CHAPTER 9

Lesson 9.1

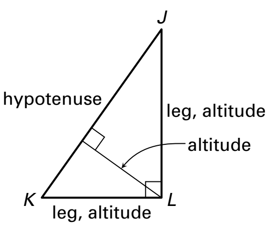
Think and Discuss (p. 525)

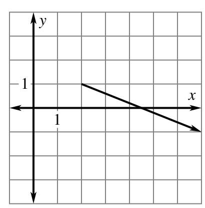
1. $\overline{AB}$ and $\overline{ED}$ are both $\perp$ to $\overline{BD}$; in a plane, 2 lines $\perp$ to the same line are $\|$.
2. According to Theorem 4.8, if the hypotenuse and leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, the two triangles are congruent.

Skill Review (p. 526)

1. $m\angle L = 180° - (30° + 60°) = 90°$

   $\triangle JKL$ is a right triangle.

2. 

3. 

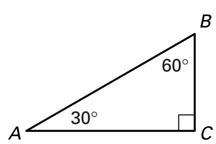
4. $\frac{x+3}{5} = \frac{x}{3}$

   $3(x+3) = 5x$

   $3x + 9 = 5x$

   $9 = 2x$

   $\frac{9}{2} = x$

5. 

   Two angles of $\triangle JKL$ are congruent to two angles of $\triangle ABC$, therefore $\triangle ABC \sim \triangle JKL$ by the AA Sim. Post.

Developing Concepts Activity (p. 527)

3. All of the triangles are similar.

9.1 Guided Practice (p. 531)

1. geometric mean **2.** KML; JMK **3.** $\overline{MK}$

4. JM **5.** JK **6.** LJ **7.** KM **8.** KM **9.** LK

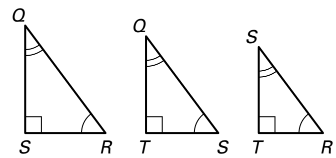
10. $\frac{97}{72} = \frac{72}{DC}$

$DC = 53.4$

$\frac{53.4}{FD} = \frac{FD}{43.6}$

$DF = 48.3$

9.1 Practice and Applications (pp. 531–534)

11. 

12. $\triangle SRQ \sim \triangle TSQ \sim \triangle TRS$

13. $\frac{x}{20} = \frac{20}{12}$

$12x = 400$

$x = 33\frac{1}{3}$

14. $\frac{4}{x} = \frac{x}{9}$

$36 = x^2$

$x = 6$

15. $\frac{5}{x} = \frac{x}{3}$

$15 = x^2$

$x = \sqrt{15}$

16. $\triangle ZYX \sim \triangle WYZ \sim \triangle WZX$; ZW

17. $\triangle SQR \sim \triangle TQS \sim \triangle TSR$; RQ

18. $\triangle GFE \sim \triangle HFG \sim \triangle HGE$; EH

19. $\triangle CBA \sim \triangle DBC \sim \triangle DCA$

$\frac{12}{16} = \frac{x}{12}$

$144 = 16x$

$x = 9$

20. $\triangle GEF \sim \triangle HGF \sim \triangle HEG$

$\frac{x}{20} = \frac{20}{25}$

$25x = 400$

$x = 16$

21. $\triangle LKJ \sim \triangle MLJ \sim \triangle MKL$

$\frac{x}{32} = \frac{32}{15}$

$15x = 1024$

$x \approx 68.3$

22. $\triangle RSQ \sim \triangle TRQ \sim \triangle TSR$

$\frac{x}{40} = \frac{40}{32}$

$32x = 1600$

$x = 50$

23. $\triangle CBA \sim \triangle DBC \sim \triangle DCA$

$\frac{x}{4} = \frac{4}{x}$

$x^2 = 16$

$x = 4$

24. $\triangle HGE \sim \triangle FHE \sim \triangle FGH$

$\frac{x}{7} = \frac{18}{x}$

$x^2 = 126$

$x = \sqrt{126} = 3\sqrt{14} \approx 11.2$

25. $\frac{x}{3} = \frac{9}{x}$

$x^2 = 27$

$x = 3\sqrt{3}$

26. $\frac{x}{12} = \frac{16}{20}$

$20x = 192$

$x = 9.6$

27. $\frac{5}{7} = \frac{m - 7}{5}$

$7m - 49 = 25$

$7m = 74$

$m = 10\frac{4}{7}$

28. $\frac{16}{14} = \frac{14}{c}$

$16c = 196$

$c = 12\frac{1}{4}$

$d = 16 - c$

$= 16 - 12\frac{1}{4}$

$= 3\frac{3}{4}$

$\frac{c}{e} = \frac{e}{d}$

$\frac{12\frac{1}{4}}{e} = \frac{e}{3\frac{3}{4}}$

$e^2 = \frac{49 \cdot 15}{4 \cdot 4}$

$e = \frac{7\sqrt{15}}{4}$

29. $\frac{24}{32} = \frac{32}{x}$

$24x = 1024$

$x = 42\frac{2}{3}$

$\frac{66\frac{2}{3}}{y} = \frac{y}{24}$

$y^2 = 1600$

$y = 40$

$\frac{32}{z} = \frac{24}{40}$

$24z = 1280$

$z = 53\frac{1}{3}$

30. $\frac{x + 9}{18} = \frac{8}{x + 9}$

$x^2 + 18x + 81 = 144$

$x^2 + 18x - 63 = 0$

$(x + 21)(x - 3) = 0$

$x + 21 = 0 \qquad x - 3 = 0$

$x = -21 \qquad x = 3$

Solution: $x = 3$

31. About 76 cm; $\triangle ABC$ and $\triangle ADC$ are congruent right triangles by the SSS Congruence Postulate, so $\overline{AC}$ is a perpendicular bisector of $\overline{BD}$. By Geometric Mean Theorem 9.3, the altitude from D to hypotenuse $\overline{AC}$ divides $\overline{AC}$ into segments of lengths about 23.7 cm and 61.1 cm. By Geometric Mean Theorem 9.2, the length of the altitude to the hypotenuse of each right triangle is about 38 cm long, so the crossbar $\overline{BD}$ should be about $2 \cdot 38$, or 76 cm long.

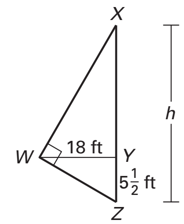
32. $xy = h - 5.5$

$\frac{xy}{wy} = \frac{wy}{zy}$

$\frac{h - 5.5}{18} = \frac{18}{5.5}$

$5.5h - 30.25 = 324$

$h \approx 64.4$ ft



Not drawn to scale

33. $\triangle DCB \sim \triangle DAC \sim \triangle CAB$

$\frac{CD}{2} = \frac{1.5}{2.5}$ $\quad$ $\frac{AC}{2} = \frac{2}{2.5}$ $\quad$ $\frac{DB}{1.5} = \frac{1.5}{2.5}$

$CD = 1.2$ m $\quad$ $AD = 1.6$ m $\quad$ $DB = 0.9$ m

Area of $\triangle DCB = \frac{1}{2}(1.2)(0.9)$

$= 0.54 \text{ m}^2$

Area of $\triangle DAC = \frac{1}{2}(1.6)(1.2)$

$= 0.96 \text{ m}^2$

Area of $\triangle CAB = \frac{1}{2}(2)(1.5)$

$= 1.5 \text{ m}^2$

34. Given $\triangle ABC$ is a right triangle and altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$; $\triangle DBC$ and $\triangle DCA$ are right triangles by the definition of right triangles; $\angle CDB \cong \angle ACB$ because all right angles are congruent; $\angle B \cong \angle B$ by reflexive property for angles; therefore $\triangle ACB \sim \triangle CDB$ by the AA Similarity Postulate; $\angle ADC \cong \angle ACB$ because all right angles are congruent; $\angle A \cong \angle A$ by the reflexive property for angles; therefore $\triangle ACB \sim \triangle ADC$ by the AA Similarity Postulate; $m\angle ACD + m\angle DCB = 90°$ by the Angle Addition Postulate; $m\angle DCB + m\angle B = 90°$ because the two acute angles in a right triangle are complementary; $m\angle ACD = m\angle B$ by the Transitive and Subtraction Properties of Equality, so $\angle ACD \cong \angle B$ by the def. of congruent ∠s; $\angle CDA \cong \angle CDB$ because all right angles are congruent; so $\triangle DCA \sim \triangle DBC$ by the AA Similarity Postulate.

35. From Ex. 34, $\triangle CBD \sim \triangle ACD$. Corresponding side lengths are in proportion, so $\frac{BD}{CD} = \frac{CD}{AD}$.

36. From Ex. 34, $\triangle ABC \sim \triangle CBD$ and $\triangle ABC \sim \triangle ACD$. Corresponding side lengths are in proportion, so $\frac{AB}{BC} = \frac{BC}{BD}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

37. Values of the ratios will vary, but will not be equal. The theorem says they are equal.

38. The ratios are equal.

39. The ratios are equal when the triangle is a right triangle but are not equal when the triangle is not a right triangle.

40. Using the right triangle, calculate the values of $\frac{AB}{CB}$, $\frac{CB}{DB}$, $\frac{AB}{AC}$, $\frac{AC}{AD}$. These proportions should be true: $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$. Now drag C to change the value of $m\angle C$ (so $m\angle C \neq 90$) and recalculate $\frac{AB}{CB}$, $\frac{CB}{DB}$, $\frac{AB}{AC}$, and $\frac{AC}{AD}$. The values of the ratios will vary but $\frac{AB}{CB} \neq \frac{CB}{DB}$ and $\frac{AB}{AC} \neq \frac{AC}{AD}$.

41. D

42. $\frac{DC}{12} = \frac{12}{24}$ $AD = 24 - 6$

$DC = \frac{144}{24}$ $AD = 18$ C

$DC = 6$

43. Method 1

Measure the distance from the ground to the person's eye level (DC) and the distance from the person to the building (AC). Use the proportion $\frac{BC}{AC} = \frac{AC}{DC}$ and solve for BC (the height of the building). One advantage of this method is you only need two measurements. One disadvantage is you need a friend to help.

Method 2

Measure the length of the building's shadow (QS), the height of the pole (NP) and the length of the pole's shadow (MP). Use the proportion $\frac{MP}{QS} = \frac{NP}{RS}$ and solve for RS (the height of the building). One advantage is it can be done by one person. One disadvantage is it must be done when the building and pole cast a shadow.

