3)^2 + 2\left(\frac{1}{2}\right)(9 + 6)(7) + (22)(5)}{(36)(24)}$

$\approx \frac{243.27}{864}$

$\approx 28.16\%$

22. $P \approx 100\% - 28.16\%$ (answer from question 21)

$\approx 71.84\%$

Chapter 11 Project (p. 714)

1. yes **2.** yes

3. No; any two great circles must intersect.

4. *Sample answer:* No. There are no parallel lines in geometry on a sphere.

5. Yes; for example, a line of longitude and the equator on a globe

6. Answers may vary. *Sample answer:* 90°, 90°, 90°

7. Answers may vary. The sum should be larger than 180°. *Sample answer:* 270°

8. Answers may vary. The angles will have measures between 60° and 180°. *Sample answer:* 90°, 90°, 90°

9. no; yes; yes

10. Answers may vary. The sum of the angles is between 180° and 540°.