9.1 Mixed Review (p. 534)

44. $n^2 = 169$

$n = \pm 13$

45. $14 + x^2 = 78$

$x^2 = 64$

$x = \pm 8$

46. $d^2 + 18 = 99$

$d^2 = 81$

$d = \pm 9$

47. If the measure of one of the angles of a triangle is greater than 90°, then the triangle is obtuse; true.

48. If the corresponding angles of two triangles are congruent, then the two triangles are congruent; false.

49. $A = \frac{1}{2}(6)(12)$

$= 36 \text{ in.}^2$

50. $A = (7)(4.5)$

$= 31.5 \text{ cm}^2$

51. $A = \frac{1}{2}(12 + 13)(5)$

$= 62.5 \text{ m}^2$

Lesson 9.2

9.2 Guided Practice (p. 538)

1. *Sample answer:* In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

2. A, C

3. $2^2 + 1^2 = x^2$

$5 = x^2$

$\sqrt{5} = x$

no

4. $x^2 + 8^2 = 10^2$

$x^2 = 36$

$x = 6$

yes

5. $4^2 + x^2 = 8^2$

$x^2 = 48$

$x = 4\sqrt{3}$

no

6. $5^2 + d^2 = 6^2$

$d^2 = 11$

$d = \sqrt{11}$

≈ 3.3 ft

9.2 Practice and Applications (pp. 538–541)

7. $65^2 + 72^2 = x^2$

$4225 + 5184 = x^2$

$9409 = x^2$

$97 = x$

yes

8. $6^2 + x^2 = 9^2$

$36 + x^2 = 81$

$x^2 = 45$

$x = 3\sqrt{5}$

no

9. $39^2 + x^2 = 89^2$

$1521 + x^2 = 7921$

$x^2 = 6400$

$x = 80$

yes

10. $9^2 + 40^2 = x^2$

$81 + 1600 = x^2$

$1681 = x^2$

$41 = x$

yes

11. $7^2 + x^2 = 9^2$

$49 + x^2 = 81$

$x^2 = 32$

$x = 4\sqrt{2}$

no

12. $2^2 + 3^2 = x^2$

$4 + 9 = x^2$

$13 = x^2$

$\sqrt{13} = x$

no

13. $8^2 + x^2 = 16^2$

$64 + x^2 = 256$

$x^2 = 192$

$x = 8\sqrt{3}$

no

14. $20^2 + x^2 = 29^2$

$400 + x^2 = 841$

$x^2 = 441$

$x = 21$

yes

15. $14^2 + 14^2 = x^2$

$196 + 196 = x^2$

$392 = x^2$

$14\sqrt{2} = x$

no

16. $8^2 + x^2 = 16^2$

$64 + x^2 = 256$

$x^2 = 192$

$x = 8\sqrt{3}$

17. $6^2 + b^2 = 10^2$

$36 + b^2 = 100$

$b^2 = 64$

$b = 8$

$12 - 8 = 4$

$4^2 + 6^2 = x^2$

$16 + 36 = x^2$

$52 = x^2$

$2\sqrt{13} = x$

18. $3^2 + b^2 = 5^2$

$9 + b^2 = 25$

$b^2 = 16$

$b = 4$

$11 - 4 = 7$

$3^2 + 7^2 = x^2$

$9 + 49 = x^2$

$58 = x^2$

$\sqrt{58} = x$

19. $12^2 + 16^2 = t^2$

$144 + 256 = t^2$

$400 = t^2$

$20 = t$

20. $9^2 + 12^2 = t^2$

$81 + 144 = t^2$

$225 = t^2$

$15 = t$

21. $18^2 + s^2 = 30^2$

$324 + s^2 = 900$

$s^2 = 576$

$s = 24$

22. $20^2 + r^2 = 101^2$

$400 + r^2 = 10{,}201$

$r^2 = 9801$

$r = 99$

23. $35^2 + s^2 = 37^2$

$1225 + s^2 = 1369$

$s^2 = 144$

$s = 12$

24. $595^2 + r^2 = 757^2$

$354{,}025 + r^2 = 573{,}049$

$r^2 = 219{,}024$

$r = 468$

25. $9^2 + b^2 = 12^2$

$81 + b^2 = 144$

$b^2 = 63$

$b = 3\sqrt{7}$

$A = \frac{1}{2}(9)(3\sqrt{7})$

$= 35.7 \text{ cm}^2$

26. $5^2 + b^2 = 14^2$

$25 + b^2 = 196$

$b^2 = 171$

$b = 3\sqrt{19}$

$A = \frac{1}{2}(3\sqrt{19})(5)$

$= 32.7 \text{ m}^2$

27. $3.5^2 + b^2 = 8^2$

$12.25 + b^2 = 64$

$b^2 = 51.75$

$b = 7.2$

$A = \frac{1}{2}(7)(7.2)$

$= 25.2 \text{ cm}^2$

28. $a^2 + 4^2 = 5^2$

$a^2 = 25 - 16 = 9$

$a = 3 \text{ m}$

$4^2 + b^2 = 8.5^2$

$b^2 = 72.25 - 16 = 56.25$

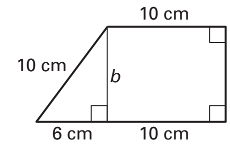
$b = 7.5 \text{ m}$

base $= a + b = 3 + 7.5 = 10.5$ m

$A = \frac{1}{2}(10.5)(4)$

$= 21 \text{ m}^2$

29.



$6^2 + b^2 = 10^2$

$b^2 = 100 - 36 = 64$

$b = 8$

$A = \frac{1}{2}(8)(10 + 16)$

$= 104 \text{ cm}^2$

30. $12^2 + b^2 = 13^2$

$144 + b^2 = 169$

$b^2 = 25$

$b = 5$

$d_1 = 12 + 8 = 20$

$d_2 = 5 + 5 = 10$

$A = \frac{1}{2}(20)(10)$

$= 100 \text{ m}^2$

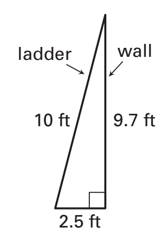
31. $65^2 + 65^2 = c^2$

$8450 = c^2$

$91.9 \text{ ft} = c$

Distance from pitcher's plate to home is 50 feet. The distance from second base to home is about 91.9 feet so the distance from second to the pitcher's plate is $91.9 - 50$ or about 41.9 feet.

32. The minimum distance of the base of the ladder from the wall is $\frac{10}{4}$ or 2.5 feet. The ladder, if placed 2.5 feet from the wall, will reach $\sqrt{100 - 6.25} \approx 9.7$ feet up the wall.



33. $3 \text{ ft} = 36 \text{ inches}$

$2 \text{ ft } 6 \text{ in.} = 30 \text{ inches}$

$36^2 + 15^2 = c^2$

$1296 + 225 = c^2$

$1521 = c^2$

$39 \text{ in.} = c$

$39 \text{ in.} + 39 \text{ in.} + 16 \text{ in.} = 94 \text{ in.}$

34. $300 \text{ ft} = 3600 \text{ in.}$

$300 \text{ ft } 1 \text{ in.} = 3601 \text{ in.}$

$3600^2 + h^2 = 3601^2$

$h^2 = 7201$

$h \approx 84.9 \text{ in.}$

35. $r = 3 \text{ in.}(4) + 6 \text{ in.}(2) + 12 \text{ in.}(2)$

$= 12 + 12 + 24$

$= 48 \text{ in.}$

36. $18^2 + 30^2 = r^2$

$324 + 900 = r^2$

$1224 = r^2$

$35.0 \text{ in.} \approx r$

Method 2 uses less ribbon.

37. The area of the large square is $(a + b)^2$. Also, the area of the large square is the sum of the areas of the four congruent right triangles plus the area of the small square, or $4\left(\frac{1}{2} \cdot a \cdot b\right) + c^2$. Thus, $(a + b)^2 = 4\left(\frac{1}{2} \cdot a \cdot b\right) + c^2$, and so $a^2 + 2ab + b^2 = 2ab + c^2$. Subtracting $2ab$ from each side gives $a^2 + b^2 = c^2$.

38. The area of the trapezoid is $\frac{1}{2}(a + b)^2$. Also, the area of the trapezoid is equal to the sum of the areas of the two congruent right triangles plus the area of the isosceles triangle or

$\left(\frac{1}{2} \cdot a \cdot b\right) + \left(\frac{1}{2} \cdot a \cdot b\right) + \left(\frac{1}{2} \cdot c^2\right)$. Thus

$\frac{1}{2}(a + b)^2 = a \cdot b + \frac{1}{2}c^2$, and so

$a^2 + 2ab + b^2 = 2ab + c^2$. Subtracting $2ab$ from each side gives $a^2 + b^2 = c^2$.

39. a. $AB = \sqrt{8^2 + 4^2}$

$= 4\sqrt{5}$

$BD = \sqrt{80 + 16}$

$= 9.8 \text{ ft}$

yes

b. $AB = \sqrt{144 + 64}$

$= 4\sqrt{13}$

$BD = \sqrt{208 + 100}$

$= 17.5 \text{ ft}$

No. The longest space in the room is the diagonal of the room which is only about 17.5 ft long.

c. $d = \sqrt{l^2 + w^2 + h^2}$

The length of the diagonal of the base is $\sqrt{l^2 + w^2}$. The length of the diagonal of the box is

$\sqrt{\sqrt{(l^2 + w^2)^2} + h^2} = \sqrt{l^2 + w^2 + h^2}$.

40. The length of one side of the rhombus is $\sqrt{\left(\frac{1}{2}a\right)^2 + \left(\frac{1}{2}b\right)^2}$ or $\frac{1}{2}\sqrt{a^2 + b^2}$. Multiplying the length of one side by 4 gives the perimeter of the rhombus, which is $4\left(\frac{1}{2}\sqrt{a^2 + b^2}\right)$ or $2\sqrt{a^2 + b^2}$.

41. $P = 2\sqrt{a^2 + b^2}$; $a = x$, $b = 0.75x$

$80 = 2\sqrt{x^2 + (0.75x)^2}$

$40 = \sqrt{x^2 + 0.5625x^2}$

$40 = \sqrt{1.5625x^2}$

$40 = 1.25x$

$32 = x$

$a = 32 \text{ cm}, b = 0.75(32) = 24 \text{ cm}$

9.2 Mixed Review (p. 541)

42. $\left(\sqrt{6}\right)^2 = 6$ **43.** $\left(\sqrt{9}\right)^2 = 9$ **44.** $\left(\sqrt{14}\right)^2 = 14$

45. $\left(2\sqrt{2}\right)^2 = 8$ **46.** $\left(4\sqrt{13}\right)^2 = 208$

47. $-\left(5\sqrt{49}\right)^2 = -1225$ **48.** $4\left(\sqrt{9}\right)^2 = 36$

49. $\left(-7\sqrt{3}\right)^2 = 147$ **50.** no **51.** no **52.** no **53.** no

54. yes

55. *Sample answer:* slope of $\overline{PQ} = -\frac{11}{2} =$ slope of $\overline{RS}$; slope of $\overline{QR} = \frac{5}{4} =$ slope of $\overline{PS}$. Both pairs of opposite sides are parallel, so $PQRS$ is a parallelogram by the definition of a parallelogram.

56. *Sample answer:* slope of $\overline{PQ} = -3 =$ slope of $\overline{RS}$; slope of $\overline{QR} = \frac{3}{8} =$ slope of $\overline{PS}$. Both pairs of opposites sides are parallel, so $PQRS$ is a parallelogram by the definition of a parallelogram.

Lesson 9.3

Activity 9.3 Investigating Sides and Angles of Triangles (p. 542)

Construct

Constructions may vary.

Investigate

Values in tables may vary.

Conjecture

4. $(AC)^2 + (BC)^2 = (AB)^2$ when $m\angle C = 90°$

$(AC)^2 + (BC)^2 < (AB)^2$ when $m\angle C > 90°$

$(AC)^2 + (BC)^2 > (AB)^2$ when $m\angle C < 90°$

9.3 Guided Practice (p. 545)

1. *Sample answer:* If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

2. acute: $c^2 < 24^2 + 18^2$

$c < 30$

right: $c^2 = 24^2 + 18^2$

$c = 30$

obtuse: $c^2 > 24^2 + 18^2$

$c > 30$

3. C **4.** D **5.** D **6.** A

7. No; the sum of $22^2 + 38^2 = 1928$, while $45^2 = 2025$. Since the two numbers are not equal, the triangles formed by the crossbars and the sides are not right triangles so the crossbars are not perpendicular.

9.3 Practice and Applications (pp. 546–548)

8. $97^2 \underline{\ ?\ } 65^2 + 72^2$

$9409 = 9409$

right

9. $89^2 \underline{\ ?\ } 80^2 + 39^2$

$7921 = 7921$

right

10. $23^2 \underline{\ ?\ } 20.8^2 + 10.5^2$

$529 < 542.89$

not right

11. $\left(\sqrt{26}\right)^2 \underline{\ ?\ } 1^2 + 5^2$

$26 = 26$

right

12. $(3\sqrt{3})^2 \underline{\ ?\ } 2^2 + 5^2$

$27 < 29$; not right

13. $(4\sqrt{35})^2 \underline{\ ?\ } 20^2 + 13^2$

$560 < 569$; not right

14. $20^2 + 99^2 \underline{\ ?\ } 101^2$

$400 + 9801 \underline{\ ?\ } 10{,}201$

$10{,}201 = 10{,}201$; right

15. $21^2 + 28^2 \underline{\ ?\ } 35^2$

$441 + 784 \underline{\ ?\ } 1225$

$1225 = 1225$; right

16. $10^2 + 17^2 \underline{\ ?\ } 26^2$

$100 + 289 \underline{\ ?\ } 676$

$389 < 676$; obtuse

17. not a triangle

18. $4^2 + (\sqrt{67})^2 \underline{\ ?\ } 9^2$

$16 + 67 \underline{\ ?\ } 81$

$83 > 81$; acute

19. $(\sqrt{13})^2 + 6^2 \underline{\ ?\ } 7^2$

$13 + 36 \underline{\ ?\ } 49$

$49 = 49$; right

20. $16^2 + 30^2 \underline{\ ?\ } 34^2$

$256 + 900 \underline{\ ?\ } 1156$

$1156 = 1156$; right

21. $10^2 + 11^2 \underline{\ ?\ } 14^2$

$100 + 121 \underline{\ ?\ } 196$

$221 > 196$; acute

22. $4^2 + 5^2 \underline{\ ?\ } 5^2$

$16 + 25 \underline{\ ?\ } 25$

$41 > 25$; acute

23. $17^2 + 144^2 \underline{\ ?\ } 145^2$

$289 + 20{,}736 \underline{\ ?\ } 21{,}025$

$21{,}025 = 21{,}025$; right

24. $10^2 + 49^2 \underline{\ ?\ } 50^2$

$100 + 2401 \underline{\ ?\ } 2500$

$2501 > 2500$; acute

25. $(\sqrt{5}) + 5^2 \underline{\ ?\ } 5.5^2$

$5 + 25 \underline{\ ?\ } 30.25$

$30 < 30.25$; obtuse

26. Rectangle; the quadrilateral has two pairs of congruent opposite sides. Each triangle formed by either diagonal is a right triangle because in each case $14^2 + 8^2 = (2\sqrt{65})^2$; $260 = 260$. Therefore, the quadrilateral has four right angles. The quadrilateral is a rectangle.

27. Square; the diagonals bisect each other, so the quadrilateral is a parallelogram. The diagonals are congruent, so the parallelogram is a rectangle. $1^2 + 1^2 = (\sqrt{2})^2$, so the diagonals intersect at right angles to form perpendicular lines; thus, the parallelogram is also a rhombus. A quadrilateral that is both a rectangle and a rhombus is a square.

28. Rhombus; the diagonals bisect each other so the quadrilateral is a parallelogram. $3^2 + 4^2 = 5^2$, so the diagonals intersect at right angles to form perpendicular lines so the parallelogram is a rhombus.

29. slope of $\overline{AC} = \dfrac{6-3}{4-0} = \dfrac{3}{4}$;

slope of $\overline{BC} = -\dfrac{7-3}{3-0} = -\dfrac{4}{3}$;

Since $\left(\dfrac{3}{4}\right)\left(-\dfrac{4}{3}\right) = -1$, $\overline{AC} \perp \overline{BC}$, so $\angle ABC$ is a right angle. Therefore, $\triangle ABC$ is a right triangle by the definition of a right triangle.

30. distance from B to $C = \sqrt{(-3-0)^2 + (7-3)^2}$

$= 5$

distance from B to $A = \sqrt{(-3-4)^2 + (7-6)^2}$

$= 5\sqrt{2}$

distance from A to $C = \sqrt{(4-0)^2 + (6-3)^2}$

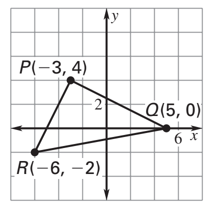
$= 5$

$5^2 + 5^2 = (5\sqrt{2})^2$

$25 + 25 = 50$; therefore $\triangle ABC$ is a right triangle by the Converse of the Pythagorean Theorem.

31. Computing slopes is easier because it involves two calculations, not three. Computing slopes also does not involve square roots.

32.



$PQ = \sqrt{(-3-5)^2 + (4-0)^2}$

$= 4\sqrt{5}$

$PR = \sqrt{(-3+6)^2 + (4+2)^2}$

$= 3\sqrt{5}$

$RQ = \sqrt{(-6-5)^2 + (-2-0)^2}$

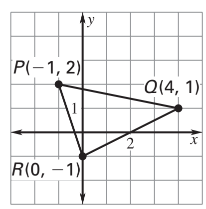
$= 5\sqrt{5}$

$(4\sqrt{5})^2 + (3\sqrt{5})^2 \underline{\ ?\ } (5\sqrt{5})^2$

$80 + 45 = 125$

The triangle is a right triangle.

33.



$PQ = \sqrt{(-1-4)^2 + (2-1)^2}$

$= \sqrt{26}$

$PR = \sqrt{(-1-0)^2 + (2+1)^2}$

$= \sqrt{10}$

$RQ = \sqrt{(0-4)^2 + (-1-1)^2}$

$= \sqrt{17}$

$(\sqrt{10})^2 + (\sqrt{17})^2 \underline{\ ?\ } (\sqrt{26})^2$

$27 > 26$

The triangle is an acute triangle.

34. Since $2^2 + 3^2 < 4^2$, $\triangle ABC$ is obtuse and $\angle ABC$ is obtuse. $\angle 1$ and $\angle ABC$ are a linear pair and are therefore supplementary. By the definition of supplementary angles, $m\angle ABC + m\angle 1 = 180°$. Since $\angle ABC$ is obtuse, $m\angle ABC > 90°$. Therefore, $m\angle 1 < 90°$. $\angle 1$ is an acute angle by definition of an acute angle.

35. Since $(\sqrt{10})^2 + 2^2 < 4^2$, $\triangle ABC$ is obtuse and $\angle C$ is obtuse. By the Triangle Sum Theorem, $m\angle A + m\angle ABC + m\angle C = 180°$. $\angle C$ is obtuse, so $m\angle C > 90°$. It follows that $m\angle ABC < 90°$. Vertical angles are congruent, so $m\angle ABC = m\angle 1$. By substitution, $m\angle 1 < 90°$. By the definition of an acute angle, $\angle 1$ is acute.

36. If a, b, and c are a Pythagorean triple, then $a^2 + b^2 = c^2$. Let k represent a positive integer. Multiplying both sides of the equation by k^2 gives the equation $k^2(a^2 + b^2) = k^2c^2$, or $k^2a^2 + k^2b^2 = k^2c^2$ by the Distributive Property. So $(ka)^2 + (kb)^2 = (kc)^2$ by a property of powers. Since $k > 0$, ka, kb, and kc represent the side lengths of a right triangle by the Converse of the Pythagorean Theorem.

37. A, C, D **38.** rectangle

39. $169^2 \overset{?}{=} 119^2 + 120^2$ $6649^2 \overset{?}{=} 4800^2 + 4601^2$

$28{,}561 = 28{,}561$ ✓ $44{,}209{,}201 = 44{,}209{,}201$ ✓

$18{,}541^2 \overset{?}{=} 13{,}500^2 + 12{,}709^2$

$343{,}768{,}681 = 343{,}768{,}681$ ✓

40. $714^2 \overset{?}{=} 599^2 + 403^2$

$509{,}796 < 521{,}210$ so the $\triangle$ is acute.

Cincinnati is not directly north of Tallahassee. It is northwest of Tallahassee.

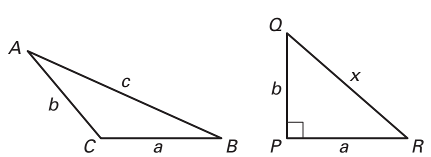
41.

Reasons
1. Pythagorean Theorem
2. Given
3. Substitution property of equality
5. Converse of the Hinge Theorem
6. Given, def. of right angle, def. of acute angle, and substitution property of equality
7. Def. of acute triangle ($\angle C$ is the largest angle of $\triangle ABC$.)

42. Given: In $\triangle ABC$, $c^2 > a^2 + b^2$

Prove: $\triangle ABC$ is an obtuse triangle.

Plan for Proof: Draw right triangle PQR with side lengths a, b, and hypotenuse x. Compare lengths c and x.



Statements	Reasons
1. $x^2 = a^2 + b^2$	**1.** Pythagorean Theorem
2. $c^2 > a^2 + b^2$	**2.** Given
3. $c^2 > x^2$	**3.** Substitution prop. of equality
4. $c > x$	**4.** A property of square roots
5. $m\angle C > m\angle P$	**5.** Converse of Hinge Thm.
6. $\angle C$ is an obtuse angle.	**6.** Given, def. of rt. angle, def. of obtuse angle, subst. prop. of equality
7. $\triangle ABC$ is an obtuse triangle.	**7.** Def. of obtuse triangle ($\angle C$ is the largest $\angle$ of $\triangle ABC$.)

43.

Statements	Reasons
1. $x^2 = a^2 + b^2$	**1.** Pythagorean Theorem
2. $c^2 = a^2 + b^2$	**2.** Given
3. $c^2 = x^2$	**3.** Substitution prop. of equality
4. $c = x$	**4.** A property of square roots
5. $m\angle N = m\angle R$	**5.** Converse of Hinge Thm.
6. $\angle N$ is a right angle.	**6.** Given, def. of rt. angle, def. of obtuse angle, subst. prop. of equality
7. $\triangle LMN$ is a right triangle.	**7.** Def. of right triangle ($\angle N$ is the largest $\angle$.)

44. $77^2 + 36^2 \overset{?}{=} 85^2$ $82^2 + 40^2 \overset{?}{=} 91^2$

$5629 + 1296 \overset{?}{=} 7225$ $6724 + 1600 \overset{?}{=} 8281$

$6925 < 7225$ $8324 > 8281$

$\triangle ABC$ is obtuse $\triangle DEF$ is acute

$\angle A$ is obtuse $\angle D$ is acute

A

45. $m\angle A > 90°$, so $m\angle B + m\angle C < 90°$

$m\angle D < 90°$, so $m\angle E + m\angle F > 90°$

B

46.

Statements	Reasons
1. $\overline{NP}$ is an altitude.	**1.** Given
2. $\overline{NP} \perp \overline{MQ}$	**2.** Def. of altitude
3. $\angle MPN$ and $\angle QPN$ are right angles.	**3.** Def. of perpendicular
4. $\triangle MPN$ and $\triangle QPN$ are right triangles.	**4.** Def. of right triangle
5. $(MN)^2 = s^2 + t^2$, $(NQ)^2 = r^2 + t^2$	**5.** Pythagorean Theorem
6. $(MN)^2 + (NQ)^2 = s^2 + t^2 + r^2 + t^2 = s^2 + 2t^2 + r^2$	**6.** Addition and Substitution properties of equality
7. t is the geometric mean of r and s.	**7.** Given
8. $\frac{r}{t} = \frac{t}{s}$	**8.** Def. of geometric mean
9. $t^2 = rs$	**9.** Cross product prop.
10. $(MN)^2 + (NQ)^2 = s^2 + 2(rs) + r^2 = (r + s)^2$	**10.** Substitution prop. of equality
11. $r + s = MQ$	**11.** Given (diagram)
12. $(MN)^2 + (NQ)^2 = (MQ)^2$	**12.** Substitution prop. of equality
13. $\triangle MQN$ is a right triangle.	**13.** Converse of the Pythagorean Thm.

9.3 Mixed Review (p. 549)

47. $\sqrt{22} \cdot \sqrt{2} = \sqrt{44} = 2\sqrt{11}$

48. $\sqrt{6} \cdot \sqrt{8} = \sqrt{48} = 4\sqrt{3}$

49. $\sqrt{14} \cdot \sqrt{6} = \sqrt{84} = 2\sqrt{21}$

50. $\sqrt{15} \cdot \sqrt{6} = \sqrt{90} = 3\sqrt{10}$

51. $\frac{3}{\sqrt{11}} = \frac{3\sqrt{11}}{11}$ **52.** $\frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$

53. $\frac{12}{\sqrt{18}} = \frac{12}{3\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

54. $\frac{8}{\sqrt{24}} = \frac{8}{2\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

55. an enlargement with center C and scale factor $\frac{7}{4}$

56. reduction with center C and scale factor $\frac{3}{5}$

57. $5x = x + 36$ $2y = y + 11$

$4x = 36$ $y = 11$

$x = 9$

Quiz 1 (p. 549)

1. $\triangle CDB \sim \triangle BDA \sim \triangle CBA$ **2.** $\overline{BD}$

3. $\frac{9}{15} = \frac{15}{AC}$

$9AC = 225$

$AC = 25$

4. $\frac{9}{15} = \frac{BD}{20}$

$180 = 15BD$

$BD = 12$

5. $3^2 + x^2 = 7^2$

$x = 2\sqrt{10}$

6. $x^2 + 12^2 = 18^2$

$x = 6\sqrt{5}$

7. $x^2 + 6^2 = 18^2$

$x = 12\sqrt{2}$

8. $219^2 \stackrel{?}{=} 168^2 + 140^2$

$47{,}961 > 47{,}824$

No; the square of the longest side is larger than the sum of the squares of the smaller sides.

Lesson 9.4

Activity 9.4 Developing Concepts (p. 550)

Exploring the Concept

1. Triangles may vary.

2. side length 3 cm: $3^2 + 3^2 = c^2$

$c = 3\sqrt{2}$ cm

side length 4 cm: $4^2 + 4^2 = c^2$

$c = 4\sqrt{2}$ cm

side length 5 cm: $5^2 + 5^2 = c^2$

$c = 5\sqrt{2}$ cm

Conjecture

3. The length of the hypotenuse is the product of the length of one side and $\sqrt{2}$.

Exploring the Concept

4. Triangles may vary.

6. triangle with side length 4 cm: side lengths: 2 cm, 4 cm, $2\sqrt{3}$ cm

triangle with side length 6 cm: side lengths: 3 cm, 6 cm, $3\sqrt{3}$ cm

triangle with side length 8 cm: side lengths: 4 cm, 8 cm, $4\sqrt{3}$ cm

Conjecture

7. $\frac{\text{hypotenuse}}{\text{shorter leg}}: \frac{4}{2} = 2; \frac{6}{3} = 2; \frac{8}{4} = 2$

$\frac{\text{longer leg}}{\text{shorter leg}}: \frac{2\sqrt{3}}{2} = \sqrt{3}; \frac{3\sqrt{3}}{3} = \sqrt{3}; \frac{4\sqrt{3}}{4} = \sqrt{3}$

ratio of hypotenuse:longer leg:shorter leg $= 2:\sqrt{3}:1$

9.4 Guided Practice (p. 554)

1. Right triangles with angle measures $45°-45°-90°$ and $30°-60°-90°$

2. According to the AA Similarity Postulate, since two angles of one triangle are congruent to two angles of the other triangle, the two triangles are similar.

3. true   **4.** false   **5.** false   **6.** true   **7.** true

8. true   **9.** $x = 4\sqrt{2}$ **10.** $a = 2; b = 2\sqrt{3}$

11. $h = k$

$9 = \sqrt{2}k$

$\frac{9}{\sqrt{2}} = k$

$\frac{9\sqrt{2}}{2} = k = h$

9.4 Practice and Applications (pp. 554–556)

12. $x = 5; y = 5\sqrt{2}$ **13.** $a = 12\sqrt{3}; b = 24$

14. $e = 2\sqrt{2}$

15. $d \cdot \sqrt{2} = 8$

$d = 4\sqrt{2}$

$c = 4\sqrt{2}$

16. $c = 5; d = 5\sqrt{3}$

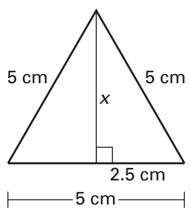
17. $q = 16\sqrt{2}; r = 16$ **18.** $m = 12; p = 6\sqrt{3}$

19. $f \cdot \sqrt{3} = 8$

$f = \frac{8\sqrt{3}}{3}$

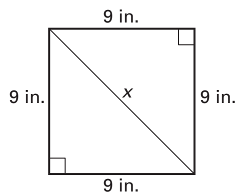
$h = \frac{16\sqrt{3}}{3}$

20. $n = 6$

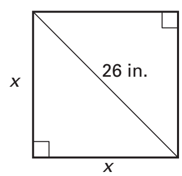
21. 

$2.5^2 + x^2 = 5^2$
$6.25 + x^2 = 25$
$x^2 = 18.75$
$x \approx 4.3$ cm

22. $4s = 36$
$s = 9$ in.
$9^2 + 9^2 = x^2$
$81 + 81 = x^2$
$162 = x^2$
$12.7 \text{ in.} \approx x$



23. $x^2 + x^2 = 26^2$
$2x^2 = 676$
$x^2 = 338$
$x \approx 18.4$ in.



24. $A = \frac{1}{2}(4\sqrt{3})(8) \approx 27.7 \text{ ft}^2$

25. $A = \frac{1}{2}(6)(6\sqrt{3}) \approx 31.2 \text{ ft}^2$

26. $A = 5(2\sqrt{3}) \approx 17.3 \text{ m}^2$

27. $A = 6\left(\frac{1}{2}(4)(2\sqrt{3})\right) \approx 41.6 \text{ ft}^2$

28. $x = \sqrt{3}$ cm ≈ 1.7 cm **29.** $x = (1.4)(\sqrt{2}) \approx 2.0$ cm

30. $y = 2\sqrt{3} \approx 3.5$ cm; $x = 2 \cdot 2 = 4$ cm

31. $r = \sqrt{2}$ $t = 2$ $v = \sqrt{6}$
$s = \sqrt{3}$ $u = \sqrt{5}$ $w = \sqrt{7}$

I used the Pythagorean theorem in each right triangle, working from left to right.

32. Going from left to right: triangle 1

33. Going from left to right: triangle 3

34. $\sqrt{n + 1}$

35. Let $DF = x$. Then $EF = x$. By the Pythagorean Theorem, $x^2 + x^2 = (DE)^2$; $2x^2 = (DE)^2$; $DE = \sqrt{2x^2} = \sqrt{2} \cdot x$ by a property of square roots. Thus the hypotenuse is $\sqrt{2}$ times as long as a leg.

36. Construct $\overline{CD}$ on $\overrightarrow{BC}$ so that $CD = BC = a$. Then $\triangle ADC \cong \triangle ABC$ by the SAS Cong. Post. $\angle B \cong \angle D$ and $\angle BAC \cong \angle CAD$, because they are corresponding parts of congruent triangles. Therefore $m\angle D = 60°$ and $m\angle CAD = 30°$. $m\angle BAD = m\angle BAC + m\angle CAD = 30° + 30° = 60°$. $\triangle BAD$ is equiangular so it is also equilateral. Since it is equilateral, $AB = 2a$. If $BC = a$ and $AB = 2a$, then $AC = \sqrt{(2a)^2 - a^2} = \sqrt{3} \cdot a$. The side lengths are in the following ratio: hypotenuse:longer leg: shorter leg $= 2a:\sqrt{3} \cdot a:a$. Therefore, in a $30°-60°-90°$ triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

37. C **38.** A; $6 + 6\sqrt{3} + 12 \approx 28.4$ cm

39. Stage 1: $x^2 + x^2 = 1^2$
$2x^2 = 1$
$x^2 = \frac{1}{2}$
$x = \frac{1}{\sqrt{2}}$

Stage 2: $x^2 + x^2 = \left(\frac{1}{\sqrt{2}}\right)^2$
$2x^2 = \frac{1}{2}$
$x^2 = \frac{1}{4}$
$x = \frac{1}{2}$

Stage 3: $x^2 + x^2 = \left(\frac{1}{2}\right)^2$
$2x^2 = \frac{1}{4}$
$x^2 = \frac{1}{8}$
$x = \frac{1}{2\sqrt{2}}$

Stage 4: $x^2 + x^2 = \left(\frac{1}{2\sqrt{2}}\right)^2$
$2x^2 = \frac{1}{8}$
$x^2 = \frac{1}{16}$
$x = \frac{1}{4}$

40. The pattern of the lengths is $\frac{1}{\sqrt{2^n}}$, where $n =$ the number of the stage.

41. $\frac{1}{\sqrt{2^8}} = \frac{1}{16}$ Substitute 8 for n into the formula $\frac{1}{\sqrt{2^n}}$ and simplify.

9.4 Mixed Review (p. 557)

42. Let $x =$ length of the third side; $14 + 9 > x$; $x + 9 > 14$; $5 \text{ cm} < x < 23 \text{ cm}$

43. $Q'(-1, 2)$ **44.** $P'(-8, 3)$ **45.** $A'(-4, -5)$

46. $B'(0, -10)$ **47.** AA Similarity Postulate

48. SAS Similarity Theorem **49.** SSS Similarity Theorem

Math & History

1. area of triangles: $4\left(\frac{1}{2}ab\right) = 2ab$

area of square: $(b - a)^2 = b^2 - 2ab + a^2$

2. $2ab + b^2 - 2ab + a^2 = c^2$
$a^2 + b^2 = c^2$

Lesson 9.5

9.5 Guided Practice (p. 562)

1. $\sin A = \frac{BC}{AB}$ $\cos A = \frac{AC}{AB}$ $\tan A = \frac{BC}{AC}$

2. The value of a trigonometric ratio depends only on the measure of the acute angle, not on the particular right triangle used to compute the value.

3. $\frac{4}{5}$ **4.** $\frac{3}{5}$ **5.** $\frac{4}{3}$ **6.** $\frac{3}{5}$ **7.** $\frac{4}{5}$ **8.** $\frac{3}{4}$

9. $\sin 25° = \frac{7}{d}$

$d \approx 16.6$, or about 17 ft

9.5 Practice and Applications (pp. 562–565)

10. $\sin R = \frac{45}{53} \approx 0.8491$ $\cos R = \frac{28}{53} \approx 0.5283$

$\tan R = \frac{45}{28} \approx 1.6071$ $\sin S = \frac{28}{53} \approx 0.5283$

$\cos S = \frac{45}{53} \approx 0.8491$ $\tan S = \frac{28}{45} \approx 0.6222$

11. $\sin B = \frac{6}{10} = 0.6$ $\cos B = \frac{8}{10} = 0.8$

$\tan B = \frac{6}{8} = 0.75$ $\sin A = \frac{8}{10} = 0.8$

$\cos A = \frac{6}{10} = 0.6$ $\tan A = \frac{8}{6} \approx 1.3333$

12. $\sin X = \frac{3}{\sqrt{13}} \approx 0.8321$ $\cos X = \frac{2}{\sqrt{13}} \approx 0.5547$

$\tan X = \frac{3}{2} = 1.5$ $\sin Y = \frac{2}{\sqrt{13}} \approx 0.5547$

$\cos Y = \frac{3}{\sqrt{13}} \approx 0.8321$ $\tan Y = \frac{2}{3} \approx 0.6667$

13. $\sin D = \frac{7}{25} = 0.28$ $\cos D = \frac{24}{25} = 0.96$

$\tan D = \frac{7}{24} \approx 0.2917$ $\sin F = \frac{24}{25} = 0.96$

$\cos F = \frac{7}{25} = 0.28$ $\tan F = \frac{24}{7} \approx 3.4286$

14. $\sin G = \frac{2}{\sqrt{5}} \approx 0.8944$ $\cos G = \frac{1}{\sqrt{5}} \approx 0.4472$

$\tan G = \frac{2}{1} = 2$ $\sin H = \frac{1}{\sqrt{5}} \approx 0.4472$

$\cos H = \frac{2}{\sqrt{5}} \approx 0.8944$ $\tan H = \frac{1}{2} = 0.5$

15. $\sin J = \frac{5}{\sqrt{34}} \approx 0.8575$ $\cos J = \frac{3}{\sqrt{34}} \approx 0.5145$

$\tan J = \frac{5}{3} \approx 1.6667$ $\sin K = \frac{3}{\sqrt{34}} \approx 0.5145$

$\cos K = \frac{5}{\sqrt{34}} \approx 0.8575$ $\tan K = \frac{3}{5} = 0.6$

16. 0.7431 **17.** 0.9744 **18.** 6.3138 **19.** 0.4540
20. 0.3420 **21.** 0.0349 **22.** 0.9781 **23.** 0.8090
24. 0.4245 **25.** 0.4540 **26.** 0.8290 **27.** 2.2460

28. $\tan 37° = \frac{6}{y}$ $\sin 37° = \frac{6}{x}$

$y \approx 8.0$ $x \approx 10.0$

29. $\sin 23° = \frac{t}{34}$ $\cos 23° = \frac{s}{34}$

$t \approx 13.3$ $s \approx 31.3$

30. $\cos 36° = \frac{4}{r}$ $\tan 36° = \frac{s}{4}$

$r \approx 4.9$ $s \approx 2.9$

31. $\sin 65° = \frac{t}{8}$ $\cos 65° = \frac{u}{8}$

$t \approx 7.3$ $u \approx 3.4$

32. $\sin 70° = \frac{9}{v}$ $\tan 70° = \frac{9}{w}$

$v \approx 9.6$ $w \approx 3.3$

33. $\sin 22° = \frac{6}{x}$ $\tan 22° = \frac{6}{y}$

$x \approx 16.0$ $y \approx 14.9$

34. $A = \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) = 4 \text{ cm}^2$

35. $A = \frac{1}{2}(12)(6.9) \approx 41.6 \text{ m}^2$

36. $A = \frac{1}{2}(11)(6 \cdot 3) \approx 34.9 \text{ m}^2$

37. $\tan 13° = \frac{h}{58.2}$

$h \approx 13.4$ m

38. $\tan 42° = \frac{d}{40}$

$d \approx 36.0$ m

39. vertical drop, $x = 5500 - 5018 = 482$ ft

$\sin 20° = \frac{482}{d}$

≈ 1409.3 ft

40. $\tan 55° = \frac{d}{500}$

$d \approx 714.1$ m

41. $\sin 45° = \frac{30}{26 + x}$

$x = \frac{30}{\sin 45°} - 26$

$x \approx 16.4$ in.

42. $\tan 20° = \frac{x}{8}$

$x \approx 2.9$ ft

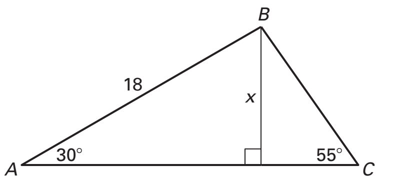
43. $\sin A = \frac{a}{c}$ $\cos A = \frac{b}{c}$ $\tan A = \frac{a}{b}$

$\sin B = \frac{b}{c}$ $\cos B = \frac{a}{c}$ $\tan B = \frac{b}{a}$

44. The tangent of one acute angle of a right triangle is the reciprocal of the tangent of the other acute angle. The sine of one acute angle of a right triangle is the same as the cosine of the other acute angle and the cosine of one acute angle of a right triangle is the same as the sine of the other acute angle.

45. Procedures may vary. One method is to reason that since the tangent ratio is equal to the ratio of the lengths of the legs, the tangent is equal to 1 when the legs are equal in length, that is, when the triangle is a $45°-45°-90°$ triangle. Tan $A > 1$ when $m\angle A > 45°$, and tan $A < 1$ when $m\angle A < 45°$, since increasing the measure of $\angle A$ increases the length of the opposite leg and decreasing the measure of $\angle A$ decreases the length of the opposite leg.

46. $\triangle ABC$ is not a right triangle, so you cannot use the trigonometric ratios.



$\sin 30° = \dfrac{x}{18}$ $\qquad$ $\sin 55° = \dfrac{9}{BC}$

$x = 9$ $\qquad$ $BC \approx 11.0$

47.

Reasons
1. Given
2. Pythagorean Theorem
3. Division property of equality
5. Substitution property of equality

48. $\sin 30° = \dfrac{1}{2}$ $\qquad$ $\cos 30° = \dfrac{\sqrt{3}}{2}$

$$(\sin 30°)^2 + (\cos 30°)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} + \frac{3}{4} = 1$$

49. $\sin 45° = \dfrac{\sqrt{2}}{2}$ $\qquad$ $\cos 45° = \dfrac{\sqrt{2}}{2}$

$$(\sin 45°)^2 + (\cos 45°)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$$
$$= \frac{2}{4} + \frac{2}{4} = 1$$

50. $\sin 60° = \dfrac{\sqrt{3}}{2}$ $\qquad$ $\cos 60° = \dfrac{1}{2}$

$$(\sin 60°)^2 + (\cos 60°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{3}{4} + \frac{1}{4} = 1$$

51. $\sin 13° \approx 0.2250$ $\qquad$ $\cos 13° \approx 0.9744$

$$(\sin 13°)^2 + (\cos 13°)^2 \approx (0.2250)^2 + (0.9744)^2$$
$$\approx 1$$

52.

Statements	**Reasons**
1. $\triangle ABC$ is a right triangle with side lengths *a*, *b*, and hypotenuse *c*.	**1.** Given
2. $\tan A = \dfrac{a}{b}$	**2.** Def. of tangent
3. $\cos A = \dfrac{b}{c}$; $\sin A = \dfrac{a}{c}$	**3.** Def. of sine and cosine
4. $\dfrac{\sin A}{\cos A} = \dfrac{\frac{a}{c}}{\frac{b}{c}} = \dfrac{a}{b}$	**4.** Subst. prop of equality and dividing and simplifying fractions
5. $\tan A = \dfrac{\sin A}{\cos A}$	**5.** Transitive property of equality

53. $\sin 25° = \dfrac{8}{CD}$; D $\qquad$ **54.** C

55. $\tan 53° = \dfrac{x}{60}$ $\qquad$ $\tan 29° = \dfrac{y}{60}$

$x \approx 79.62$ $\qquad$ $y \approx 33.26$

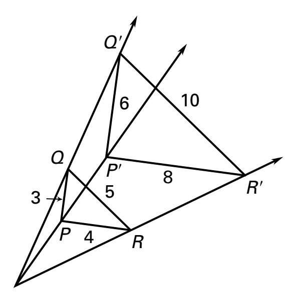
$x - y = h$

$79.62 - 33.26 \approx h$

$h \approx 46$ ft

9.5 Mixed Review (p. 566)

56.



enlargement;

scale factor $= \dfrac{6}{3}$

$= 2$

$Q'R' = 10$

$P'R' = 8$

57. $\triangle MNP \sim \triangle NQP \sim \triangle MQN$

$\dfrac{7}{15} = \dfrac{QP}{7}$ $\qquad$ $\dfrac{18.27}{NP} \approx \dfrac{NP}{3.27}$

$49 = 15(QP)$ $\qquad$ $(NP)^2 \approx 59.7429$

$QP \approx 3.27$ $\qquad$ $NP \approx 7.73$

58. $x^2 + 95^2 = 193^2$

$x^2 + 9025 = 37{,}249$

$x^2 = 28{,}224$

$x = 168$

yes

59. $x^2 + 50^2 = 65^2$

$x^2 + 2500 = 4225$

$x^2 = 1725$

$x \approx 5\sqrt{69}$

no

60. $42.9^2 + 70^2 = x^2$

$1840.41 + 4900 = x^2$

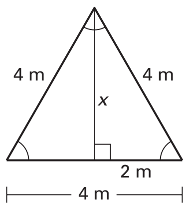
$6740.41 = x^2$

$82.1 = x$

no

Quiz 2 (p. 566)

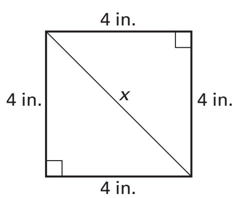
1.



$x = 2\sqrt{3}$ m

≈ 3.5 m

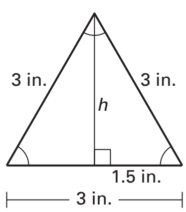
2.



$x = 4\sqrt{2}$ in.

≈ 5.7 in.

3.



$h = 1.5\sqrt{3}$ in. $A \approx \frac{1}{2}(3)(1.5\sqrt{3})$

≈ 3.9 in.^{2}

4. $\sin 40° = \frac{10}{x}$ $\quad$ $\tan 40° = \frac{10}{y}$

$x \approx 15.6$ $\quad$ $y \approx 11.9$

5. $\cos 62° = \frac{x}{18}$ $\quad$ $\sin 62° = \frac{y}{18}$

$x \approx 8.5$ $\quad$ $y \approx 15.9$

6. $\tan 25° = \frac{x}{20}$ $\quad$ $\cos 25° = \frac{20}{y}$

$x \approx 9.3$ $\quad$ $y \approx 22.1$

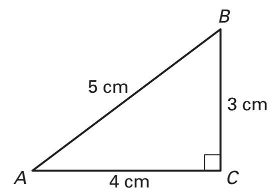
7. $\tan 11° = \frac{950}{d}$

$d \approx 4887.3$ ft

Lesson 9.6

Activity 9.6 Developing Concepts (p. 567)

1.



2. $\sin A = \frac{3}{5} = 0.6$ $\quad$ $\cos A = \frac{4}{5} = 0.8$ $\quad$ $\tan A = \frac{3}{4} = 0.75$

3. $\sin^{-1} 0.6 \approx 36.9°$ $\quad$ $\cos^{-1} 0.8 \approx 36.9°$ $\quad$ $\tan^{-1} 0.75 \approx 36.9°$

4. The values are approximately equal.

9.6 Guided Practice (p. 570)

1. To solve a right triangle is to find the measures of all angles and the lengths of all sides of the triangle.

2. true **3.** false **4.** $m\angle A \approx 35.0°$ **5.** $m\angle A \approx 79.5°$

6. $m\angle A \approx 64.2°$ **7.** $m\angle A \approx 84.3°$

8. $33^2 + 56^2 = c^2$ $\quad$ $\sin A = \frac{56}{65}$ $\quad$ $\sin B = \frac{33}{65}$

$4225 = c^2$ $\quad$ $m\angle A \approx 59.5°$ $\quad$ $m\angle B \approx 30.5°$

$65 = c$

9. $91^2 + d^2 = 109^2$ $\quad$ $\sin D = \frac{60}{109}$ $\quad$ $\sin E = \frac{91}{109}$

$d^2 = 3600$ $\quad$ $m\angle D \approx 33.4°$ $\quad$ $m\angle E \approx 56.6°$

$d = 60$

10. $\sin 60° = \frac{y}{4}$ $\quad$ $\cos 60° = \frac{x}{4}$ $\quad$ $m\angle X = 30°$

$y \approx 3.5$ $\quad$ $x = 2$

9.6 Practice and Applications (pp. 570–572)

11. $48^2 + 55^2 = QS^2$

$5329 = QS^2$

$73 = QS$

12. $\sin Q = \frac{55}{73}$

$m\angle Q \approx 48.9°$

13. $\sin S = \frac{48}{73}$

$m\angle S \approx 41.1°$

14. $m\angle A \approx 26.6°$ **15.** $m\angle A = 45°$ **16.** $m\angle A = 30°$

17. $m\angle A \approx 20.5°$ **18.** $m\angle A \approx 81.4°$ **19.** $m\angle A \approx 50.2°$

20. $m\angle A \approx 65.6°$ **21.** $m\angle A \approx 6.3°$

22. $20^2 + 21^2 = AB^2$ $\quad \sin B = \frac{20}{29}$ $\quad \sin A = \frac{21}{29}$

$841 = AB^2$ $\quad m\angle B \approx 43.6°$ $\quad m\angle A \approx 46.4°$

$29 = AB$

23. $7^2 + 7^2 = DE^2$ $\quad \sin E = \frac{7}{\sqrt{98}}$ $\quad \sin D = \frac{7}{\sqrt{98}}$

$98 = DE^2$ $\quad m\angle E = 45°$ $\quad m\angle D = 45°$

$9.9 \approx DE$

24. $2^2 + 6^2 = GH^2$ $\quad \sin G = \frac{6}{\sqrt{40}}$ $\quad \sin H = \frac{2}{\sqrt{40}}$

$40 = GH$ $\quad m\angle G \approx 71.6°$ $\quad m\angle H \approx 18.4°$

$6.3 \approx GH$

25. $8^2 + ML^2 = 9.2^2$ $\quad \sin L = \frac{8}{9.2}$ $\quad \cos K = \frac{8}{9.2}$

$ML^2 = 20.64$ $\quad m\angle L \approx 60.4°$ $\quad m\angle K \approx 29.6°$

$ML \approx 4.5$

26. $4^2 + NQ^2 = 13.6^2$ $\quad \sin N = \frac{4}{13.6}$ $\quad \cos P = \frac{4}{13.6}$

$NQ^2 = 168.96$ $\quad m\angle N \approx 17.1°$ $\quad m\angle P \approx 72.9°$

$NQ \approx 13.0$

27. $6^2 + TS^2 = 12.5^2$ $\quad \sin S = \frac{6}{12.5}$ $\quad \cos R = \frac{6}{12.5}$

$TS^2 = 120.25$ $\quad m\angle S \approx 28.7°$ $\quad m\angle R \approx 61.3°$

$TS \approx 11.0$

28. $\sin 26° = \frac{q}{4.5}$ $\quad \cos 26° = \frac{p}{4.5}$

$q \approx 2.0$ $\quad p \approx 4.0$

$m\angle P = 90° - 26°$

$m\angle P = 64°$

29. $\sin 20° = \frac{s}{12}$ $\quad \cos 20° = \frac{t}{12}$

$s \approx 4.1$ $\quad t \approx 11.3$

$m\angle T = 90° - 20°$

$m\angle T = 70°$

30. $\tan 52° = \frac{x}{8.5}$ $\quad \cos 52° = \frac{8.5}{z}$

$x \approx 10.9$ $\quad z \approx 13.8$

$m\angle Y = 90° - 52°$

$m\angle Y = 38°$

31. $\cos 56° = \frac{5}{c}$ $\quad \tan 56° = \frac{a}{5}$

$c \approx 8.9$ $\quad a \approx 7.4$

$m\angle B = 90° - 56° = 34°$

32. $\tan 51° = \frac{d}{3}$ $\quad \cos 51° = \frac{3}{e}$

$d \approx 3.7$ $\quad e \approx 4.8$

$m\angle F = 90° - 51°$

$m\angle F = 39°$

33. $\sin 34° = \frac{4}{m}$ $\quad \tan 34° = \frac{4}{\ell}$

$m \approx 7.2$ $\quad \ell \approx 5.9$

$m\angle L = 90° - 34°$

$m\angle L = 56°$

34. $\tan B = \frac{69}{36}$

≈ 1.9167

35. $\tan^{-1} B = \frac{69}{36}$

$m\angle B \approx 62.4°$

36. $69^2 + 36^2 = AB^2$

$6057 = AB^2$

$77.8 \text{ in.} \approx AB$

37. $\sin A = \frac{BC}{AB} = \frac{36}{\sqrt{6057}}$

$\sin A \approx 0.4626$

38. $\tan x = \frac{4855}{17{,}625}$

$x \approx 15.4°$

39. $240^2 = 17^2 + x^2$

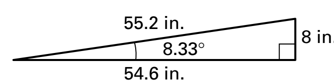
$57{,}311 = x^2$

$x \approx 239.4$ in. or 19 ft 11 in.

$\sin y = \frac{17}{240}$

$y \approx 4.1°$

40.



41. Answers may vary.

42. $\tan x = \frac{7}{11}$

$x \approx 32.5°$

43. $\tan x = \frac{8.25}{9}$

$x \approx 42.5°$

44. *Sample answer:* riser length: 6 in.; tread length: 12 in.

$\tan x = \frac{6}{12}$

$x \approx 26.6°$

45. *Sample answer:* The riser-to-tread ratio affects the safety of the stairway in several ways. First, the deeper the tread the more of a person's foot can fit on the step. This makes a person less likely to fall. Also, the smaller the angle of inclination the less steep the stairway. This makes the stairs less tiring to climb, and therefore, safer.

46. Draw an altitude, $\overline{CD}$, from C to $\overline{AB}$, and let $CD = h$. In rt. $\triangle ACD$, $\sin A = \frac{h}{b}$. In rt. $\triangle BCD$, $\sin B = \frac{h}{a}$. Thus, $h = b \cdot \sin A$ and $h = a \cdot \sin B$. By the substitution prop. of equality, $b \cdot \sin A = a \cdot \sin B$. Dividing both sides by $\sin A \cdot \sin B$ gives $\frac{b}{\sin B} = \frac{a}{\sin A}$, or $\frac{a}{\sin A} = \frac{b}{\sin B}$.

9.6 Mixed Review (p. 572)

47. $\langle 3, 2\rangle$ **48.** $\langle -2, 2\rangle$ **49.** $\langle -1, -3\rangle$

50. $\langle 1, 0\rangle$ **51.** $\langle 1, -2\rangle$ **52.** $\langle 3, 1\rangle$

53. $\frac{x}{30} = \frac{5}{6}$

$6x = 150$

$x = 25$

54. $\frac{7}{16} = \frac{49}{y}$

$7y = 784$

$y = 112$

55. $\frac{3}{10} = \frac{g}{42}$

$10g = 126$

$g = 12.6$

56. $\frac{7}{18} = \frac{84}{k}$

$7k = 1512$

$k = 216$

57. $\frac{m}{2} = \frac{7}{1}$

$m = 14$

58. $\frac{8}{t} = \frac{4}{11}$

$88 = 4t$

$t = 22$

59. not a triangle

60. $228^2 \ \underline{?}\ 220^2 + 60^2$

$51{,}984 < 52{,}000$

acute

61. $8.5^2 \ \underline{?}\ 7.7^2 + 3.6^2$

$72.25 = 72.25$

right

62. $263^2 \ \underline{?}\ 250^2 + 80^2$

$69{,}169 > 68{,}900$

obtuse

63. $113^2 \ \underline{?}\ 112^2 + 15^2$

$12{,}769 = 12{,}769$

right

64. not a triangle

Lesson 9.7

9.7 Guided Practice (p. 576)

1. The magnitude of a vector is the distance from its initial point to its terminal point. The direction of a vector is the angle it makes with a horizontal line.

2. $\overrightarrow{AB}$: $\langle -2, -2\rangle$; $\overrightarrow{PQ}$: $\langle 3, 3\rangle$; $\overrightarrow{MN}$: $\langle 0, -3\rangle$; $\overrightarrow{UV}$: $\langle 0, 2\rangle$

3. $\overrightarrow{UV}$ is parallel to $\overrightarrow{MN}$; $\overrightarrow{AB}$ is parallel to $\overrightarrow{PQ}$

4. $(2, 2)$

5. $\langle 4, 5\rangle$ $|\overrightarrow{AB}| = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{41} \approx 6.4$

6. $\langle -4, -2\rangle$ $|\overrightarrow{PQ}| = \sqrt{(5-1)^2 + (4-2)^2} = 2\sqrt{5} \approx 4.5$

7. $\langle 2, -5\rangle$ $|\overrightarrow{MN}| = \sqrt{(-3+1)^2 + (4+1)^2} = \sqrt{29} \approx 5.4$

8. $\tan x = \frac{5}{4}$

$x \approx 51.3°$ north of east

9. $\langle 4, 5\rangle + \langle -4, -2\rangle = \langle 0, 3\rangle$

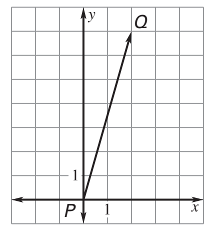
9.7 Practice and Applications (pp. 576–579)

10. $\langle 4, 1\rangle$ $|\overrightarrow{RS}| = \sqrt{(1-5)^2 + (1-2)^2} = \sqrt{17} \approx 4.1$

11. $\langle -3, 6\rangle$ $|\overrightarrow{JK}| = \sqrt{(2+1)^2 + (-2-4)^2} = 3\sqrt{5} \approx 6.7$

12. $\langle -4, -4\rangle$ $|\overrightarrow{EF}| = \sqrt{(0+4)^2 + (-1+5)^2} = 4\sqrt{2} \approx 5.7$

13.



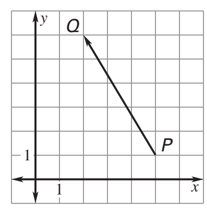
$\langle 2, 7\rangle$

$|\overrightarrow{PQ}| = \sqrt{(2-0)^2 + (7-0)^2}$

$= \sqrt{53}$

≈ 7.3

14.



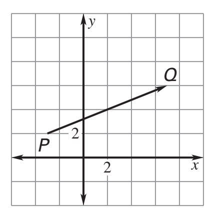
$\langle -3, 5\rangle$

$|\overrightarrow{PQ}| = \sqrt{(5-2)^2 + (1-6)^2}$

$= \sqrt{34}$

≈ 5.8

15.



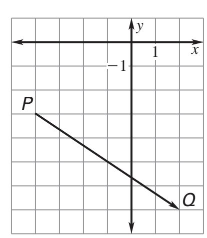
$\langle 10, 4\rangle$

$|\overrightarrow{PQ}| = \sqrt{(-3-7)^2 + (2-6)^2}$

$= \sqrt{116}$

≈ 10.8

16.

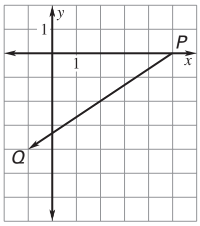


$\langle 6, -4\rangle$

$|\overrightarrow{PQ}| = \sqrt{(-4-2)^2 + (-3+7)^2}$

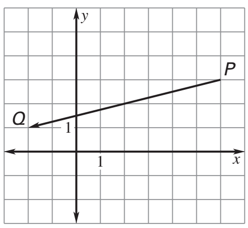
$= \sqrt{52}$

≈ 7.2

17. 

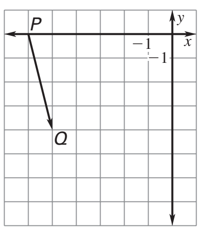
$\langle -6, -4 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(5 + 1)^2 + (0 + 4)^2}$
$= \sqrt{52}$
≈ 7.2

18. 

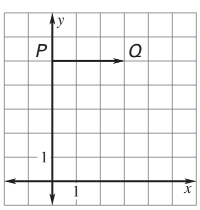
$\langle -8, -2 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(6 + 2)^2 + (3 - 1)^2}$
$= \sqrt{68}$
≈ 8.2

19. 

$\langle 1, -4 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(-6 + 5)^2 + (0 + 4)^2}$
$= \sqrt{17}$
≈ 4.1

20. 

$\langle 3, 0 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(0 - 3)^2 + (5 - 5)^2}$
$= \sqrt{9}$
$= 3$

21. (0, 20) and (60, 30)

$|\overrightarrow{ST}| = \sqrt{(60 - 0)^2 + (30 - 20)^2}$
≈ 61 miles per hour

$\tan x = \frac{10}{60}$

$x \approx 9.5°$ north of east

22. (0, 0) and (40, −50)

$|\overrightarrow{AB}| = \sqrt{(40 - 0)^2 + (-50 - 0)^2}$
≈ 64 miles per hour

$\tan x = \frac{50}{40}$

$x \approx 51.3°$ south of east

23. (−10, 10) and (−50, 50)

$|\overrightarrow{LM}| = \sqrt{(-10 + 50)^2 + (10 - 50)^2}$
≈ 57 miles per hour

$\tan x = \frac{40}{40}$

$x \approx 45°$ north of west

24. (0, 0) and (−50, −40)

$|\overrightarrow{OP}| = \sqrt{(0 + 50)^2 + (0 + 40)^2}$
≈ 64 miles per hour

$\tan x = \frac{40}{50}$

$x \approx 38.7°$ south of west

25. $\overrightarrow{EF}$, $\overrightarrow{CD}$, and $\overrightarrow{AB}$

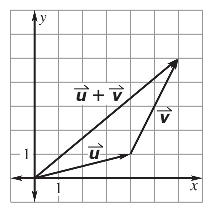
26. $\overrightarrow{EF}$ and $\overrightarrow{CD}$

27. $\overrightarrow{EF}$ and $\overrightarrow{CD}$

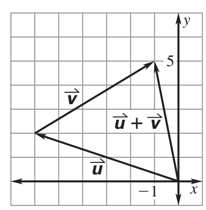
28. $\overrightarrow{GH}$ and $\overrightarrow{JK}$

29. yes; no

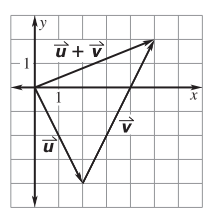
30. Round 2; the vectors have the same magnitude and opposite directions. In Round 1, team A won; since $\overrightarrow{CA}$ has a greater magnitude than $\overrightarrow{CB}$, $\overrightarrow{CA}$ represents a greater force applied.

31. 

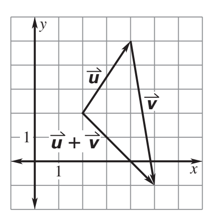
$\vec{u}$: $\langle 4, 1 \rangle$
$\vec{v}$: $\langle 2, 4 \rangle$
$\vec{u} + \vec{v} = \langle 6, 5 \rangle$

32. 

$\vec{u}$: $\langle -6, 2 \rangle$
$\vec{v}$: $\langle 5, 3 \rangle$
$\vec{u} + \vec{v} = \langle -1, 5 \rangle$

33. 

$\vec{u}$: $\langle 2, -4 \rangle$
$\vec{v}$: $\langle 3, 6 \rangle$
$\vec{u} + \vec{v} = \langle 5, 2 \rangle$

34. 

$\vec{u}$: $\langle 2, 3 \rangle$
$\vec{v}$: $\langle 1, -6 \rangle$
$\vec{u} + \vec{v} = \langle 3, -3 \rangle$

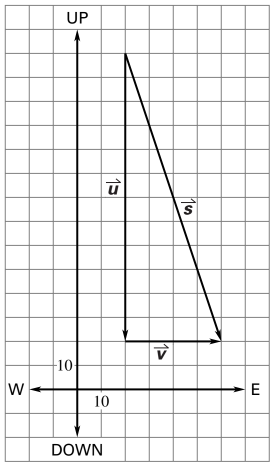
35. $\langle 4, 11 \rangle$ **36.** $\langle 8, 7 \rangle$ **37.** $\langle 10, 10 \rangle$

38. $\langle -2, -3 \rangle$ **39.** $\langle 4, -4 \rangle$ **40.** $\langle 0, 0 \rangle$

41. $\vec{u}$: $\langle 0, -120 \rangle$

$\vec{v}$: $\langle 40, 0 \rangle$

42.



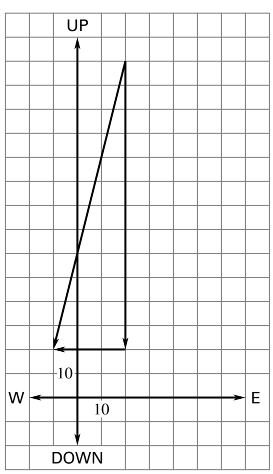
43. $|\vec{s}| = \sqrt{(20 - 60)^2 + (140 - 20)^2} \approx 126.5$ mi/h

the speed at which the skydiver is falling, taking into account the breeze.

44. $\tan x = \dfrac{120}{40}$

$x \approx 71.6°$

45.



The new velocity is $\langle -30, -120 \rangle$.

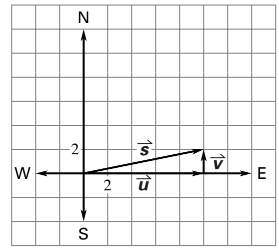
46. $|\overrightarrow{JK}| = \sqrt{10}$

Sample answer: $\overrightarrow{AB} = \langle 3, 1 \rangle$

The component form must give the same magnitude as $\overrightarrow{JK}$. $|\overrightarrow{AB}| = \sqrt{10}$.

47. When $k > 0$, the magnitude of $\vec{v}$ is k times the magnitude of $\vec{u}$ and the directions are the same. When $k < 0$, the magnitude of $\vec{v}$ is $|k|$ times the magnitude of $\vec{u}$ and the direction of $\vec{v}$ is opposite the direction of $\vec{u}$. Justifications may vary.

48. a.



b. $|\vec{s}| = \sqrt{(10 - 0)^2 + (2 - 0)^2} \approx 10.2$ mi/h

$\tan x = \dfrac{2}{10}$

$x \approx 11.3°$ north of east

c. Answers may vary.

49. $\overrightarrow{AB}$: $\langle 54, 24 \rangle$

$\overrightarrow{BC}$: $\langle -36, 36 \rangle$

$\overrightarrow{AB} + \overrightarrow{BC} = \langle 18, 60 \rangle$

50. $\overrightarrow{CA}$: $\langle -18, -60 \rangle$

$\langle -18, -60 \rangle + \langle 18, 60 \rangle = \langle 0, 0 \rangle$

51. $|\overrightarrow{AB}| = \sqrt{54^2 + 24^2} \approx 59.1$ ft

$|\overrightarrow{BC}| = \sqrt{(-36)^2 + (36)^2} \approx 50.9$ ft

$|\overrightarrow{CA}| = \sqrt{(-18)^2 + (-60)^2} \approx 62.6$ ft

Total distance $\approx 59.1 + 50.9 + 62.6 = 172.6$ ft

52. The answer to Ex. 50 is a vector which gives the final position of the bumper car, while the answer to Ex. 51 is a number which gives the total distance traveled by the bumper car.

53. Since $\angle D$ and $\angle E$ are right angles and all right angles are congruent, $\angle D \cong \angle E$. Since $\triangle ABC$ is equilateral, $\overline{AB} \cong \overline{BC}$. $\overline{DE} \parallel \overline{AC}$, so $\angle DBA \cong \angle BAC$ and $\angle EBC \cong \angle BCA$ by the Alternate Interior Angles Theorem. An equilateral triangle is also equiangular, so $m\angle BAC = m\angle BCA = 60°$. By the definition of congruent angles and the substitution property of equality, $\angle DBA \cong \angle EBC$. $\triangle ADB \cong \triangle CEB$ by the AAS Congruence Theorem. Corresponding parts of congruent triangles are congruent, so $\overline{DB} \cong \overline{EB}$. By the definition of midpoint, B is the midpoint of $\overline{DE}$.

54. $x = 45$, $y = 90$ **55.** $x = 120$, $y = 30$ **56.** $x = 30$, $y = 60$

57. $(x + 1)^2 = x^2 + 2x + 1$

58. $(x + 7)^2 = x^2 + 14x + 49$

59. $(x + 11)^2 = x^2 + 22x + 121$

60. $(7 + x)^2 = 49 + 14x + x^2$

Quiz 3 (p. 580)

1. $\sin 25° = \frac{b}{46}$ $\cos 25° = \frac{a}{46}$

$b \approx 19.4$ $a \approx 41.7$

$m\angle A = 90° - 25° = 65°$

2. $\sin 45° = \frac{12}{z}$ $\tan 45° = \frac{12}{y}$

$z \approx 17.0$ $y = 12$

$m\angle Y = 45°$

3. $\tan 40° = \frac{m}{16}$ $\cos 40° = \frac{16}{q}$

$m \approx 13.4$ $q \approx 20.9$

$m\angle N = 90° - 40° = 50°$

4. $\sin 75° = \frac{p}{8}$ $\cos 75° = \frac{q}{8}$

$p \approx 7.7$ $q \approx 2.1$

$m\angle Q = 90° - 75° = 15°$

5. $\sin G = \frac{6}{7.6}$ $6^2 + f^2 = 7.6^2$

$m\angle G \approx 52.1°$ $f^2 = 21.76$

$f \approx 4.7$

$m\angle F \approx 90° - 52.1° \approx 37.9°$

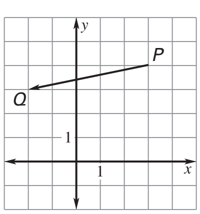
6. $\sin K = \frac{3}{12.4}$ $3^2 + \ell^2 = 12.4^2$

$m\angle K \approx 14.0°$ $\ell^2 = 144.76$

$\ell \approx 12.0$

$m\angle L \approx 90° - 14.0° \approx 76.0°$

7.

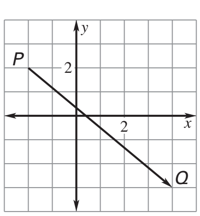


$\overrightarrow{PQ}: \langle -5, -1 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(3 + 2)^2 + (4 - 3)^2}$

≈ 5.1

8.

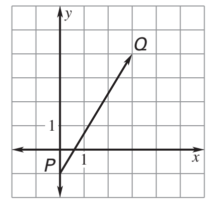


$\overrightarrow{PQ}: \langle 6, -5 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(-2 - 4)^2 + (2 + 3)^2}$

≈ 7.8

9.

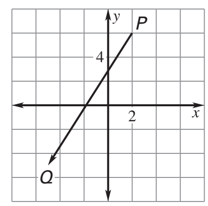


$\overrightarrow{PQ}: \langle 3, 5 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(3 - 0)^2 + (4 + 1)^2}$

≈ 5.8

10.

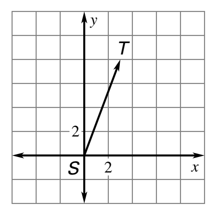


$\overrightarrow{PQ}: \langle -7, -11 \rangle$

$|\overrightarrow{PQ}| = \sqrt{(2 + 5)^2 + (6 + 5)^2}$

≈ 13.0

11.



$\tan x = \frac{8}{3}$

$x \approx 69.4°$ north of east

12. $\langle 4, 2 \rangle$ **13.** $\langle 2, 4 \rangle$ **14.** $\langle -2, -8 \rangle$

15. $\langle 2, 1 \rangle$ **16.** $\langle 6, 13 \rangle$ **17.** $\langle 0, 3 \rangle$

Chapter 9 Review (pp. 582–584)

9.1 Similar Right Triangles

1. $\frac{6}{x} = \frac{9}{6}$ $\frac{5}{y} = \frac{y}{9}$

$36 = 9x$ $y^2 = 45$

$x = 4$ $y = 3\sqrt{5}$

2. $\frac{25}{x} = \frac{x}{9}$ $\frac{y}{16} = \frac{9}{y}$

$x^2 = 225$ $y^2 = 144$

$x = 15$ $y = 12$

3. $\frac{36}{27} = \frac{x}{36}$ $48 - 27 = y$ $\frac{z}{27} = \frac{21}{z}$

$27x = 1296$ $y = 21$ $z^2 = 567$

$x = 48$ $z = 9\sqrt{7}$

≈ 23.8

9.2 The Pythagorean Theorem

4. $12^2 + 16^2 = t^2$

$400 = t^2$

$20 = t$

yes

5. $8^2 + s^2 = 12^2$

$s^2 = 80$

$s = 4\sqrt{5} \approx 8.9$

no

6. $r^2 + 16^2 = 34^2$
$r^2 = 900$
$r = 30$
yes

7. $4^2 + 6^2 = t^2$
$52 = t^2$
$t = 2\sqrt{13} \approx 7.2$
no

9.3 The Converse of the Pythagorean Theorem

8. $10^2 \ \underline{?}\ 6^2 + 7^2$
$100 > 85$
obtuse

9. $41^2 \ \underline{?}\ 40^2 + 9^2$
$1681 = 1681$
right

10. not a triangle

11. $9^2 \ \underline{?}\ 3^2 + (4\sqrt{5})^2$
$81 < 89$
acute

9.4 Special Right Triangles

12. hypotenuse $= \sqrt{2}(3\sqrt{2}) = 6$

13. leg $= \dfrac{6}{\sqrt{2}} = 3\sqrt{2}$

$P = 4(3\sqrt{2})$
$= 12\sqrt{2}$ in.

$A = (3\sqrt{2})^2$
$= 18$ in.^{2}

14. shorter leg $= \dfrac{1}{2}(12) = 6$ in.; longer leg $= 6\sqrt{3}$ in.

15. altitude $= 9\sqrt{3}$ cm

$A = \dfrac{1}{2}(18)(9\sqrt{3}) = 81\sqrt{3} \approx 140.3$ cm^2

9.5 Trigonometric Ratios

16. $\sin J = \dfrac{11}{61} \approx 0.1803$ $\quad \cos J = \dfrac{60}{61} \approx 0.9836$

$\tan J = \dfrac{11}{60} \approx 0.1833$ $\quad \sin L = \dfrac{60}{61} \approx 0.9836$

$\cos L = \dfrac{11}{61} \approx 0.1803$ $\quad \tan L = \dfrac{60}{11} \approx 5.4545$

17. $\sin P = \dfrac{35}{37} \approx 0.9459$ $\quad \cos P = \dfrac{35}{37} \approx 0.3243$

$\tan P = \dfrac{35}{12} \approx 2.9167$ $\quad \sin N = \dfrac{12}{37} \approx 0.3243$

$\cos N = \dfrac{35}{27} \approx 0.9459$ $\quad \tan N = \dfrac{12}{35} \approx 0.3429$

18. $\sin B = \dfrac{4\sqrt{2}}{9} \approx 0.6285$ $\quad \cos B = \dfrac{7}{9} \approx 0.7778$

$\tan B = \dfrac{4\sqrt{2}}{7} \approx 0.8081$ $\quad \sin A = \dfrac{7}{9} \approx 0.7778$

$\cos A = \dfrac{4\sqrt{2}}{9} \approx 0.6285$ $\quad \tan A = \dfrac{7}{4\sqrt{2}} \approx 1.2374$

9.6 Solving Right Triangles

19. $8^2 + x^2 = 12^2$
$x^2 = 80$
$x = 4\sqrt{5}$
≈ 8.9

$\cos X = \dfrac{8}{12}$
$m\angle X \approx 48.2°$

$\sin Z = \dfrac{8}{12}$
$m\angle Z \approx 41.8°$

20. $\sin 50° = \dfrac{d}{20}$
$d \approx 15.3$

$\cos 50° = \dfrac{f}{20}$
$f \approx 12.9$

$m\angle F = 90° - 50°$
$= 40°$

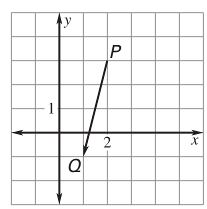
21. $\tan R = \dfrac{8}{15}$
$m\angle R \approx 28.1°$

$8^2 + 15^2 = s^2$
$289 = s^2$
$17 = s$

$\sin T = \dfrac{15}{17}$
$m\angle T \approx 61.9°$

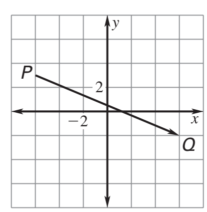
9.7 Vectors

22.



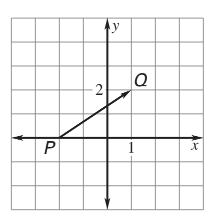
$\overrightarrow{PQ}: \langle -1, -4 \rangle$
$|\overrightarrow{PQ}| = \sqrt{1 + 16} = \sqrt{17} \approx 4.1$

23.



$\overrightarrow{PQ}: \langle 12, -5 \rangle$
$|\overrightarrow{PQ}| = \sqrt{144 + 25} = 13$

24.



$\overrightarrow{PQ}: \langle 3, 2 \rangle$
$|\overrightarrow{PQ}| = \sqrt{9 + 4} = \sqrt{13} \approx 3.6$

25. $\vec{u} + \vec{v} = \langle 14, 9 \rangle$

$|\vec{u} + \vec{v}| = \sqrt{196 + 81} = \sqrt{277} \approx 16.6$

$\tan x = \frac{9}{14}$

$x \approx 32.7°$ north of east

Chapter 9 Chapter Test (p. 585)

1. E **2.** A **3.** C **4.** D **5.** B

6. *DBA*; *DAC*

7. *WXYZ* is a kite. The diagonals are perpendicular and the quadrilateral has two pairs of consecutive congruent sides, but opposite sides are not congruent.

8. $PQ = \sqrt{25 + 4} = \sqrt{29}$

$QR = \sqrt{9 + 16} = 5$

$PR = \sqrt{4 + 36} = 2\sqrt{10}$

$(2\sqrt{10})^2 \underset{}{\overset{?}{\_\_}} (\sqrt{29})^2 + 5^2$

$40 < 54$

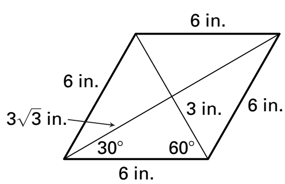
acute

9. $15^2 + b^2 = 113^2$

$b^2 = 12{,}544$

$b = 112$

10.



$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(6)(6\sqrt{3})$

$= 18\sqrt{3}$

$\approx 31.2 \text{ in.}^2$

side length = 6 in.

11. $\sin 30° = \frac{KL}{9}$ $\cos 30° = \frac{JL}{9}$

$KL = 4.5$ $JL \approx 7.8$

$m\angle K = 90° - 30° = 60°$

12. $\sin 25° = \frac{12}{DF}$ $\tan 25° = \frac{12}{DE}$

$DF \approx 28.4$ $DE \approx 25.7$

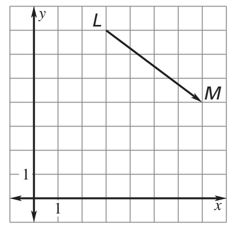
$m\angle F = 90° - 25° = 65°$

13. $4^2 + QR^2 = 6^2$ $\cos P = \frac{4}{6}$ $\sin R = \frac{4}{6}$

$QR = 2\sqrt{5}$ $m\angle P \approx 48.2°$ $m\angle R \approx 41.8°$

≈ 4.5

14.



$\overrightarrow{LM}: \langle 4, -3 \rangle$

$|\overrightarrow{LM}| = \sqrt{16 + 9} = 5$

$\tan x = \frac{3}{4}$

$x \approx 36.9°$ south of east

15. $\sin 40° = \frac{CD}{10}$ $\sin 50° = \frac{10}{AB}$

$CD \approx 6.4$ $AB \approx 13.1$

16. $m\angle BCA = 90° - 35° = 55°$

$\sin 35° = \frac{BC}{40}$ $\tan 35° = \frac{22.9}{DE}$

$BC \approx 22.9$ $DE \approx 32.7$

17. $\langle -2, -8 \rangle$ **18.** $\langle 4, 1 \rangle$ **19.** $\langle 2, 3 \rangle$

Chapter 9 Standardized Test (pp. 586–587)

1. $\frac{x + 1}{20} = \frac{5}{x + 1}$

$(x + 1)^2 = 100$

$x^2 + 2x - 99 = 0$

$(x + 11)(x - 9) = 0$

$x = -11$ $x = 9$ C

2. $A = 11(14) = 154 \text{ in.}^2$ C

3. $P = 2(\sqrt{125}) + 2(14) \approx 50.4$ in. D

4. B **5.** D

6. $x^2 + x^2 = 16^2$

$2x^2 = 256$

$x = \sqrt{128} = 8\sqrt{2}$ in. D

$P = 4(8\sqrt{2}) = 32\sqrt{2}$ in.

7. $\tan 67° = \frac{x}{8}$ $\cos 67° = \frac{8}{y}$

$x \approx 18.8$ $y \approx 20.5$

E

8. $\tan x = \frac{12}{9}$

$x \approx 53.1°$

A

9. $\sin A = \frac{8}{13}$

$m\angle A \approx 38.0°$

B

10. $-2 + x = 6$ $\quad$ $y + 4 = 11$

$x = 8$ $\quad$ $y = 7$

B

11. $|\overrightarrow{AB}| = \sqrt{(-8-1)^2 + (3+9)^2} = 15$ $\quad$ D

12. $24^2 + x^2 = 25^2$

$x = 7$

$p = 7 + 24 + 25 = 56$

13. $\sin x = \frac{7}{25}$ $\quad$ $\sin y = \frac{24}{25}$

$x \approx 16.3°$ $\quad$ $y \approx 73.7°$

14. $A = (25 \cdot 12) - \left(\frac{1}{2}(7)(24)\right) = 216$ square units

15. $BD = (10\sqrt{3})(\sqrt{3}) = 30$ $\quad$ $BC = 30\sqrt{2} \approx 42.4$

$FG = 15$ $\quad$ $GC = \sqrt{450} \approx 21.2$

$DC = 30$ $\quad$ $AF = 10\sqrt{3} + 15 \approx 32.3$

16. $m\angle ABC = 75°$; $m\angle FEA = 60°$; $m\angle BGF = 135°$

17. $\tan 30° \approx \frac{FE}{32.3}$ $\quad$ $\cos 30° \approx \frac{32.3}{AE}$

$FE \approx 18.6$ $\quad$ $AE \approx 37.3$

18. $A \approx \frac{1}{2}(30)(32.3 + 15) = 709.5$ square units

19. **a.** $\tan 30° = \frac{5}{b}$ $\quad$ **b.** $\tan 40° = \frac{5}{b}$

$b \approx 8.7$ cm $\quad$ $b \approx 6.0$ cm

c. $\tan 50° = \frac{5}{b}$ $\quad$ **d.** $\tan 60° = \frac{5}{b}$

$b \approx 4.2$ cm $\quad$ $b \approx 2.9$ cm

e. $\tan 70° = \frac{5}{b}$

$b \approx 1.8$ cm

20. As the sun rises, the value of b decreases.

21. $\tan x = \frac{5}{5.25}$

$x \approx 43.6°$

22. The value of the expression increases as the sun approaches the horizon.

Chapter 9 Cumulative Practice (pp. 588–589)

1. No; if two planes intersect, then their intersection is a line. The three points must be collinear, so they cannot be the vertices of a triangle.

2. always $\quad$ **3.** never $\quad$ **4.** always

5. $\overline{BD}$ is the median from point B, $\overline{AD} \cong \overline{CD}$, $\overline{BD} \cong \overline{BD}$, and it is given that $\overline{AB} \cong \overline{CB}$. Thus, $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Postulate. Also, $\angle ABD \cong \angle CBD$ since corresponding parts of congruent triangles are congruent. By the definition of an angle bisector, $\overline{BD}$ bisects $\angle ABC$.

6. *Sample answer:* Given quadrilateral $ABCD$ where $m\angle A = 37°$, $m\angle B = 143°$, and $m\angle C = 37°$. Since there are 360° in a quadrilateral, $m\angle D = 360° - (37° + 143° + 37°) = 143°$. $\angle A$ and $\angle C$ are opposite angles, and $\angle B$ and $\angle D$ are opposite angles. $\angle A \cong \angle C$ and $\angle B \cong \angle D$. Since both pairs of opposite angles are congruent, quadrilateral $ABCD$ is a parallelogram.

7. Yes; clockwise and counterclockwise rotational symmetry of 120°.

8. $26 + 4y = 90$ $\quad$ $26 + x + 10 = 90$

$4y = 64$ $\quad$ $x = 54$

$y = 16$

9. $3x - 5 + 5x = 7 = 180$ $\quad$ $y + 3x - 5 = 180$

$8x - 12 = 180$ $\quad$ $y + 3(24) - 5 = 180$

$8x = 192$ $\quad$ $y + 67 = 180$

$x = 24$ $\quad$ $y = 113$

10. $z = 55$; $x = 90$; $y = 65$

11. $m = \frac{6 + 2}{-5 - 1} = -\frac{8}{6} = -\frac{4}{3}$

slope of perpendicular bisector $= \frac{3}{4}$

midpoint: $\left(\frac{-5 + 1}{2}, \frac{6 - 2}{2}\right) = (-2, 2)$

$y - 2 = \frac{3}{4}(x + 2)$

$y - 2 = \frac{3}{4}x + \frac{6}{4}$

$y = \frac{3}{4}x + \frac{7}{2}$

12. $AB = \sqrt{16 + 9} = 5$

$AC = \sqrt{36 + 64} = 10$

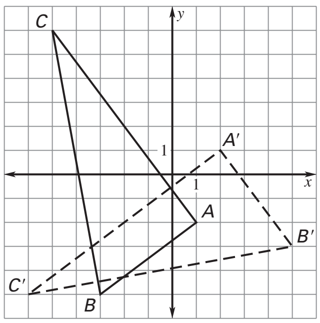
$BC = \sqrt{4 + 121}$

$= \sqrt{125} = 5\sqrt{5}$

scalene right triangle

13. $A'(-1, -2)$, $B'(3, -5)$, $C'(5, 6)$

14.



$A'(2, 1)$

$B'(5, -3)$

$C'(-6, -5)$

15. $A'(-3, 6), B'(-7, 9), C'(-9, -2)$

16. $\dfrac{12}{x} = \dfrac{5}{2}$

$24 = 5x$

$x = 4\frac{4}{5}$

17. $\dfrac{3}{7} = \dfrac{x}{8}$

$7x = 24$

$x = 3\frac{3}{7}$

18. $\dfrac{7}{9} = \dfrac{y}{y + 3}$

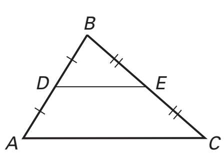
$9y = 7y + 21$

$2y = 21$

$y = 10\frac{1}{2}$

19. No; in *ABCD*, the ratio of the length to width is 8:6 or 4:3. In *APQD*, the ratio of length to width is 4:6, or 2:3. Since these ratios are not equal, the rectangles are not similar.

20.



$\overline{DE}$ is a midsegment of $\triangle ABC$. By the Midsegment Theorem, $\overline{DE} \parallel \overline{AC}$. Since the lines containing these segments are parallel, $\angle BDE \cong \angle BAC$ and $\angle BED \cong \angle BCA$, by the Corresponding Angles Postulate. Since two angles of $\triangle BDE$ are congruent to two angles of $\triangle BAC$, the two triangles are similar by the AA Similarity Postulate.

21. Yes; the ratios $\frac{6}{9}$, $\frac{8}{12}$, and $\frac{12}{18}$ all equal $\frac{2}{3}$, so the triangles are similar by the SSS Similarity Theorem.

22. $\dfrac{x}{9 - x} = \dfrac{7}{8}$

$8x = 63 - 7x$

$15x = 63$

$x = 4.2$

segment 1 = 4.2 cm

segment 2 = 4.8 cm

23. The image with scale factor $\frac{1}{3}$ has endpoints $\left(2, -\frac{4}{3}\right)$ and (4, 3); its slope is $\dfrac{\frac{13}{3}}{2} = \dfrac{13}{6}$. The image with scale factor $\frac{1}{2}$ has endpoints (3, −2) and (6, 4.5); its slope is $\dfrac{13}{6}$. The two image segments are parallel.

24. $\dfrac{ZP}{6} = \dfrac{2}{ZP}$

$ZP^2 = 12$

$ZP = 2\sqrt{3}$

$(2\sqrt{3})^2 + 2^2 = ZX^2$

$16 = ZX^2$

$4 = ZX$

25. $\dfrac{2\sqrt{3}}{XY} = \dfrac{3}{2\sqrt{3}}$

$12 = 3XY$

$4 = XY$

26. $10^2 + 24^2 = XY^2$

$676 = XY^2$

$26 = XY$

27. $19^2 \underline{\ ?\ } 15^2 + 12^2$

$361 < 369$

acute

28. $\dfrac{8\sqrt{2}}{4\sqrt{3}} = \dfrac{2\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{2\sqrt{6}}{3}$

29. Let x = the measure of the smaller acute angle.

$\sin x = \dfrac{8}{17}$ $\qquad \cos x = \dfrac{15}{17}$

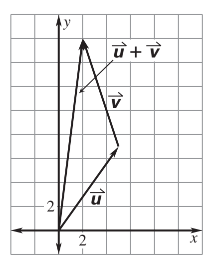
$\tan x = \dfrac{8}{15}$

30. $\sin 57° = \dfrac{ST}{20}$ $\qquad \cos 57° = \dfrac{TR}{20}$

$ST \approx 16.8$ in. $\qquad TR \approx 10.9$ in.

$m\angle S = 90° - 57° = 33°$

31.



$\vec{u} + \vec{v} = \langle 2, 16 \rangle$

$|\vec{u} + \vec{v}| = \sqrt{4 + 256} \approx 16.1$

$\tan x = \dfrac{2}{16}$

$x \approx 7.1°$ south of east

32. Construct a circle inscribed in the triangle by bisecting two angles of the triangle. The point at which the bisectors intersect is the center of the circle. Construct a segment from the center of the circle perpendicular to a side of the triangle. The length of this segment is the radius of the desired circle.

33. $\dfrac{376}{16} = \dfrac{470}{x}$

$376x = 7520$

$x = 20$ gallons

34. $P = 3 + 3 + 5 + 5 = 16$

$\dfrac{16}{36} = \dfrac{3}{w}$

$16w = 108$

$w = 6.75$ in.

$36 = 13.5 + 2\ell$

$\ell = 11.25$ in.

35. $d^2 = 127.5^2 + 140^2$

$d^2 = 35{,}856.25$ mi

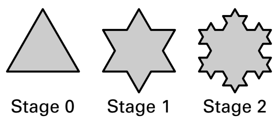
$d \approx 189.4$ mi

Project: Investigating Fractals (pp. 590–591)

Investigation

1. stage 1: $\frac{4}{3}$; stage 2: $\frac{16}{9}$
2. stage 3: $\frac{64}{27}$; stage 4: $\frac{256}{81}$; the length at stage 1 is $\frac{4}{3}$ times the length at stage 0, and similarly the length at stage 2 is $\frac{4}{3}$ times the length at stage 1.
3. At each stage the length would be $\frac{4}{3}$ times the previous stage, so the length gets increasingly large.
4. Yes; the graph is a curve that increases sharply as n, the number of stages, increases

Stages of a Koch Snowflake

5. 

6.

Stage, *n*	0	1	2	3	4
Perimeter, *P*	3	4	$\frac{16}{3}$	$\frac{64}{9}$	$\frac{256}{27}$

7. $P = 3\left(\frac{4}{3}\right)^n$